Reference-Frame Independent Dynamics, Or How to Get Off Einstein's Train

Abstract: *Einstein spent his life trying to develop a system of dynamics independent of reference frame. But this lofty goal demands a hard look at the very meaning of reference frame, a starting point for this paper. Einstein's fruitless attempts were based on his own observer-based theory of relativity, in turn based on Lorentzian covariant paradigms. Covariant mathematics admits transformations between coordinate systems and reference frames, but does not suggest a path towards invariance. However, the del operator provides a means of expressing spatial derivatives independent of coordinate system. Likewise, if time is independent of space, as this paper suggests, the total time derivative operator, d/dt, is also independent of reference frame. Therefore, physical equations that can be expressed with the del operator and the total time derivative, as opposed to reference-frame dependent partial derivatives, are naturally independent of reference frame. In particular, Maxwell's equations can be correct only if expressible by the del and d/dt operators, as proposed by Hertz and more recently advocated by Phipps. From fluid dynamics, developed by Bernoulli, Euler, Langrange, and many others, over a hundred years before Maxwell, we have the convective or Lagrangian time derivative d/dt = d/dt +* ***v*** *• del. Both sides of this equality are independent of observer, but the two right-hand terms differ in weight depending on how an observer moves with respect to the quantity being differentiated. One observer might see a buildup of material, while another sees an altered flow; one observer might see a changing field, another an acceleration. But the total change, the sum, remains invariant. With the convective derivative, we can readily derive Maxwell's famous "displacement current" term in Ampere's Law and clear up mysteries surrounding Faraday's Law, particularily relating to unipolar induction and the Sagnac experiment. Invariant Hertzian dynamics provides a means to finally get off Einstein's covariant train.*

A Gedanken-experiment:

The first step off Einstein's train is a clear conception of reference frame. As with all truth, of course, the greatest hindrance to such a conception is the belief that we already have it. Most of us were introduced to the idea of "inertial frame" long before we reached high school, though we might not have called it such. By the time we studied physics in college, we'd have been offended by anyone who condescended to explain to us the meaning of "reference frame." We knew that frame B moved with respect to frame A at velocity **v**. What more was there to know?

Perhaps much more. Laying preconceptions aside, let's begin with a ridiculously simple Gedanken-experiment. It will go quick, embarrassingly quick:

1. Imagine a point P in physical space at time t.
2. Imagine the same point P at a different time t'.

That's it! You could close your eyes and do it again, just to make sure you didn't miss anything.

Did you have any trouble? You should have. If you didn't, then you knowingly or unknowingly made some assumptions about space and time, assumptions regarding reference frame, the whole point of this discussion. To begin, what is the meaning of P except in relation to matter? P could represent the center of the earth, the tip of your finger, the location of particle X, or Alpha Centauri. But in physical space, can P be meaningfully described in terms of space itself? Not without first assuming something about space.

If you are an Einsteinian and believe in something called "space-time", then you couldn't even take step one without first assuming a reference frame. To you, time itself would differ depending on how you traveled through space, depending on your reference frame. You would need to establish a frame and your location P_{ob} before time t would even have meaning at P. To you, the "instant", a snapshot of reality at a particular time, is not universal. Simultaneity doesn't exist, you say. You can't speak merely of point P, but of the "space-time" combination P-t in reference frame R, observed from point P_{ob}.

On the other hand, you could reject "space-time" and declare the independence of space and time. You would argue, like Peter Erickson, that time is serial, with each "instant" flowing continuously into the next.[1] The serial nature of time, you say, demands the existence of the "instant", the simultaneous universal snapshot in time. The existence of the universal instant is equivalent to the independence of space and time. To you, simultaneity reigns.

Players in both camps accuse the other side of fallacies, paradoxes, and contradictions. Nonetheless, both should agree that independence of space and time is the simplest paradigm, the one a child would assume without much thought, the one that would satisfy Occam's Razor *if* it could explain the experimental results. Einsteinians claim that "space-time" is necessary because there is no consistent way to explain the propagation of light without it. Their 4-vector invariance is "beautiful" and useful, but is it fundamental? Could 4-vectors arise from some other more fundamental notions? Perhaps. Their paradigm assumes a property of space itself, that it somehow impedes motion by the constant 'c'. From whence did space obtain this magical property? The Machian paradigm, that motion is only meaningful with respect to (wrt)• matter, not wrt space itself or observer, interprets 'c' as a property of matter: not as the 'speed of light' but as the 'speed of charge'. That is, 'c' may be understood as the speed (wrt matter) at which moving charge experiences a balance between the repulsion of like elements of matter and the Amperian attraction due to parallel flows or currents. With this conception of 'c', space can be liberated from its dependence on this enigmatic constant. We needn't assume 'c' as a property of space at all, in which case the concept of "space-time" becomes ridiculous.

Consider now the universal instant, our snapshot of all space at a particular time t. Under space and time independence, this instant is the same for all points in space, no matter how far apart and no matter how any observer moves. That's what "universal" means, and what the independence of space and time demands. In this snapshot, can we say anything about reference frame? Is the concept of reference frame even meaningful in this universal instant? Consider this question carefully, since it will affect further arguments. In the universal instant, there is no passage of time, so how can we speak of a reference frame? Indeed, there is no reference frame in the universal instant, only space and matter.

