Maxwell's Maxima

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This paper looks in detail at the situation that develops with Maxwell's coupled field equations when the initial condition constitutes a pair of field pulses, in **E** and **B**, with finite total energy, such as would be needed to plausibly model a light signal for SRT, or a photon for QM. What emerges from the analysis is that, during propagation, the initial pulses always tend to spread longitudinally into complex waveforms exhibiting oscillation. So 'light in flight' is never a simple pair of pulses. It is a pair of spread-out waveforms, with maxima in the middle and long oscillating tails fore and aft. The waveform centroid may be said to travel at light speed c, but that fact alone does not at all adequately characterize light signals for SRT, or photons for QM.

1. Introduction

Despite centuries of investigation, light propagation remains to this day a mysterious business. Many of us in the NPA have long believed that Einstein's Second Postulate for Special Relativity Theory (SRT), asserting constant light speed c, is not right, and have sought to identify what about it is wrong, and what remedy should be applied. This author has talked and written about her own struggles with this problem many times in NPA meetings and publications. The present paper continues in this vein, with a potentially devastating problem finally uncovered.

Einstein's Second Postulate [1,2] is generally believed to capture the essence of Maxwell's electromagnetic theory (EMT) as it pertains to light. This paper argues that this belief is entirely wrong. Einstein's Second Postulate is in fact a violation of Maxwell's coupled field equations.

The paper looks in detail at the situation that develops with Maxwell's differential equations when the initial condition constitutes a pair of field pulses, in $\mathbf{E}$ and $\mathbf{B}$, with finite total energy. Such a pair of energy-limited pulses would be needed to plausibly model a light signal for SRT, or a photon for QM.

What emerges from this analysis is that, during propagation, the initial pulses always tend to spread longitudinally into complex waveforms exhibiting oscillation. So 'light in flight' is never a simple pair of pulses. It is a pair of spread-out waveforms, with maxima in the middle and long oscillating tails fore and aft.

The waveform centroid may be said to travel at light speed c, but that fact alone does not at all adequately characterize light signals for SRT, or photons for QM. Thus both SRT and QM are founded on an inaccurate appraisal of Maxwell, and for that reason they very well deserve to be revisited.

2. Maxwell's Propagation Process

This Section looks in detail at the situation that develops with Maxwell's differential equations for fields when the initial condition constitutes E and B field pulses with finite total energy.

Both electromagnetic signaling and quantum emission of light are usually rather laser-like, with multiple photons coming from a system of multiple atoms and traveling in a fairly well defined direction. So consider propagation along direction x, fore or aft across this page. Use the coordinate frame indicated by the drawings

$$\begin{array}{l} y \\ \uparrow \\ \odot\rightarrow\ \ x \quad (z \text{ out of the page}) \ , \\ z \end{array}$$

and, in another view,

$$\begin{array}{l} z \\ \uparrow \\ \otimes\rightarrow\ \ x \quad (y \text{ into the page}) \ . \\ y \end{array}$$

Let the Poynting vector $\mathbf{P} = \mathbf{E}\times\mathbf{B}$ of any radiation be nominally along the x direction. Let the electric field $\mathbf{E}$ be in the y direction, and let the magnetic field be $\mathbf{B}$ along the z direction.

It is generally expected that launching pulses in both $\mathbf{E}$ and $\mathbf{B}$ together results in travel of both pulses. The mechanism for this has to reside to Maxwell's equations. Maxwell's equations in free space, in differential form, in modern notation, and in Gaussian units, are [3]:

$$\begin{array}{ll} \nabla\bullet\mathbf{E} = 0 \ , & \nabla\times\mathbf{E} + \frac{1}{c}\partial\mathbf{B}/\partial t = 0 \ , \\ \nabla\bullet\mathbf{B} = 0 \ , & \nabla\times\mathbf{B} - c\partial\mathbf{E}/\partial t = 0 \ . \end{array} \quad (2.1)$$

Observe that $\mathbf{E}$ and $\mathbf{B}$ couple through Maxwell's Equations: a curl in one field creates a time derivative in the other field:

$$\partial\mathbf{E}/\partial t = \frac{1}{c}\nabla\times\mathbf{B} \ , \ \partial\mathbf{B}/\partial t = -c\nabla\times\mathbf{E} \quad (2.2)$$

This kind of coupling situation is familiar in engineering system design. In the language of that discipline, we have a classic 'feedback loop' in which a six-dimensional 'state vector' $\mathbf{E}(t), \mathbf{B}(t)$ is operated upon by a 'system function' involving differential and integral operators to generate an update that modifies the state vector. Figure 2.1 shows this in the way engineers would likely display it.

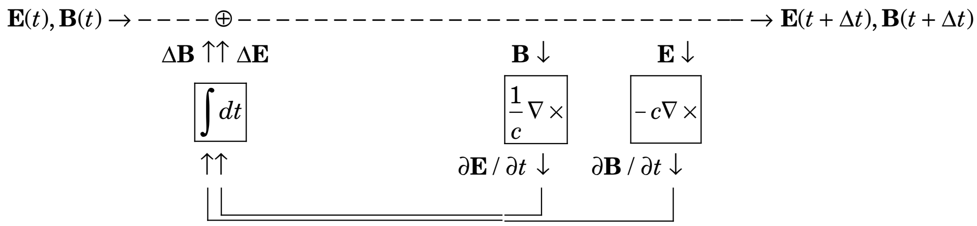

Figure 2.1. Maxwell's coupled field equations as a classic feedback loop.

The feedback and coupling between $\mathbf{E}$ and $\mathbf{B}$ can be shown to cause travel of an input pair of pulses $\mathbf{E}(0), \mathbf{B}(0)$. Consider what happens to the $\mathbf{E}$ pulse because of the feedback and coupling. Given the definition $\nabla = \hat{\mathbf{x}}\partial/\partial x + \hat{\mathbf{y}}\partial/\partial y + \hat{\mathbf{z}}\partial/\partial z$, we have

$$\begin{aligned}\nabla\times\mathbf{B} = \hat{\mathbf{x}}&\left[\partial B_z/\partial y - \partial B_y/\partial z\right]\\ &+\hat{\mathbf{y}}\left[\partial B_x/\partial z - \partial B_z/\partial x\right]\\ &\quad+\hat{\mathbf{z}}\left[\partial B_y/\partial x - \partial B_x/\partial y\right]\end{aligned}$$

and given $B_x = B_y = 0$, we have

$$\nabla\times\mathbf{B} = \hat{\mathbf{x}}\left[\partial B_z/\partial y\right] + \hat{\mathbf{y}}\left[-\partial B_z/\partial x\right] \tag{2.3}$$

So $\partial E_y/\partial t = \dfrac{1}{c}\left[-\partial B_z/\partial x\right]$. Therefore the E_y pulse grows on the leading side, and declines on the trailing side. (Note that if B_z is limited in y, as it must be, there also exists $\partial E_x/\partial t$. I will return to this point.)

