Moving clock does not run slow

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Abstract
This note is not intended to show that the time dilation formula in special relativity is wrong. Instead, I will show that the formula has been misinterpreted. In particular, I will show that the time dilation formula does not imply a moving clock runs slower compared to a stationary clock. As a corollary, there is no twin paradox.

Consider an inertial reference frame S' which moves uniformly with velocity +v parallel to the x-axis of a rest frame S. Assuming that $t=t'=0$ when the origins of the two frames coincide, the Lorentz transformation equations, which relates the spacetime coordinates (x,y,z,t) of an event to the spacetime coordinates (x',y',z',t') of the same event, are given by

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma(t' + vx'/c^2) \tag{1}$$

where

$$\gamma = [1 - (v/c)^2]^{-\frac{1}{2}}$$

is the Lorentz factor, a dimensionless quantity which is greater than 1. Let the clocks in the two frames show the same time $t=t'=0$ when the origins of the two frames coincide.

Consider two events: Event 1 and Event 2. Suppose ($x_1'=0$, $t_1'=0$) is the spacetime coordinates of Event 1 according to Jill in the S' frame. The time coordinate of Event 1 according to Jack in the S frame is

$$t_1 = \gamma(t_1' + vx_1'/c^2) = 0 = t_1'. \tag{2}$$

In other words, the time coordinates of Event 1 according to Jack and Jill are the same. Suppose $(x_2'=0, t_2'>0)$ is the spacetime coordinates of Event 2 according to Jill in the S' frame. That is, Event 1 and Event 2 happen at the same location in the S' frame. The time coordinate of Event 2 according to Jack in the S frame is

$$t_2 = \gamma(t_2' + vx_2'/c^2) = \gamma t_2'. \tag{3}$$

According to Jack, the time interval Δt between the two events is given by $t_2 - t_1$. According to Jill, the time interval $\Delta t'$ between the same two events is given by $t_2' - t_1'$. Eq. (2) and Eq. (3) imply

$$\Delta t = \gamma \Delta t'. \tag{4}$$

Since $\Delta t > \Delta t'$, it is conventionally [1,2] concluded that the synchronized clocks in the moving S' frame must therefore be running slower than the synchronized clocks in the rest frame S.

However the conventional conclusion, that moving clock runs slow, is not a logical necessity. This is because the Newtonian view that clocks in the two frames keep the same time *does not* contradict Jack's and Jill's different time coordinates for Event 2. Here's how: the clocks in the two frames show the same time t_2' when Jill observes Event 2, and the clocks in the two frames show the same time t_2 when Jack observes Event 2. Jill observes Event 2 at time t_2'. Jack observes Event 2 at time $t_2 = \gamma t_2'$. This means that Jack observes Event 2 later than Jill since $\gamma t_2' > t_2'$. The time interval Δt between Event 1 and Event 2 according to Jack is longer than the time interval $\Delta t'$ between the same two events according to Jill simply because Jack observes Event 2 later than Jill. What is called time dilation, $\Delta t = \gamma \Delta t' > \Delta t'$, therefore does not imply moving clock runs slow. Hence there is also no twin paradox.

References

[1] A. Einstein and L. Infeld, The Evolution of Physics (Simon and Schuster, 1966), pp. 183-184.

[2] W. D. McComb, Dynamics and Relativity (Oxford University Press, 1999), p. 197.