What about point P? How do we even identify this point? According to Mach's Principle, it is meaningful only in relation to matter. As noted above, we can regard P in relation to the earth, your finger, or particle X, but P as a mere abstraction is meaningless in the physical universe. Now in the universal instant, the distance between P and P_{ob} is invariant, independent of whatever coordinate system we establish to measure it, and not dependent on 'c' in any way. But how do we physically measure this distance? In relation to matter, of course. We don't measure space itself, but only compare distances with other distances. We compare the distance between P and P_{ob} with some standard distance, some ruler, like a meter (one ten billionth the circumference of the earth) or a light-year (the distance something would travel in one year at speed c). Likewise we don't measure time itself, but only compare cycles of time with standard cycles of time or 'clocks', like the rotation (day) or revolution (year) of the earth, or the oscillation of a caesium atom. This important idea will arise again.

Now on to step two of our Gedanken-experiment. What is meant by the word "same" when referring to the "same point P" at a different time t'? If we determine P wrt matter, then we really mean the "same point P" wrt some object(s), the earth, the mass center of a thermodynamic system, etc. We mean the point that travels in the reference frame of the defining object(s). But wait! We just introduced a reference frame. Can't we just refer to point P at two different times without introducing a frame of reference? No, we can't. Once time gets involved, there's no help for it. Reference frames follow. That is, reference frames are inseparably linked with the passage of time.

Let's now distinguish clearly between spatial coordinate systems and reference frames. Descartes made an astonishing intellectual leap with his invention of the coordinate system. Not only could he plot one spatial dimension against another, but also the position of an object along some axis versus time. Indeed we may plot variations in any physical quantity versus variations in another, whether continuous or discreet.

• I will abbreviate "with respect to" as "wrt" throughout this paper.

However, in the *universal instant*, when no time passes, we can plot the distribution of matter, whether expressed as energy, charge, mass, or something else, wrt space, but not wrt time. That is, we can establish a 3D coordinate system, but can't establish a "reference frame" until some amount of time passes. So a physical reference frame is not just any coordinate system, but one that "slides" with matter over time. For example, in the apocryphal story of Descartes' fly, we plot the motion of the fly across the ceiling in the coordinate system that "slides" with the earth. By arbitrarily choosing some other reference frame(s), we could view the fly's motion however we wish, though it's hard to imagine a frame more physically meaningful than that of the earth. To repeat, reference frames are spatial coordinate systems that slide with matter over time.

We might introduce greater degrees of freedom into our expressions in order to simplify complex coupling relationships between the objects of our system, or to express the "movement" of light. Our cleverness might induce us to perform useful mathematics in this invented higher-dimensional space, but none of this would cause physical space to exceed three dimensions, nor allow time to pass in the universal instant. Degrees of freedom have nothing to do with reference frames.

One last point relates to kinematics. The distance between P and P_{ob} is invariant to coordinate system only for a given instant. To ask the distance between P at time t and P_{ob} at time t' first requires a reference frame. Transformations between reference frames constitute the essence of kinematics. We cannot intelligently speak of the distance object X traveled from P_{ob} to P until we first establish a reference frame

Let's summarize the ideas derived from our perhaps not quite so trivial Gedanken-experiment. For brevity the term "instant" will henceforth mean "universal instant", the simultaneous snapshot in time that Einstein denied.

1. Point P in physical space is meaningful only wrt matter.
2. The serial, flowing nature of time demands the existence of the instant.
3. The existence of the instant is equivalent to the independence of time and space.
4. By definition, no time passes during an instant.
5. Reference frames are spatial coordinate systems that slide with matter over time.
6. For a given instant, spatial coordinate systems are meaningful, but reference frames are not.
7. For a given instant, the distance between P and P_{ob} is invariant to coordinate system.
8. To conceive of P at different times requires a reference frame.
9. The distance between P at time t and P_{ob} at time t' requires a reference frame.
10. The distance an object travels from P_{ob} to P requires a reference frame.
11. We don't measure space itself, but only compare matter with standard matter (rulers).
12. We don't measure time itself, but only compare cycles with standard cycles (clocks).
13. Degrees of freedom are not physical spatial dimensions.

If you are an Einsteinian, and have made it this far, you surely must be teeming with objections. To this I request that you reserve judgment, and explore the possibility of the universal instant, which was assumed by people for millennia prior to Einstein. Also ask yourself why space should somehow impede motion by the constant "c" and why "c" alone of all the fundamental constants (h, e, m, k…) should operate as a property of space rather than of matter. Finally if the so-called invariance of 4D "space-time" could be *derived* from space and time separately rather than *assumed*, wouldn't such a conception be preferable?

Do Inertial Frames Exist?

Before tackling reference-frame independent dynamics, we need to explore further the very meaning of reference frame, as distinguished from the concept of "inertial frame". By "reference frame" is meant the observer's "point of view," and naturally everything that's viewed must be viewed from some point of view or reference frame. However, by "inertial frame" most people tacitly assume a reference frame from which we determine how much matter "moves". By definition matter is "at rest" if it does not move wrt this particular frame. Moreover, two objects traveling linearly along "parallel" paths at the same velocity are assumed to exist in the same inertial frame as each other, both moving at speed **v** wrt the “inertial frame”. These conceptions conceal several assumptions about the nature of space itself, namely:

1. An object can somehow be “at rest” wrt space itself.
2. There exist frame(s) in which all matter could be “at rest.”
3. “At rest” defined as “not moving” is unambiguous.
4. Objects separated by a constant distance over time are “at rest” in the same frame.
5. Elements of the same body of matter are “at rest” in the same frame.
6. Reference frames are linear in nature.
7. Parallel paths exist in physical space.