Similarly, consider what happens to the $\mathbf{B}$ pulse because of the feedback and coupling. We have

$$\begin{aligned}\nabla\times\mathbf{E} = \hat{\mathbf{x}}&\left[\partial E_z/\partial y - \partial E_y/\partial z\right]\\ &+\hat{\mathbf{y}}\left[\partial E_x/\partial z - \partial E_z/\partial x\right]\\ &\quad+\hat{\mathbf{z}}\left[\partial E_y/\partial x - \partial E_x/\partial y\right]\end{aligned}$$

and given $E_x = E_z = 0$, we have

$$\nabla\times\mathbf{E} = \hat{\mathbf{x}}\left[-\partial E_y/\partial z\right] + \hat{\mathbf{z}}\left[\partial E_y/\partial x\right] \tag{2.4}$$

So $\partial B_z/\partial t = -c\left[-\partial E_y/\partial x\right] = c\,\partial E_y/\partial x$. Therefore the B_z pulse grows on the leading side, and declines on the trailing side. (Note that if E_y is limited in z, as it must be, there also exists $\partial B_x/\partial t$. I will return to this point.)

What we have so far is the first essential part of the propagation story: Any pulse has sides, and the existence of the sides means there is curl in that field. A curl in one field causes a time derivative in the other field. The time derivatives make the backsides of the pulses shrink and the front sides of pulses grow. This amounts to overall displacement along x, or travel. The speed of the travel is the only speed there is in Maxwell EMT: the 'speed of light', c.

But there is also an interesting little wrinkle: a bit of E_x and a bit of B_x has developed. Such 'longitudinal fields' (fields pointing in the nominal propagation direction x) constitute a much-discussed and controversial subject. [4] The subject should never have been so controversial. There exists a very familiar phenomenon that mandates the presence of longitudinal field components: diffraction. It is well known that light emanating from a finite aperture cannot be focused to a point; it is limited to a finite Airy spot, with rings around it. Light arriving to slightly off-axis locations implies propagation in directions slightly off the nominal direction, x, and that in turn implies Poynting vectors pointing slightly off the nominal propagation direction x, and that in turn implies components of $\mathbf{E}$ and/or $\mathbf{B}$ *in* the nominal propagation direction, *i.e.* E_x and/or B_x.

Diffraction amounts to waveform spreading in directions transverse to the nominal propagation direction x, arising from initial waveform limitation in the y and z directions transverse to x. Diffraction is a part of the story of waveform spreading that is already very well known. Its ubiquity invites one to ask the so-far neglected complementary question: What about possible waveform spreading *along* the nominal propagation direction x, occasioned by initial limitation along x, which is necessary to define a light signal pulse or a quantum photon with finite total energy? That kind of waveform spreading is the second, and previously neglected, part of the propagation story, and it is to be developed in the following Sections.

3. Waveform Development – Pictorial Model

Signal travel or photon travel is often at first imagined to proceed simply as in Fig. 3.1:

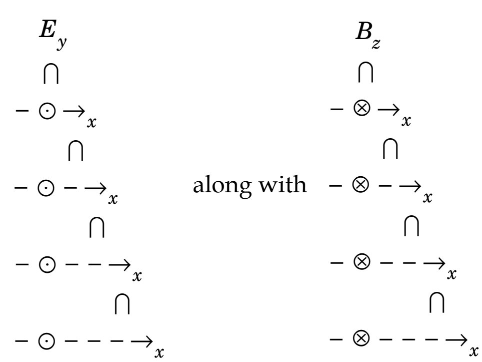

Figure 3.1. Naïve idea of signal or photon travel.

But that is not quite possible, because $\mathbf{B}$ is a solenoidal field. So the initial B_z has to be at least a doublet of the form $\overset{\cap}{\underset{\cup}{-\otimes\rightarrow}}$. And then to match that, the initial E_y has to be a doublet of the form $\overset{\cap}{\underset{\cup}{-\odot\rightarrow}}$. But to make both E_y and B_z symmetric about the origin, like the impossible singlet pair imagined initially, let E_y be a triplet of the form $\overset{\cap}{\underset{\cup\ \cup}{-\odot\rightarrow_x}}$ and let B_z be a triplet of the form $\overset{\cap}{\underset{\cup\ \cup}{-\otimes\rightarrow_x}}$. Starting from these initial E_y and B_z triplets, the subsequent process must go as follows: The triplet E_y and B_z must induce quadruplet $\partial B_z / \partial t$ and $\partial E_y / \partial t$, leading to quadruplet field increments ΔB_z and ΔE_y. And so on from there. That is, the induction process works to increase the number of peaks. Every quarter cycle adds another peak. What was originally an input pulse starts to look like a wave train.

In addition to the phenomenon of increasing pulse count, there is also a phenomenon of general spreading. Observe that E_y and B_z are in phase, as are ΔE_y and ΔB_z, so the matched vector cross products $\mathbf{E}_y \times \mathbf{B}_z$ and $\Delta\mathbf{E}_y \times \Delta\mathbf{B}_z$ both point in the positive x direction, together make a steady of Poynting vector, which one can consider a standard unit for comparison. But the $\mathbf{E}_y$ and $\Delta\mathbf{B}_z$ are a quarter cycle out of phase, as are the $\Delta\mathbf{E}_y$ and $\mathbf{B}_z$, so the mixed cross products $\mathbf{E}_y \times \Delta\mathbf{B}_z$ and $\Delta\mathbf{E}_y \times \mathbf{B}_z$ are each oscillatory in time, like $\frac{1}{2}\cos(\omega t)\sin(\omega t) = \frac{1}{4}\sin(2\omega t)$, and over a quarter cycle each mixed cross product has root mean square magnitude of $\frac{1}{8}$ unit of Poynting vector. And the mixed cross products occur both fore and aft on the initial waveform, and they point forward on the fore side and back on the aft side; *i.e.* both point *away* from the waveform centroid. So they provide an on-going mechanism for waveform spreading. Over a quarter cycle, $\mathbf{E}_y \times \Delta\mathbf{B}_z$ and $\Delta\mathbf{E}_y \times \mathbf{B}_z$ will spread the initial waveform by $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ pulse width, and over a full cycle, they will spread the initial waveform by a full pulse width.

