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The Sagnac effect explained

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It is remarkable that almost a century after the discovery of the Sagnac effect no justification based on the two relativistic theories has been found. Hasselbach and Nicklaus, describing their own experiment, list about twenty different explanations of the effect and comment: "This great variety (if not disparity) in the derivation of the Sagnac phase shift constitutes one of the several controversies ... that have been surrounding the Sagnac effect since the earliest days of studying interferences in rotating frames of reference."

In the Sagnac 1913 experiment a platform was made to rotate uniformly around a vertical axis at a rate of 1-2 full rotations per second. In an interferometer mounted on the platform, two interfering light beams, reflected by four mirrors, propagated in opposite directions along a closed horizontal circuit defining a certain area. The rotating system included also the luminous source and a photographic plate recording the interference fringes.

Sagnac observed a shift of the interference fringes every time the rotation was modified. This is strictly tied to the relative time delay with which the two light beams reach the detector. Considering his experiment conceptually similar to the Michelson-Morley one, he informed the scientific community with two papers (in French) bearing the titles "*The existence of the luminiferous ether demonstrated by means of the effect of a relative ether wind in an uniformly rotating interferometer*" and "*On the proof of reality of the luminiferous ether with the experiment of the rotating interferometer*"

My assumptions:

(**i**) Relative to the system S_0 the velocity of light is "c" in all directions, so that clocks can be synchronized in S_0 with the Einstein method and the one way velocities relative to S_0 can be measured;

(**ii**) Space is homogeneous and isotropic and time homogeneous, at least from the point of view of observers at rest in S_0;

(**iii**) The origin of S, observed from S_0, moves according to the eq. $x_0 = v\,t_0$;

(**iv**) The axes of S and S_0 coincide for $t = t_0 = 0$.

To (**i**) - (**iv**) we can add two points based on solid empirical evidence:

(**v**) The two way velocity of light is the same in all directions and in all inertial systems: $c_2(\theta) = c$.

(**vi**) Clock retardation takes place with the usual velocity dependent square root factor when clocks move with respect to S_0. We have eliminated ambiguities by saying that R in the formula $\tau = \tau_0 / R$ has to be calculated relatively to S_0.

These six conditions were shown to reduce the transformations of the space and time variables from S_0 to S to the form

$$\begin{cases} & x = (x_0 - v\,t_0)/R & \\ y = y_0 & ; & z = z_0 \\ & t = Rt_0 + e_1(x_0 - v\,t_0) & \end{cases} \tag{1}$$

The physically free parameter e_1 can be fixed conventionally by defining in S the simultaneity of distant events, or, which is the same, by choosing a clock synchronization method in S. The velocity of light consequence of (1) is:

$$c_1(\theta) = \frac{c}{1 + \Gamma\cos\theta} \tag{2}$$

with

$$\Gamma = \frac{v}{c} + c e_1 R \tag{3}$$

The transformations (1) represent the set of theories "equivalent" to the TSR: if e_1 is varied, different elements of the set are obtained. They should be equivalent for the explanation of experimental results. The Lorentz transformations are recovered as a particular case with $e_1 = -v / c^2 R$, whence $\Gamma = 0$ and $c_1(\theta) = c$. Different values of e_1 are obtained from different synchronisation conventions.

In order to simplify calculations, we consider now a monochromatic light source placed on the disk emitting two coherent beams of light in opposite directions. These travel along a circumference concentrical with the disk, until they reunite in a point *A* and interfere, after a propagation. The circular path can be obtained by forcing the light to propagate tangentially to the internal surface of a cylindrical mirror. The positioning of the interference figure depends on the disk rotational velocity ("Sagnac effect"). Most textbooks deduce the Sagnac formula (our Eq. (8) below) in the laboratory, but say nothing about the description of the phenomenon given by an observer placed on the rotating platform: we will see that SRT predicts a null effect on the platform, while an approach based on the inertial transformations gives the right answer. For simplicity we will assume that the laboratory is at rest in the privileged frame.

Sagnac effect seen from the laboratory. Light propagating in the rotational direction of the disk must cover a distance larger than the disk circumference length L_0 by a quantity $\ell_1 = v\,t_{01}$ equaling the shift of *A* during the time taken by light to reach the interference region. Therefore

$$L_0 + \ell_1 = c\,t_{01} \qquad ; \qquad \ell_1 = v t_{01} \tag{4}$$

From these equations it is easy to get:

$$t_{01} = \frac{L_0}{c - v} \tag{5}$$

Light propagating in the direction opposite to that of rotation must instead cover a distance smaller than L_0 by a quantity $\ell_2 = v\, t_{02}$ equaling the shift of A during the time taken by light to reach the interference region. Therefore

$$L - \ell_2 = c\, t_{02} \qquad ; \qquad \ell_2 = v\, t_{02} \tag{6}$$

One now gets

$$t_{02} = \frac{L_0}{c+v} \tag{7}$$

The time difference Δt_0 between the two propagations is the parameter fixing the phase difference in the considered interference point. From (5) and (7) it follows

$$\Delta t_0 = t_{01} - t_{02} = \frac{2L_0}{c^2} \frac{v}{1 - v^2/c^2} = \frac{2L}{c^2} \frac{v}{R} \tag{8}$$

Obviously $L_0 = LR$ is the disk circumference length reduced in the laboratory by the usual relativistic factor, if L is the rest length of the same disk. The consistency of Eq. (5) with experimental data has been checked in many experiments.

Sagnac effect seen from the disk. As a preliminary to the solution of the problem on the disk consider a clock marking the time t fixed in the origin of the moving inertial system S. Seen from S_0 it therefore satisfies the equation $x_0 = v\, t_0$. Substituting this equation into the equivalent transformations (1) we get $x = 0$ (the fixed coordinate of the clock in S) and $t = R\, t_0$. Therefore, all the ETs lead to the same relationship between the times $t,\ t_0$. For time intervals we write

$$\Delta t = R\, \Delta t_0 \tag{9}$$

Eq. (6) will be assumed to hold also for a clock on the rim of a disk rotating with speed v. This is consistent with our general philosophy that every small portion of the circumference of the rotating platform can be considered instantaneously at rest in a moving inertial frame of reference locally "tangent" to the disk. Therefore Eq. (2) applies for the velocity of light on the disk. Only the cases of light moving parallel and antiparallel to the local absolute velocity must be considered. It follows that the inverse velocity of light for these two cases is respectively given by:

$$\frac{1}{\tilde{c}(0)} = \frac{1+\Gamma}{c} \qquad ; \qquad \frac{1}{\tilde{c}(\pi)} = \frac{1-\Gamma}{c} \tag{10}$$

with Γ given by (3). The time difference on the disk is given by

$$\Delta t = t_1 - t_2 = \frac{L}{\tilde{c}(0)} - \frac{L}{\tilde{c}(\pi)} = \frac{2L\Gamma}{c} \tag{11}$$

Substituting (8) in the right hand side of (11) we get

(12)

where is the usual square root factor describing the dilation of time intervals in a moving frame. Only the <u>inertial transformations</u>, corresponding to $e_1 = 0$ allow us to get (9) from (12). For all other values of e_1 one will get wrong results from (9). In particular, the TSR with its $e_1 = -v/c^2R$ predicts $\Delta t = 0$.

We have reached the conclusion that of all theories having different values of only one ( ) gives a rational description of the Sagnac effect on the rotating platform. In the case of the calculated time difference on the platform disagrees with the prediction (8) in the laboratory, prediction which is of course the same for all theories satisfying the equivalent transformations (SRT included), since in the laboratory (assumed to be at rest in the privileged frame) Einstein's synchronization was used.