As a good Einsteinian, if indeed you are, you will naturally scoff at the first two assumptions, recognizing them as following space-based paradigms. You will say that the inertial frame isn’t universal, but a choice of the observer. Something is “at rest” if you happen to choose the frame that moves with the object. Its motion and energy are determined wrt the observer. Motion and energy are not fundamental physical concepts, not facts of nature, but merely an observer choice, so you say. You define “at rest” as “not moving wrt the observer” rather than as “experiencing no net force or torque,” the Machian definition.

A matter-based Machian (like myself in case you missed that) will agree with you and with Einstein that space itself doesn’t provide us with a unique frame of reference from which to determine motion. From whence would space obtain this surreal and privileged frame? But from here we part ways. The Machian doesn’t deny a privileged frame as you do; he simply attributes this frame to the location and movements of matter, not to space itself. Further, once your observer-based frame is chosen, you assume that anything not moving *in this frame* is by definition “at rest”, while the Machian defines “at rest” as “experiencing no net force or torque.” The Einsteinian definition of “at rest,” though not based on some absolute frame of space, is nonetheless still based on space, not matter. It considers only the observer and cares not a whit for the influence of matter on the objects observed. Let’s take some examples to see which definition makes more sense.

Imagine driving down a freeway at 60 miles (or 90 km) per hour (mph). You suddenly notice a fly on your windshield, hanging on for dear life, and realize how good it feels to be safe and snug behind the wheel. Having accelerated to cruising speed, you experience no net force or torque, and so consider yourself “at rest” wrt the car. On the other hand, the fly definitely experiences the force of the wind. That force is real, and doesn’t depend on the observation of your friend, who waves to you as you pass. He sees you both moving at 60 mph, with the distance between you and the fly remaining constant over time. Being a good Einsteinian, your friend declares that both you and the fly “move” at 60 mph with respect to his “inertial frame”, but remain “at rest” in the frame of the car. But a Machian, who defines “at rest” as “experiencing no net force or torque”, sees the fly as moving absolutely wrt the earth and sees you absolutely “at rest” wrt the car (almost). He bases his findings on matter, independent of your friend’s point of view, the view of another car passing in the opposite direction, or anyone else’s view. The difference between you and the fly, he says, depends on the material comprising the car. With the exception of the molecules on or near the surface and of the tires, which circulate and contact the road, he declares the majority of the car to be “at rest.” He would say the same of the interior of the spaceship carrying Einstein’s travelling twin. If it weren’t “at rest” it would experience some net compression, expansion or torsion.

After a long drive, you finally get home and decide to do some star gazing. You notice a satellite that never seems to move, and realize that it orbits the earth in a geo-stationary cycle, traveling around the earth exactly once every twenty-four hours. Because it travels at precisely the right height and velocity, the force of gravity is exactly balanced by the “centrifugal” force of its motion. Since the distance between you and the satellite remains constant over time, you conclude that both you and the satellite are “at rest” wrt the inertial frame of the earth. The problem is that you experience a net force called gravity while the satellite does not. You now take the role of the fly that experiences a real net force, independent of how anyone looks at, while the satellite resembles you behind the wheel of your car. Barring perturbations from the moon or of turmoil within the earth, it could travel indefinitely along its orbital path without any addition of energy. Machians would say that it is “at rest” and that the “centrifugal” force, like the Coriolis force, is merely an abstraction to account for less “gravity.”

Translating or Rotating? Infinite or Finite?

Next you realize that the earth's frame isn't really linear or translational anyway, but rotational. From the point of view of the sun, there exists no linear "inertial frame" from which either you or the satellite could remain "at rest" for more than an instant. And the sun isn't "at rest" in any inertial frame either because it orbits around the galaxy. After pondering the orbital motion of the galaxy and so on, you wonder if anything anywhere is ever "at rest" wrt a linear "inertial frame". If not, what is the meaning of "inertial frame"? Perhaps the only physically meaningful reference frames are orbital or rotational frames. Only objects circulating in balanced orbits ever seem to be "at rest" in the Machian sense of "experiencing no net force or torque."

Attempting to remedy the situation, you next conclude that the satellite is not in the earthbound inertial frame after all, because it travels at velocity $v_s = \omega R_s$, not at earth velocity $v_e = \omega R_e$. To be in the same inertial frame, you argue, two objects must move in the same direction *and* with the same linear velocity. But here you make an unwarranted assumption about space. You tacitly assume infinite space. Yes, when you assume that it's possible for two objects to travel in exactly "parallel" paths (the same direction), you assume that those paths will never meet. If they met, they would not be parallel. "Parallel" lines and "flat" planes both require infinite space or bounded space. Such paths or planes either go on forever or the bump into a boundary. Though this paper isn't intended to argue the case fully, a rational argument for finite, unbounded space or "closed space" can be made, as follows:

1. The universe is governed by conservation.
2. Conservation of energy, matter, etc. implies a fixed amount of whatever is conserved, and thus a finite amount.
3. Finite energy (or matter) in infinite space implies zero density (infinity is *really* big).
4. Energy (or matter) *density* is not the conserved quantity, so we may not argue for an average constant density in an infinite space.
5. Space itself exhibits no preferred frame.
6. Bounded space would necessarily prefer the frame of the boundary.
7. By Mach, matter interacts only with other matter, not with boundaries.
8. The absence of matter across the boundary would alter matter interactions near a boundary.
9. There is no evidence of such differences in interaction.