To better focus on the waveform-spreading phenomenon alone, without the complication of waveform travel, let us consider a single pulse in E_y alone. The initial E_y singlet $\overset{\cap}{-\odot\rightarrow_x}$ induces $\partial B_z / \partial t$ of the doublet form $\overset{\cap}{\underset{\cup}{-\otimes\rightarrow_x}}$, leading to B_z of that same doublet form a quarter cycle later, which then induces $\partial E_y / \partial t$ of the triplet form $\overset{\cap\ \cap}{\underset{\cup}{-\odot\rightarrow_x}}$, leading to E_y of that same triplet form a quarter cycle later, and so on. The E_y waveform starts and remains symmetric about the x origin. The B_z waveform emerges anti-symmetric about the x origin and remains so. More and more peaks emerge, but the energy centroid of the waveform goes nowhere.

Similarly, if we start with a triplet pulse in B_z alone, then the B_z waveform starts and remains symmetric about the x origin, and the E_y waveform emerges anti-symmetric about the x origin and remains so. More and more peaks emerge, but the energy centroid of the waveform goes nowhere.

With only a single field waveform launched, waveform spreading happens, but waveform travel does not happen. Waveform travel can be avoided by launching just one field pulse alone, not both together. But waveform spreading cannot be avoided, no matter what. So the conventional idea of a light signal pulse or a quantum photon as a 'bullet' is not viable with Maxwell's coupled field equations. It becomes viable only when the coupled field equations are reduced to wave equations, by substituting one field equation into another. That procedure obscures the coupling mathematically, and thereby *seems* to allow a solution with behavior that is not in fact physically possible; namely, the hypothetical 'bullet' that does not spread longitudinally.

4. Waveform Regression

Maxwell's equations generally admit conjugate pairs of solutions, in one of which the temporal evolution goes one way (think of it as $+t$), and in the other of which the temporal evolution goes the opposite way (think of it as $-t$, but not really). Thus for each of the waveform-developing solutions mentioned above, there exists a conjugate solution that exhibits waveform regression; *i.e.* contraction back to a pulse.

The idea of 'contracting' solutions is somewhat reminiscent of 'advanced' solutions going backwards in time, which were introduced many times in the early 20th century, but particularly popularized in the mid 20th century by Wheeler and Feynman [5,6], who were looking to time symmetry as the basis for an electromagnetic generalization of instantaneous (Newtonian) gravitational interaction. There are important differences between the contracting waveforms introduced above and such advanced solutions: **1)** Wheeler and Feynman were looking at interactions between essentially point sources and receivers, and so had to be looking at spherically expanding retarded solutions and spherically contracting advanced solutions, not at essentially one-dimensional expanding and contracting beams. **2)** The Wheeler-Feynman expansion or contraction is related to the spherical area of a wave front, not the waveform in the radial propagation direction. **3)** A lengthy discussion of the paradox of advanced actions is necessitated in the Wheeler-Feynman work, whereas the 'contracting' solutions introduced here are not in fact 'advanced' at all; they are just contracting, in real time, in the longitudinal direction.

The main similarity between the present work and the Wheeler-Feynman work is this: both involve two parts to make a full problem description. The Wheeler-Feynman work used retarded solutions and advanced solutions simultaneously to describe inter-particle interaction. The present work uses an expanding solution and a contracting solution in sequence to describe launching and delivery of an electromagnetic signal pulse or a quantum photon.

One big problem about advanced solutions in the Wheeler-Feynman context of spherical waves is the establishment of initial conditions. It seems impossible to imagine how initial conditions could occur correctly everywhere all over a big sphere. This problem disappears in the present context of a transversely limited laser-like light signal pulse or quantum photon. The expanding solution automatically sets up the initial conditions for the subsequent contracting solution: the field spatial profiles of the expanding solution are the same as the field spatial profiles of the contracting solution.

5. Two Step Light

At the beginning of the 20th century, attention was focused on the solutions to the wave equations derived from Maxwell's coupled field equations, and on their single parameter, light speed c. Now in the 21st century, it is appropriate to exploit to the solutions provided directly by Maxwell's original coupled field equations. The representation of the delivery of an electromagnetic signal pulse as the sequence of an expanding solution followed by a contracting solution has been called 'Two Step Light' (TSL) [7].

The moment of switch from the solution expanding from the source to the solution contracting to the receiver involves at most two changes, each of which is totally non-traumatic, as follows.

The first thing only *possibly* changes. It is the frequency of oscillations. The frequency changes if, and only if, the receiver is moving relative to the source. The frequency change occurs because the reference for light speed c changes from the source to the receiver. If the receiver is moving at speed V relative to the source, then the light speed 'c relative to the source' was '$c - V$ relative to the receiver'. Upon chance of reference, that situation changes to 'c relative to the receiver', and '$c + V$ relative to the source'.

The second thing *definitely* changes. It is the sign of all time derivatives. Without fail, time derivatives change at the moment of switch, because this switch is fundamentally what it takes to change from a solution that is expanding to one that is contracting. It is worth noting here that there is no 'mechanism' that causes this switch. The switch occurs because we set the boundary conditions for the math to fulfill: "start at this particular source, and end up at this particular receiver, and do it by combining solutions of the Maxwell differential equations." If exactly the needed switch did not occur, then the particular solution produced would fit some other set of boundary conditions instead. The subject of boundary conditions is actually a subtle one, rather like the subject of quantum entanglement. Boundary conditions often extend over space, and, though they may move over time, they are eternal in time. What does that mean in the point-particle, elastic-time world of SRT? I shall just leave that question hanging for interested readers to take up.