Recognizing that many venerable scholars fall into both camps on the finite versus infinite universe debate, this summary is not intended as a comprehensive "proof" of a finite, unbounded universe. But whether you agree with the summary arguments above or not, you should recognize that Euclidean space, containing "parallel" lines and "flat" planes, is necessarily infinite or bounded. It by no means necessarily represents physical space, experimental evidence of "flat space" notwithstanding. The conclusion of the "flatness" of space from these experiments results from another assumption, that light could somehow "travel" from a source in multiple directions around a geodesic, reaching a receiver from many directions. This assumption clearly violates Fermats' principle, that light follows only one path, the path of least time. Fermat's principle also boasts ample experimental evidence and predicts that we will never see light from the same source arriving via different paths. Thus, finite, unbounded space is conceivable, and it is therefore unwise to tacitly assume that linear "inertial frames", in which objects follow parallel paths, even exist at all.

You don't have to become a believer in finite, unbounded (closed) space to visualize such a thing in 2D. Simply picture the surface of a globe. There is very little difference between the path of the equator and that of 1^{o}N latitude around the earth. Both paths circulate with respect to the poles, as do the paths at 10^{o}N, 45^{o}S, and 89^{o}N. With a sufficiently large globe, we would regard the equator path as "straight" and the paths slightly north or south as "parallel", though technically the 0.01^{o} path is slightly shorter and slightly more curved. The 1^{o}, 10^{o}, 45^{o} and 89^{o} paths become ever shorter and increasingly curved, but all represent closed orbits. There is no qualitative difference between the "translational" path along the equator and the "rotational" path at 89.99^{o}, only the curvature of the orbit. Thus in a closed 3D space there is no essential difference between rotation and translation. Everything that translates also follows an orbital path. And the components of all bodies that rotate also translate according to the kinematic expression $v = \omega R$. The idea that translation and rotation are two different ways of viewing the same motion is both unifying and consistent with a physical understanding of Maxwell's Equations.[2]

You might object that closed Reimannian space is also an axiom of general relativity. However, we are not thus bound to accept that theory's other assumptions or conclusions merely by accepting closed space. How can immaterial physical space "bend" under the influence of material objects? Why is the size or expansion of the universe or any other property of space connected in any way with the constant "c"? Is there even sense in the idea of space expansion except in relation to matter? As noted before, we don't access space directly, but only compare matter with matter in space. So though his intuition relating gravity (and other forces) to the curvature of space may have been on the right track, Einstein and his assumptions constitute unnecessary excess baggage on our trip through finite, unbounded space.

The purpose of our discussion on closed space is to demonstrate a need for conceiving of reference frames in terms of orbits or rotation rather than mere linear translation. Orbital frames are not "inertial frames," as commonly understood in terms of translation alone. Indeed real physical objects can be found "at rest" in the Machian sense of "experiencing no net force or torque" only in an appropriate orbit, not by somehow merely "standing still", whatever that might mean. A rotational frame or "gauge" tracks not only how fast something moves, but also about which point it orbits. Is it even possible to move along the surface of the globe without rotating wrt some point (actually two polar points)? No, all motion is orbital in closed space, and thus a gauge exists even for so-called linear motion. The common notion of a linear "inertial frame" is inconceivable is finite, unbounded space. To quote Peter Woit's book, it's *Not Even Wrong*.[3]

Moreover, objects rotating about a common center with a common frequency do not necessarily occupy the same frame, just as the satellite is not in the same frame as the stargazer. Nor do all the mirrors and equipment in the Sagnac or Michelson-Gale experiments occupy the same rotational frame. It would behoove science to examine these and other effects in terms of rotational reference frame, a concept that can liberate as well as enlighten. So next time you ride on Einstein's train, complete the round trip, and ask yourself whether the side of the train on the inside track occupies the same reference frame as the side on the outside. If not, you've identified different rotational reference frames or gauges.

Our discussion on "inertial frame" can be summarized by a restatement of Newton's First Law: "Objects in motion stay in motion and objects at rest stay at rest." To begin, objects maintaining constant velocity **v** wrt Newton would be "at rest" in the frame moving at speed **v**, so we could restate the law as follows: "Objects at rest in some frame stay at rest in that frame." This is consistent with the Machian definition of "at rest" as "experiencing no net force or torque." If you don't push it or turn it, it doesn't move, regardless of what frame it's in. Moreover, when he proclaimed his new law, Newton imagined a universe in which an object could travel forever without any influence from other matter, though in the physical universe nothing travels forever except in steady-state orbits. We might then reformulated his law as: "Objects in stable orbits stay in orbit." This is what we observe in the real universe, and indicates the direction physics needs to take in the future.

The idea of an "inertial frame" with respect to space, whether absolute or observer-based, misses the point of what is moving with respect to what. It does not distinguish between the fly and driver or between the surface and the interior of the rocket. Moreover, reference frames based on a naïve conception of linear translation miss the importance of rotation in real world physics. In short, the term "inertial frame," whether uttered by an Einsteinian relativist or an absolute space devotee, ought to signal an alert in your mind: "Warning! Space-based linear thinking ahead!"

The Meaning and Purpose of Partial Derivatives

Like all real journeys, ours has taken twists and turns and even bifurcated, only to reconnect later on. It has included rotation and radiation as well as translation. In an effort to get off Einstein's linear train, we've first had to develop an understanding of several fundamental concepts, not limited to the following:

1. the universal, simultaneous instant
2. the independence of time and space
3. reference frame as a spatial coordinate system that "slides" (rotates) with matter over time
4. the necessary connection between time and reference frame
5. "at rest" as "experiencing no net force or torque"
6. the inadequacy of the space-based "inertial frame" concept
7. the greater generality of a rotational reference frame or "gauge" over a linear "inertial frame"
8. the infinite or bounded nature of Euclidean space
9. the inherent closed-path, rotational nature of closed space

We now need to turn our attention toward the mathematics of coordinate systems and a lesson in or two in history. Descartes' coordinate system concept spawned a revolution in the way we visualize the physical world. Though it influenced Newton, the idea didn't really begin to mature until tackled by the masterful Euler over a hundred years later. Euler and his contemporaries realized that a physical system could be described via any number of coordinate systems. Not limited to Cartesian systems, they discovered spherical, cylindrical, and other coordinate systems, beneficial for exploiting the symmetries of particular distributions of matter or characteristic motions. They derived transformations between coordinate systems, fully aware that the physics they described did not depend on their choice of coordinates.

With notation unfamiliar to moderns, Euler nonetheless clearly communicated the concept of the partial derivative both wrt space and to time. He understood that no physical ensemble depends on the coordinate system employed to describe it. Thus a differential change in something wrt a particular coordinate or degree of freedom• depends only on that arbitrarily chosen variable. It is not a fact of nature. For example, a diverging field could be described with partial derivatives of (x, y, z) even though the field doesn't depend on x, y or z individually at all. This is the magic of partial derivatives Euler recognized. Transform the variables of any system of equations and only the explicit variations wrt each new variable need be considered. If a variation wrt x, say, creates a corresponding variation wrt y, the y variation is already accounted for in the complete expression, and does not need to be considered as part of the variation wrt x. By the chain rule, the *total* derivative wrt x includes variations in y, z and other variables resulting from the variation in x. However, because x is not a property of nature, the total derivative wrt x is physically meaningless. If the system depended on x independently, x would necessarily be a property of nature.

When we choose a coordinate system and/or set of variables to describe a system, we don't care about the variables themselves. We care about the physical system, and choose coordinates or variables that best describe it. Frequently variations wrt one coordinate result in variations wrt others, so a complete description incorporates the interplay of all the coordinates and variables. Now in a system described by only one coordinate or variable, the partial change wrt to that variable equals the total change wrt it. The chain rule doesn't apply and partial derivatives serve no purpose. Moreover that variable, or at least a change wrt it, is physically meaningful. However, any system described by two or more variables requires more care, especially if the variables are not independent. Then variations in one affect variations in others and we must employ partial derivatives. By definition, partial derivatives are employed when a coupling between variables exists, or we would simply use total derivatives. No physical system should ever be described in terms of only one partial derivative. With what would it couple? Moreover, any partial derivative by itself is necessarily incomplete, requiring information from other partial derivatives for physical meaning. In short, any isolated partial derivative is physically meaningless. We'll return to this important point later.

Now partial derivatives apply to time as well as to space. What you observe as a change wrt time depends on your perspective or reference frame. Thus to a merry-go-round enthusiast, another rider's position does not change with time in spite of what an earthbound observer sees. An inner tube flowing down a quiet stream is "at rest" wrt the stream, though the shore might seem to move backwards. Variations wrt time may or may not completely describe a system, depending on whether the system is "at rest" wrt to a given

• In the context of this discussion, coordinate is not necessarily limited to "spatial coordinate", but could include degrees of freedom, such as rotation angles or expansion rates of a translating object.

perspective or reference frame. The driver of the car doesn't see the fly "move," so the variation he measures wrt time is zero. However, he doesn't observe the fly from the frame in which the fly moves, the frame of the earth. That is, his partial derivative wrt to time is insufficient to describe the fly's motion completely. To obtain the total variation in the fly's movement, we must also consider variations wrt to other coordinates over time, in this case the velocity of the fly's frame (the earth) wrt the driver. As emphasized above, the partial variation wrt t, like all partial variations, is physically meaningless by itself. By definition it implies a coupling with the partial derivatives of other variables or coordinates, or else we could simply employ the total derivative wrt time. To describe the system completely, we must employ the chain rule and consider variations wrt *all* the relevant variables in the system. We need the total time derivative to intelligently determine how much the fly moves, not merely the partial time derivative which depends on our reference frame.