Note that the discontinuity in time derivatives needed here is feasible for the original Maxwell coupled field equations, but not for the wave equations derived from them. That is because Maxwell's coupled field equations are first order in derivatives, and so require only field continuity, but the wave equations are second order in derivatives, and so require not only field continuity, but also continuity of the first derivatives of the fields. That requirement is violated when the time derivatives switch sign in making the switch from a solution that is expanding to one that is contracting.

Again we see that the wave equations have a set of possible solutions that is not identical to the set of possible solutions for Maxwell's original coupled field equations. That is why it is so important to investigate the solutions for the original coupled field equations in order to model light signal pulses and quantum photons.

In the Two Step Light formulation, the speed of light throughout the first step, the expansion step, is c relative to the source. In Einstein's SRT [1,2], the speed of light is c for any observer, and any observer has his own reference frame, so the speed of light is c in any reference frame, and that includes the frame of a hypothetical observer resident with the source. But the actual observer is the receiver. So the first step in Two Step Light is not clearly compatible with Einstein. It is more compatible with Ritz [8], but only so long as the motion of the source is inertial. If the motion of the source is *not* inertial (usually the case), then the first step in Two Step Light is more compatible with Moon and Spencer *et al*. [9-11]. Their vitally important idea was on-going connection between the light and its source, even *after* the propagation process starts. The present work adds to the Moon-Spencer idea a second step, wherein there is on-going connection between light and its receiver, even *before* the propagation process concludes. The speed of light throughout the second step, the contraction step, is c relative to the receiver. This idea resembles Einstein's idea, but it doesn't just *follow* Einstein; it goes *further*, allowing even non-inertial motion of the receiver (certainly the case for all human observers!).

Ref. [7] began by postulating Two Step Light, as an alternative to Einstein's Second Postulate from [1,2]. The present work invites the reader to omit the use of a postulate there altogether, establishing that Maxwell's coupled differential equations for fields determine that Two Step Light has to happen. The whole strategy is just this: always follow where the energy goes.

6. Pascal's Triangle and Energy Redistribution

The preceding Section featured some sketches of waveforms spreading out over time, like $\overset{\cap}{-\odot\rightarrow_x}$, and $\underset{\cup}{\overset{\cap}{-\otimes\rightarrow}}$, and $\underset{\cup}{\overset{\cap\ \ \cap}{-\odot\rightarrow_x}}$, and the reader can well imagine others with more peaks, but the author finds them too difficult to draw in this way. It is necessary to move on from sketches to numerical models.

We begin, not with the continuous waveforms themselves, but with the energy contents of their peaks. That is, we think about $E^2/2$ and $B^2/2$. When an E_y single pulse generates a B_z double pulse, with the E_y pulse thereby disappearing, it has to be at least approximately true that the dying E_y pulse casts half its energy to each of its neighboring two emerging B_z pulses. This 'bequeath equally to both thy neighbors below' energy-redistribution algorithm is related to the arithmetic algorithm that generates Pascal's famous triangle of binomial coefficients, $n!\big/\left[(n-j)!\,j!\right]$, $j = 0,1,\ldots n$, which arise in the expansion of 2^n as $(1+1)^n$. The Pascal algorithm is 'sum both neighbors above'. The energy redistribution algorithm differs only in that all the rows of Pascal's triangle have to be normalized to sum to unity. Thus we have Fig. 6.1:

Pascal's Triangle in its Original Form:

$$
\begin{array}{ccccccccccccccccc}
&&&&&&&&1&&&&&&&& \\
&&&&&&&1&&1&&&&&&& \\
&&&&&&1&&2&&1&&&&&& \\
&&&&&1&&3&&3&&1&&&&& \\
&&&&1&&4&&6&&4&&1&&&& \\
&&&1&&5&&10&&10&&5&&1&&& \\
&&1&&6&&15&&20&&15&&6&&1&& \\
&1&&7&&21&&35&&35&&21&&7&&1& \\
1&&8&&28&&56&&70&&56&&28&&8&&1
\end{array}
$$

etc.

Pascal's Triangle Normalized:

$$
\begin{array}{ccccccccccccc}
&&&&&&1&&&&&& \\
&&&&&\frac{1}{2}&&\frac{1}{2}&&&&& \\
&&&&\frac{1}{4}&&\frac{2}{4}&&\frac{1}{4}&&&& \\
&&&\frac{1}{8}&&\frac{3}{8}&&\frac{3}{8}&&\frac{1}{8}&&& \\
&&\frac{1}{16}&&\frac{4}{16}&&\frac{6}{16}&&\frac{4}{16}&&\frac{1}{16}&& \\
&\frac{1}{32}&&\frac{5}{32}&&\frac{10}{32}&&\frac{10}{32}&&\frac{5}{32}&&\frac{1}{32}& \\
\frac{1}{64}&&\frac{6}{64}&&\frac{15}{64}&&\frac{20}{64}&&\frac{15}{64}&&\frac{6}{64}&&\frac{1}{64}
\end{array}
$$

etc., etc.

Pascal's Triangle Normalized and Decimalized:

$$
\begin{array}{ccccccccc}
&&&&1&&&& \\
&&&.5&&.5&&& \\
&&.25&&.5&&.25&& \\
&.125&&.375&&.375&&.125& \\
.0625&&.25&&.375&&.25&&.0625
\end{array}
$$

etc., etc., etc.

Figure 6.1. Pascal's triangle approaching a Gaussian distribution.

Number strings, like 1, and $.5 \;\; .5$, and $.25 \;\; .5 \;\; .25$, represent waveform peak energy contents. Waveform amplitude sketches, like $-\overset{\cap}{\odot}\rightarrow_x$, and $\underset{\cup}{-}\overset{\cap}{\otimes}\rightarrow_x$, and $\overset{\cap}{-}\underset{\cup}{\odot}\overset{\cap}{\rightarrow}_x$, can be matched in our minds with number strings like 1, and $-\sqrt{.5}\;\sqrt{.5}$, and $\sqrt{.25}\;\; -\sqrt{.5}\;\; \sqrt{.25}$. Longer waveform amplitudes sketches, too difficult to draw, can be replaced in our minds with longer number strings.