The Convective Derivative and Continuums

Unlike x above, time t IS a fact of nature, as discussed at length. The total change wrt time IS physically meaningful if time is independent of space. This is where Euler employed his genius over 250 years ago. In modern notation, here is Euler's derivation of the total variation of function **f** wrt time:

$$\frac{d\vec{\mathbf{f}}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{f}}(\vec{\mathbf{r}} + \vec{\mathbf{v}}(\vec{\mathbf{r}},t)\Delta t, t + \Delta t) - \vec{\mathbf{f}}(\vec{\mathbf{r}},t)}{\Delta t} = \frac{\partial \vec{\mathbf{f}}}{\partial t} + \left(\vec{\mathbf{v}} \bullet \vec{\nabla}\right)\vec{\mathbf{f}} \qquad [1]$$

where **f** could be replaced by a scalar, tensor, or any other order function. We can also obtain this result in Cartesian coordinates by means of the chain rule, though the result does not depend on Cartesian or any other choice of coordinates.

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y}\frac{\partial y}{\partial t}\frac{\partial}{\partial y} = \frac{\partial}{\partial t} + \vec{\mathbf{v}} \bullet \vec{\nabla} \qquad [2]$$

This important operator for the total time derivation has several aliases: the *convective*, *Lagrangian*, or *hydrodynamic* derivative. The second term on the right, **v** • del, is called the *advective* derivative. Of course, Euler didn't use the del operator, but he clearly understood the concept of a gradient, and he understood that partials of (x, y, z) wrt time represent a velocity **v**. But what velocity? The velocity of the observer (the driver or the friend) wrt the proper frame (the earth) of the system (the fly). The critical point about this expression is that both terms on the right depend on the reference frame of the observer, but their sum does not. The total time derivative, a fact of nature, does not depend on the reference frame of the observer. So in our fly example, the friend sees only changes wrt time (the partial time derivative term) while the driver sees only changes due to his own motion wrt the proper frame (the advective term). Both observers, in fact all observers, agree that the fly moves wrt the proper frame of the earth. They just derive it differently.

The del operator, expressing div, grad and curl with a single symbol, wasn't used until Heaviside and Gibbs in the late 1800s, though the concept of coordinate independence was understood 150 earlier in the work of Euler, Langrange and many others. If space is independent of time, then div, grad and curl operations must be facts of nature. They don't depend on the coordinates used to express them. We can physically grasp the concepts of divergence as "spreading out", gradient as "topological variation" and curl as "curvature" without requiring any particular coordinate system. Now if del expresses facts of nature wrt space and d/dt expresses facts of nature wrt time, we should expect any fact of nature operated on by del and d/dt to also be a fact of nature. The difficulty, as we shall see, is discovering the facts of nature on which to operate.

Euler, Lagrange, and many others applied the profoundly simple convective derivative successfully to many problems in fluid dynamics and hydrodynamics. The term "fluid" really means continuum, an infinitely divisible distribution of matter. Thus the mathematics of fluid dynamics also applies to gases or anything else that can be treated as a continuum. Over the course of the two hundred years following

Euler, many difficult problems related to flow, waves, vortices, and turbulence were solved, and it seems reasonable that these solutions would also apply to other continuums. In the 1950s, for example, Hannes Alfven created the new science of magneto-hydrodynamics (MHD), which added magnetic interactions to the continuum behavior of matter. MHD is now standard physics for astronomers today.

Ironically, because matter at some fundamental level is composed of particles, what once were thought continuums are in fact discreet-ums. But a double irony follows from the fact that particles themselves have structure, and this structure requires a continuum of matter. If this were not so, and the structure of particles was itself composed of discreet sub-particles, what would be the structure of the sub-particles? And the sub-sub-particles? Below any level that is fundamentally discreet, there exists no structure. Even a fractal distribution in the limit becomes a continuum. Therefore, the mathematics of continuums could and should be fruitfully applied to the very structure of matter. This hasn't been seriously attempted because mainstream science convinced the world that matter is fundamentally discreet and thus structure can't be known. A third irony arises because continuum mathematics has not been fully applied to electrodynamic fields, which are naturally continuous in nature. This occurred primarily because of the confusion caused by Einstein with his distortion of the basic reality of space and time. It's most unfortunate because it *almost* happened in the work of one of Einstein's immediate predecessors, Heinrich Hertz.

The Lorentz Conundrum: From Euler to Hertz to Phipps

To get off Einstein's train, we must first depart from Lorentz's. Hendrik Lorentz received credit for his famous transformation **E** = **v** x **B**, though in fact Hermann Grassmann derived essential the same thing from Faraday's Law fifty years prior to Lorentz. There is a complementary expression resulting from Ampere's Law, **H** = -**v** x **D**, though this doesn't receive much attention. The problem with these expressions is their suggestion that electrodynamic fields somehow depend on **v**, the velocity of the observer. What is **B** to one observer is **E** to another. Electrodynamic fields don't appear to be facts of nature at all, but the whims of an observer. Something is invariant, something is a fact of nature, but what? Surely the reference frame in which **v** = 0 is the "proper" frame, but what is this frame? The problem really comes down to this: wrt what do we determine **v**?

Most 19th century scientists answered, wrt space. They believed in a special privileged frame from which all motion could be determined in an absolute sense. They believed that space itself somehow provided us with one special reference frame that told us how to determine **v**. The problem is that experiments like the Michelson-Morley experiment (MMX) didn't seem to confirm the idea, throwing science for a major loop. Einstein came along with a new answer: **v** is determined wrt observer. The fact of nature isn't **E** or **B**, but a property of light and the constant 'c'. But both of these solutions accepted the notion of the "inertial frame," the frame which defines "at rest" for all matter. But neither of these answers made a distinction between the driver and the fly, or between the inner tube and the eddy currents near the shore of the stream. Neither considered the matter that created **E** and **B** in the first place. Neither said, **v** is determined wrt matter. Neither employed the magic convective derivative of fluid dynamics.

By the time Einstein waltzed into fame in 1905, Heinrich Hertz had been dead for nearly ten years, departing from this earth at the astonishingly young age of 36. Though scientific history might have been different had he lived, Hertz nonetheless did propose a version of Maxwell's equations based on invariant total time derivatives and the Eulerian concept of the convective derivative.[4] Ironically, everybody missed it, until Thomas E. Phipps, Jr. rediscovered it in the 1970s. Though even then few seemed to grasp the importance of this first-order difference in Maxwell's equations, Phipps has been crusading for over thirty years on this very point. Here are the Maxwell's equations expressed with total time derivatives:

$$\vec{\nabla} \bullet \vec{\mathbf{D}} = \rho \qquad\qquad \vec{\nabla} \times \widetilde{\mathbf{H}} = \frac{d\vec{\mathbf{D}}}{dt} \qquad [3a,b]$$

$$\vec{\nabla} \bullet \widetilde{\mathsf{B}} = 0 \qquad\qquad \vec{\nabla} \times \vec{\mathsf{E}} = -\frac{d\widetilde{\mathsf{B}}}{dt} \qquad [4a,b]$$

As mentioned earlier, both the del operator and the total time derivative d/dt are invariant facts of nature, but a difficulty arises from the fields themselves. How do we determine which fields are invariant? If these are invariant expressions of nature itself, then the "true" value of $\mathcal{D}$ produces the "true" value of $\mathcal{H}$ and likewise with $\mathcal{B}$ and $\mathcal{E}$. Therefore let's declare $\mathcal{D}$, $\mathcal{H}$, $\mathcal{B}$ and $\mathcal{E}$, in script notation, to be the "true" invariant fields and derive the observer-dependent fields **D**, **H**, **B** and **E** from them. We now need to apply vector identities to expand the advective term **v** • del. From any textbook covering vector calculus, we find that for any vector or higher-order tensor **A**:

$$\vec{\nabla}\left(\vec{\mathbf{v}} \bullet \vec{\mathbf{A}}\right) = \vec{\mathbf{v}} \times \left(\vec{\nabla} \times \vec{\mathbf{A}}\right) + \vec{\mathbf{A}} \times \left(\vec{\nabla} \times \vec{\mathbf{v}}\right) + \vec{\mathbf{v}} \bullet \vec{\nabla}\vec{\mathbf{A}} + \vec{\mathbf{A}} \bullet \vec{\nabla}\vec{\mathbf{v}} \qquad [5]$$

$$\vec{\nabla} \times \left(\vec{\mathbf{v}} \times \vec{\mathbf{A}}\right) = \vec{\mathbf{A}} \bullet \vec{\nabla}\vec{\mathbf{v}} - \vec{\mathbf{v}} \bullet \vec{\nabla}\vec{\mathbf{A}} + \vec{\mathbf{v}}\left(\vec{\nabla} \bullet \vec{\mathbf{A}}\right) - \vec{\mathbf{A}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [6]$$

[5] =>
$$\vec{\mathbf{v}} \bullet \vec{\nabla}\vec{\mathbf{A}} = \vec{\nabla}\left(\vec{\mathbf{v}} \bullet \vec{\mathbf{A}}\right) - \vec{\mathbf{v}} \times \left(\vec{\nabla} \times \vec{\mathbf{A}}\right) - \vec{\mathbf{A}} \times \left(\vec{\nabla} \times \vec{\mathbf{v}}\right) - \vec{\mathbf{A}} \bullet \vec{\nabla}\vec{\mathbf{v}} \qquad [7]$$

[6] =>
$$\vec{\mathbf{v}} \bullet \vec{\nabla}\vec{\mathbf{A}} = -\vec{\nabla} \times \left(\vec{\mathbf{v}} \times \vec{\mathbf{A}}\right) + \vec{\mathbf{A}} \bullet \vec{\nabla}\vec{\mathbf{v}} + \vec{\mathbf{v}}\left(\vec{\nabla} \bullet \vec{\mathbf{A}}\right) - \vec{\mathbf{A}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [8]$$

Equations [7-8] offer two completely different expansions of the advective term, and each term in both expansions deserves a physical interpretation, though that is beyond the scope of this paper. For now, let's be satisfied to apply [1-2] and [8] with **A** = $\mathcal{D}$ to Ampere's Law ([4a]) and **A** = $\mathcal{B}$ to Faraday's Law ([4b]):

$$\vec{\nabla} \times \widetilde{\mathsf{H}} = \frac{\partial\vec{\mathsf{D}}}{\partial t} - \vec{\nabla} \times \left(\vec{\mathbf{v}} \times \vec{\mathsf{D}}\right) + \vec{\mathsf{D}} \bullet \vec{\nabla}\vec{\mathbf{v}} + \vec{\mathbf{v}}\left(\vec{\nabla} \bullet \vec{\mathsf{D}}\right) - \vec{\mathsf{D}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [9]$$

$$\vec{\nabla} \times \vec{\mathsf{E}} = -\frac{\partial\widetilde{\mathsf{B}}}{\partial t} + \vec{\nabla} \times \left(\vec{\mathbf{v}} \times \widetilde{\mathsf{B}}\right) - \widetilde{\mathsf{B}} \bullet \vec{\nabla}\vec{\mathbf{v}} - \vec{\mathbf{v}}\left(\vec{\nabla} \bullet \widetilde{\mathsf{B}}\right) + \widetilde{\mathsf{B}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [10]$$