The deeper one goes into the normalized, decimalized Pascal's triangle, the more the string of numbers in a row resembles a Gaussian curve. For the above case of $n = 4$, let the 5 numbers present be identified by an index $i = \; -2,-1,0,+1,+2$. The best-fit Gaussian is $\exp(-i^2/2)\big/\sqrt{2\pi}$, which yields point values:

Gaussian point values

.0540	.2420	.3989	.2420	.0540

that approximate Pascal numbers

.0625	.2500	.3750	.2500	.0625

Written with a continuous variable x, the fitting function $\exp(-x^2/2)\big/\sqrt{2\pi}$, constitutes a Gaussian $\exp(-x^2/2\sigma^2)\big/\sqrt{2\pi}\,\sigma$ with standard deviation $\sigma = \sqrt{2/2} = 1$. This result suggests the more general model $\exp\left[-x^2/(n/2)\right]\big/\sqrt{(n/2)\pi}$ where n is the power of 2 for which the normalized binomial coefficients from Pascal are being fit. The standard deviation of the general fitting Gaussian is $\sigma = \sqrt{n/4}$.

Knowing full well that this model works better and better the further one goes with it, I shall leave the reader to play with it for larger n.

The parameter $n/4$ will recur later in the present work.

7. Input Pulse - Math Models

The previous Section developed a rough discrete numerical model for energy content of waveform peaks. That exercise prepares and informs one about what to expect from a more detailed continuous model to cover both waveform amplitude profile and energy density profile.

We shall assume just one pulse input, to suppress the travel aspect of waveform emergence and focus on the waveform-spreading aspect. There is an initial choice to make: What shall be the shape of the input pulse? Reasonable candidates include a Lorenzian like $1\big/(1+x^2)$, a hyperbolic function like $1\big/\cosh(x)$, or a Gaussian like $\exp(-x^2)$, and no doubt many others the reader may think of. But so far, we have seen Gaussian units in Maxwell's equations and Gaussian waveforms from Pascal's spreading algorithm. A definite theme is developing here: 'Gaussian everything'. So let us suppose that the shape of an initiating pulse is a Gaussian function e^{-x^2}. Detailed numerical results may depend on this choice, but general character should not. As with the strings of Pascal numbers in the last Section, a pretty good Gaussian fit can be found for most 'bump-like' functions. So the study of just the Gaussian function is of value more generally.

8. Waveform Development – Math Model

The continuous model for the waveform as it emerges is developed next. That means we shall in effect carry out the repetitive calculation process indicated by Fig. 2.1. The input is the Gaussian e^{-x^2}, normalized for unit total energy:

$$\text{energy before normalization} = \int_{-\infty}^{\infty} e^{-2x^2} dx = \sqrt{\pi / 2} \tag{8.1a}$$

$$\text{energy-normalized Gaussian} = e^{-x^2} \Big/ \sqrt{\sqrt{\pi / 2}} \tag{8.1b}$$

The spatial derivatives successively applied to an input Gaussian by Maxwell's propagation process generate successive Hermite polynomials. These are defined by the formula [12]

$$H_n(x) = \frac{1}{(-1)^n e^{-x^2}} \frac{d^n}{dx^n} \left\{ e^{-x^2} \right\} \tag{8.2}$$

The Gaussian e^{-x^2} is the so-called 'generating function' for the Hermite polynomials. Given the definition, one can initialize $H_0(x) = 1$, evaluate $H_1(x) = 2x$, and generate all the rest from the recursion relation [12]

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) \tag{8.3}$$

It is very well known that distinct Hermite polynomials are 'orthogonal', where 'orthogonality' is defined according to the relation [12]

$$\int_{-\infty}^{\infty} H_n(x) H_m(x) e^{-x^2} dx = 0 \text{ for } m \neq n \quad . \tag{8.4}$$

However, that fact is not at all relevant for the present analysis. We need to establish some other facts instead. For representing electric E and magnetic B fields at successive times in the waveform development process, we are really interested in the functions analogous of the form $F_n(x) \propto \frac{d^n}{dx^n}\left\{ e^{-x^2} \right\} = A_n H_n(x) e^{-x^2}$, with the proportionality factor A_n such that the $F_n(x)$ functions are normalized to represent according to $\int_{-\infty}^{\infty} \left[F_n(x) \right]^2 dx = 1$. That requirement makes

$$A_n = 1 \Big/ \sqrt{\int_{-\infty}^{\infty} \left[H_n(x) \right]^2 e^{-2x^2} dx} \tag{8.5}$$

Inasmuch as $H_0(x) = 1$, Eq. (8.5) is just a generalization from Eqs. (8.1). Note the $\exp(-2x^2)$ appearing in the integral, which is not the $\exp(-x^2)$ factor that appeared in the standard, but here irrelevant, orthogonality definition (8.4).

There has to exist a recursion relation to evaluate $\int_{-\infty}^{\infty} \left[H_n(x) \right]^2 e^{-2x^2} dx$. It should be possible to develop this recursion relation analytically, but numerical integration also suffices to do the job, and more immediately reveals the pattern involved. We begin with just $\int_{-\infty}^{\infty} \left[H_0(x) \right]^2 e^{-2x^2} dx \equiv \int_{-\infty}^{\infty} e^{-2x^2} dx$ $= \sqrt{\pi / 2} \;\; = 1.2533136080008$. The recursion for subsequent $n = 1, 2, 3...$ is:

$$\int_{-\infty}^{\infty} \left[H_n(x) \right]^2 e^{-2x^2} dx = (2n-1) \int_{-\infty}^{\infty} \left[H_{n-1}(x) \right]^2 e^{-2x^2} dx \tag{8.6}$$

So, for example, for $n = 4$ we have

$$\int_{-\infty}^{\infty} \left[H_4(x) \right]^2 e^{-2x^2} dx = 7 \times 5 \times 3 \times 1 \times \sqrt{\pi / 2}$$

which comes to $105 \times 1.2533136080008 = 131.59792884008$. That makes the normalizing factor

$$\begin{aligned} A_4 &= 1 \Big/ \sqrt{131.59792884008} \\ &= 1 / 11.471614046859 \\ &= 0.087171691439 \end{aligned} .$$

For representing the contribution $\mathbf{E} \times \Delta \mathbf{B}$ or $\mathbf{B} \times \Delta \mathbf{E}$ to waveform spreading, we are interested in correlations between successive functions $F_n(x)$ and $F_{n+1}(x)$. Because successive functions $H_n(x)$ and $H_{n+1}(x)$ are even and odd, or else odd and even, $F_n(x)$ and $F_{n+1}(x)$ are orthogonal on the interval $-\infty$ to $+\infty$ by any plausible orthogonality definition. But on the *half* intervals 0 to $+\infty$ and $-\infty$ to 0, the $F_n(x)$ and $F_{n+1}(x)$ can yield non-zero correlations that oppose each other, and so can drive waveform spreading. These correlations are defined by

$$\int_0^{\infty} F_n(x) F_{n+1}(x) dx = -\int_{-\infty}^{0} F_n(x) F_{n+1}(x) dx$$

Since the $F_n(x)$ are proportional to the $H_n(x)$, the integral $\int_0^{\infty} F_n(x) F_{n+1}(x) dx$ is proportional to the integral $\int_0^{\infty} H_n(x) H_{n+1}(x) e^{-2x^2} dx$. This integral can be evaluated using integration by parts:

$$\begin{aligned} \int_0^{\infty} H_n(x) H_{n+1}(x) e^{-2x^2} dx &= \int_0^{\infty} H_n(x) e^{-x^2} \frac{d}{dx} H_n(x) e^{-x^2} dx \\ &= -\left[H_n(x) e^{-x^2} \right]^2 \Big|_0^{\infty} - \int_0^{\infty} H_n(x) H_{n+1}(x) e^{-2x^2} dx \end{aligned}$$

or in other words

$$\int_0^\infty H_n(x)H_{n+1}(x)e^{-2x^2}dx = -\tfrac{1}{2}\left[H_n(x)e^{-x^2}\right]^2\Big|_0^\infty = \tfrac{1}{2}\left[H_n(0)\right]^2 \quad (8.7)$$

Observe that this result is zero for n odd. These zeros occur because for odd n the product $H_n(x)H_{n+1}(x)e^{-2x^2}$ has an even number of peaks in the half interval, and these peaks cancel each other in pairs of plus/minus area. Figure 8.1 shows this for $n = 1$.

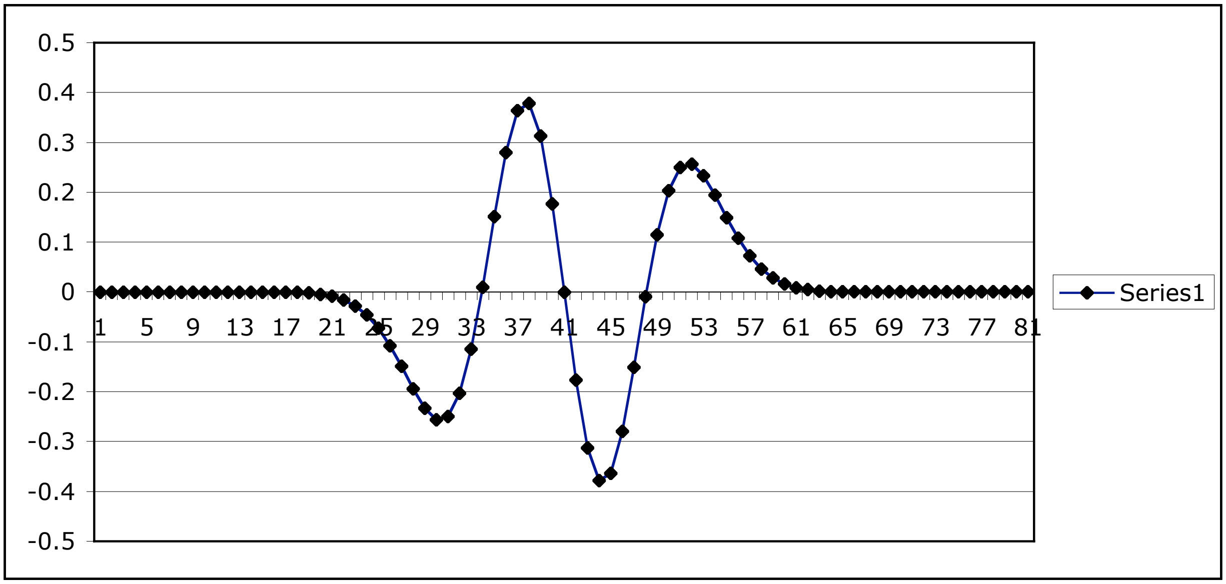

Figure 8.1. Correlation between $F_1(x)$ and $F_2(x)$.

For n even, (8.7) yields a positive result. This occurs because the product $H_n(x)H_{n+1}(x)e^{-2x^2}$ has an odd number of peaks on each half interval, so one peak is always left over to make an uncancelled non-zero contribution. Figure 8.2 shows this for $n = 2$.

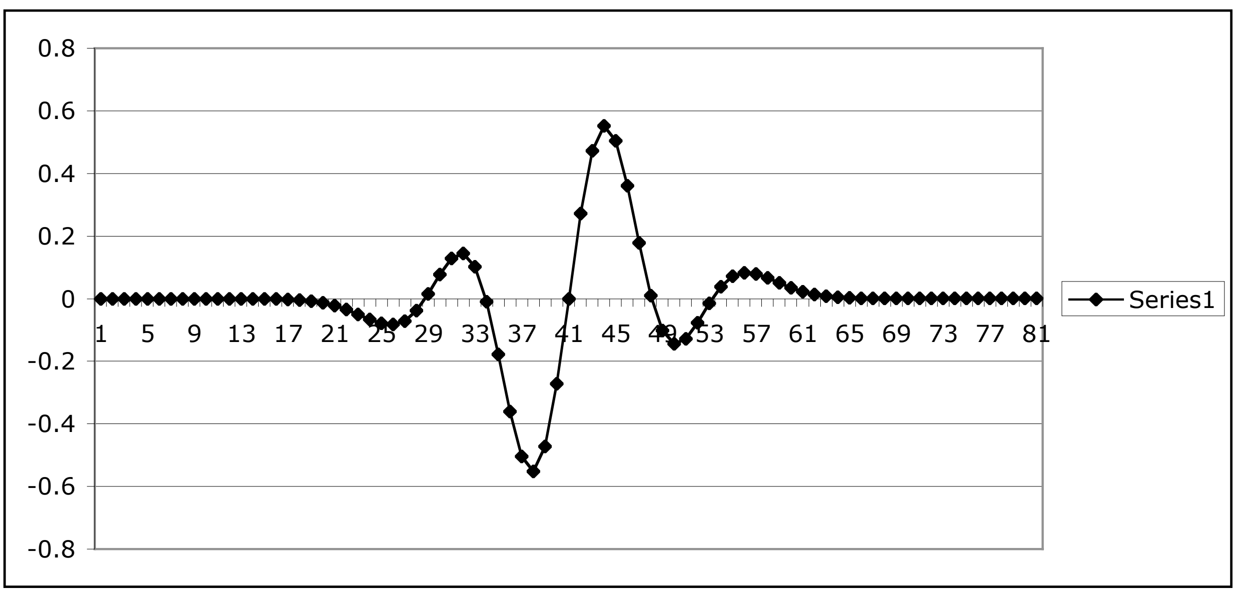

Figure 8.2. Correlation between $F_2(x)$ and $F_3(x)$.

In the case of even n, the correlation $\int_0^\infty F_n(x)F_{n+1}(x)dx$ causes spreading of the wave pattern. The rate of spread is addressed next. Since the $F_n(x)$ are proportional to the $H_n(x)$, they share in the oscillatory character of the $H_n(x)$, which is evident in [12] as

$$H_n(x) = e^{x^2}\frac{2^{n+1}}{\sqrt{\pi}}\left\{\int_0^\infty e^{-t^2}t^n\cos\left(2xt - \frac{n}{2}\pi\right)dt\right\} \quad (8.8)$$

That makes $F_n(x)$ and $F_{n+1}(x)$ a quarter cycle, or $\pi/2$ radians, out of phase. We know that

$$\int_0^\infty F_n(x)^2 dx = \int_0^\infty F_{n+1}(x)^2 dx = \frac{1}{2}$$

So we infer that

$$\int_0^\infty F_n(x) F_{n+1}(x) dx = \tfrac{1}{2}\left\langle \cos(2xt)\cos(2xt - \pi/2) \right\rangle = \tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{4}$$

This driver for waveform spreading is active half the time; *i.e.*, when the waveform has $F_n(x)$ with an even n fully developed and waning, while it builds $F_{n+1}(x)$. That makes for a rather 'staccato' development. The staccato development makes for *time average* $\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$ unit spreading, on each side of the waveform, for total spreading of $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ unit. This is a specific realization of the spreading process discussed in general terms in Sect. 3.

The Hermite peak-generation process leads to the development of a wave train. Figure 8.3 illustrates this wave train for $n = 4$, meaning 4 quarter-cycles of evolution, or $4/4 = 1$ full cycle of evolution (here we see $n/4$ again). The plot contains three curves. Series 1 is the original Gaussian pulse input to the process. Series 2 is this same Gaussian after 1 cycle of spreading. Its width has had 1 unit added, so its width is a factor of 2 larger. And of course its height is correspondingly down, by a factor of $\sqrt{2}$. This spread-out wider, slumped-over lower, Gaussian is the generating function for the Hermite polynomial of order 4, which, when scaled by the same Gaussian, gives Series 3. Series 3 is the emergent wave train.

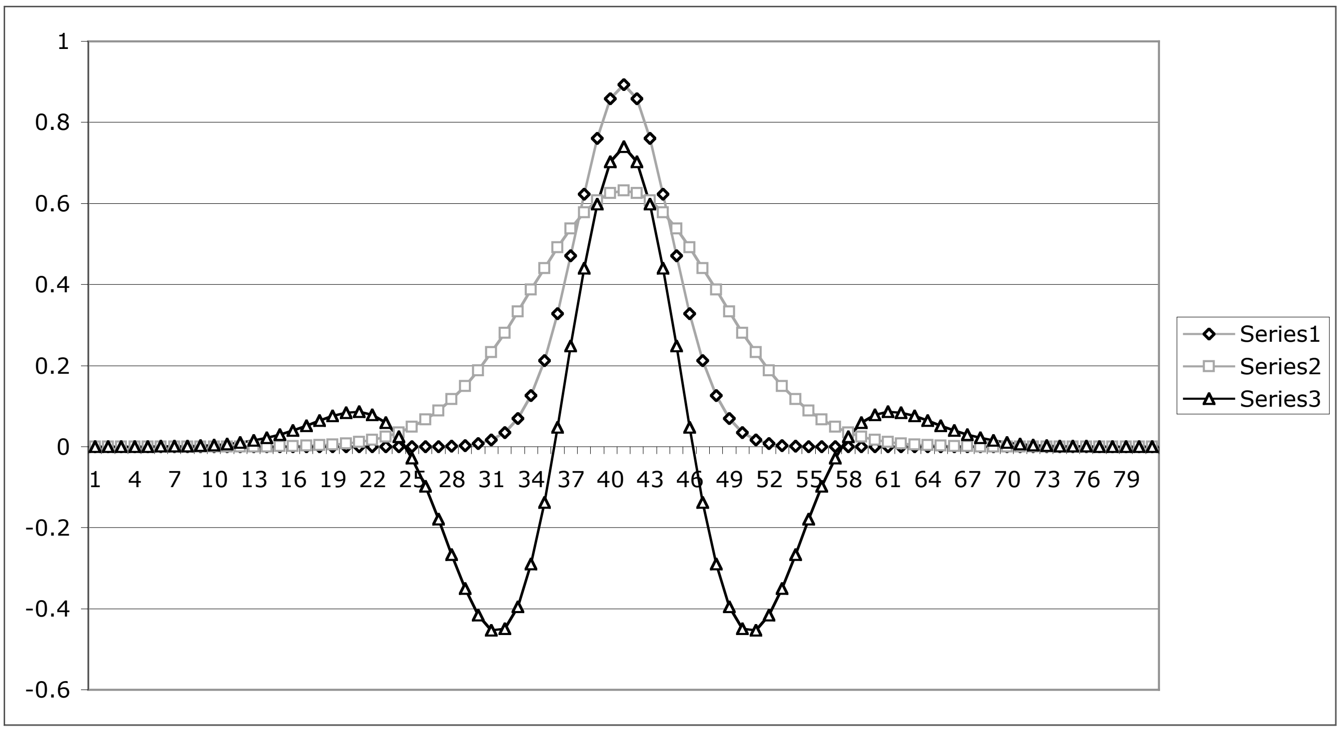

Figure 8.3. Input Gaussian pulse (Series 1), spread-out, slumped-over Gaussian after a full cycle of evolution (Series 2), and the wave train that it generates (Series 3).

At $n = 4$, the emergent wave train (Series 3) has altogether 4 zero crossings and 5 peaks. Observe that all the peaks in the emergent wave train (Series 3) are visually the same width as the original input Gaussian pulse. The width of the Gaussian input to the Hermite peak generation process determines what the wavelength of the ultimate wave train will be.

The emergence of the wave train recalls the QM uncertainty relationship. Recall that under Fourier transformation, Gaussians map into Gaussians, and that the product of the spreads of such Gaussians is a constant. In the process of wave train development, a Gaussian in position space x spreads out, while its corresponding Gaussian in wave number space in k sharpens up.

Observe that 'light in flight' develops its wavelength only during its flight. It doesn't have it to start with, and it gives it up at the end. So light at emission, or reception, has a position, but no wavelength, whereas light in flight has a wavelength, but no position. This could be the true meaning of the 'duality' of light, or 'complementarity', so often discussed in QM.

The information that underlies Fig. 8.3 can also be displayed in terms of wave energy, or squared amplitude. Figure 8.4 gives this view. Observe that the input Gaussian pulse (Series 1), the spread-out slumped-over Gaussian (Series 2), and the wave train it generates (Series 3) all have the same area, meaning they all contain the same total energy.

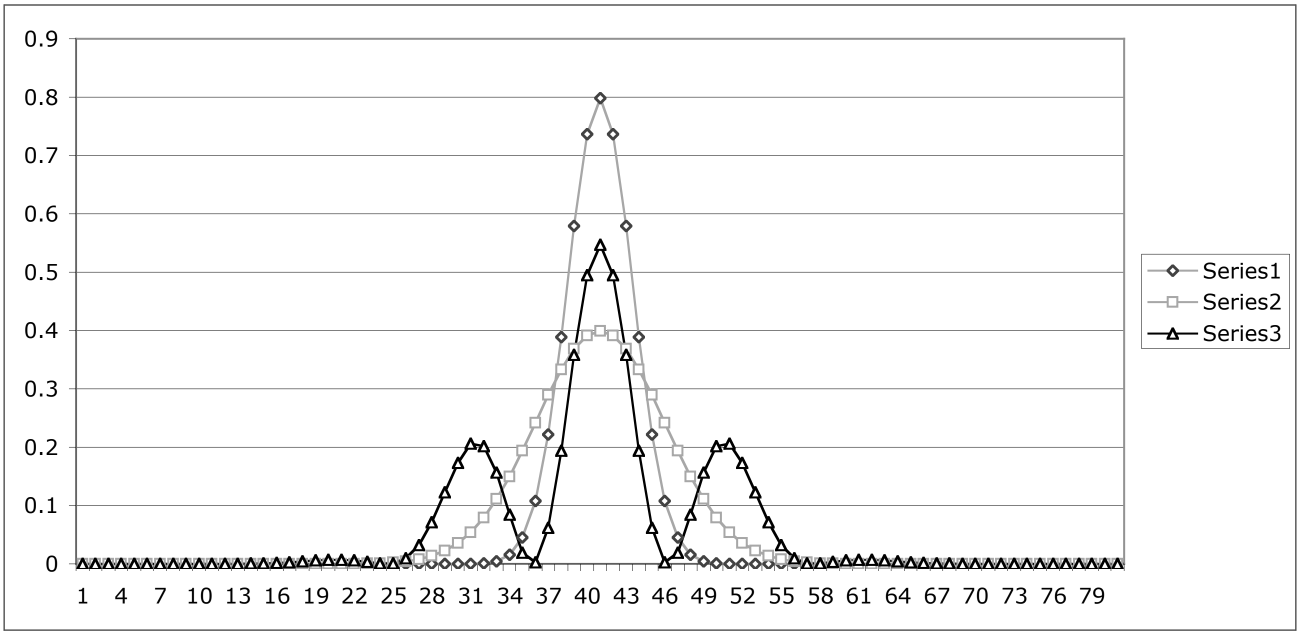

Figure 8.4. Information from Fig. 8.3 recast in terms of energy.

Basically, the Hermite peak generation process does for its generating Gaussian what the normalized, decimalized Pascal triangle algorithm does for the integer 1: H_n divides the energy of the Gaussian into separate ‘bins‘ (peaks) that are the same in number as the entries in the normalized, decimalized Pascal triangle at order n. The numbers in these bins correspond to an energy redistribution rather more spread out than the Gaussian that generates the Hermite polynomials, as one can plainly see from Fig. 8.4. The reason why the Hermite polynomials produce additional spreading is that they are exactly that: polynomials. Their amplitudes diverge in their wings. But their Gaussian weighting function soon tames them.

9. Conclusions

This paper demonstrates a ‘disconnect’ between Maxwell and Einstein. Einstein’s Second Postuate for SRT definitely does not follow from Maxwell’s EMT. What is the philosophically correct thing to say about this situation?

Especially since many of us in NPA distrust SRT in the first place, the obvious conclusion is that Maxwell was right, and Einstein was wrong. But it *could* be the reverse; Einstein could have been right (by inspiration, without foundation) and Maxwell would then have been wrong (despite many years of meticulous work). But to what extent Maxwell could be wrong we cannot imagine, since Maxwell’s EMT is very broadly and successfully applied throughout our civilization. And SRT certainly is not.

The minimum thing to say is that there exists a problem about ‘truth in advertising’ concerning SRT. There is not now, and never has been, any sort of ‘band-wagon‘ effect in NPA, but perhaps we can agree on that minimum conclusion concerning “Maxwell’s Maxima”!

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