Applying [3a-4a] we find:

[9] =>
$$\vec{\nabla} \times \left(\widetilde{\mathsf{H}} + \vec{\mathbf{v}} \times \vec{\mathsf{D}}\right) = \frac{\partial\vec{\mathsf{D}}}{\partial t} + \rho\vec{\mathbf{v}} + \vec{\mathsf{D}} \bullet \vec{\nabla}\vec{\mathbf{v}} - \vec{\mathsf{D}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [11]$$

[10] =>
$$\vec{\nabla} \times \left(\vec{\mathsf{E}} - \vec{\mathbf{v}} \times \widetilde{\mathsf{B}}\right) = -\frac{\partial\widetilde{\mathsf{B}}}{\partial t} - \widetilde{\mathsf{B}} \bullet \vec{\nabla}\vec{\mathbf{v}} + \widetilde{\mathsf{B}}\left(\vec{\nabla} \bullet \vec{\mathbf{v}}\right) \qquad [12]$$

Assume (for now) that div **v** and the dyad grad **v** are both zero and let **J** ≡ ρ**v**:

$$\widetilde{\mathbf{H}} \equiv \widetilde{\mathsf{H}} + \vec{\mathbf{v}} \times \vec{\mathsf{D}} \qquad\qquad \vec{\mathbf{E}} \equiv \vec{\mathsf{E}} - \vec{\mathbf{v}} \times \widetilde{\mathsf{B}} \qquad [13a,b]$$

[11] =>
$$\vec{\nabla} \times \widetilde{\mathbf{H}} = \frac{\partial\vec{\mathsf{D}}}{\partial t} + \vec{\mathbf{J}} \qquad [14]$$

[10] =>
$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial\widetilde{\mathsf{B}}}{\partial t} \qquad [15]$$

These look like the familiar forms of Ampere's and Faraday's Laws in terms of the reference-frame dependent fields **D**, **H**, **B** and **E**. We still need to eliminate $\mathcal{D}$ and $\mathcal{B}$ in favor of **D** and **B**, though they are nearly equal for the non-relativistic case v << c (see below). Notice how Maxwell's displacement current **J** naturally spills out from the advective term. Maxwell was certainly brilliant for realizing that a changing field in one frame equates to a current in another, but Euler beat him to it by over a hundred years, and it took Hertz to realize it. From repeated use of [13], the relationships $\mathbf{D} = \varepsilon_0\mathbf{E}$, $\mathbf{B} = \mu_0\mathbf{H}$, $\varepsilon_0\mu_0 = 1/c^2$, the definition $\boldsymbol{\beta} \equiv \mathbf{v}/c$, and the vector identity:

$$\vec{\mathbf{v}} \times \left(\vec{\mathbf{v}} \times \vec{\mathbf{A}}\right) = \vec{\mathbf{v}}\left(\vec{\mathbf{v}} \bullet \vec{\mathbf{A}}\right) - \vec{\mathbf{A}}v^2 \qquad [16]$$

$$\vec{\mathsf{E}} = \vec{\mathbf{E}} + \vec{\mathbf{v}} \times \left(\widetilde{\mathbf{B}} - \frac{\vec{\mathbf{v}}}{c^2} \times \vec{\mathsf{E}}\right) = \vec{\mathbf{E}} + \vec{\mathbf{v}} \times \widetilde{\mathbf{B}} - \vec{\boldsymbol{\beta}}\left(\vec{\boldsymbol{\beta}} \bullet \vec{\mathsf{E}}\right) + \vec{\mathsf{E}}\beta^2 \qquad \Rightarrow \qquad \vec{\mathsf{D}} \approx \vec{\mathbf{D}} + \frac{\vec{\mathbf{v}}}{c^2} \times \widetilde{\mathbf{H}} \qquad [17]$$

$$\widetilde{\mathsf{H}} = \widetilde{\mathbf{H}} - \vec{\mathbf{v}} \times \left(\vec{\mathbf{D}} + \frac{\vec{\mathbf{v}}}{c^2} \times \widetilde{\mathsf{H}}\right) = \widetilde{\mathbf{H}} - \vec{\mathbf{v}} \times \vec{\mathbf{D}} - \vec{\boldsymbol{\beta}}\left(\vec{\boldsymbol{\beta}} \bullet \widetilde{\mathsf{H}}\right) + \widetilde{\mathsf{H}}\beta^2 \qquad \Rightarrow \qquad \widetilde{\mathsf{B}} \approx \widetilde{\mathbf{B}} - \frac{\vec{\mathbf{v}}}{c^2} \times \vec{\mathbf{E}} \qquad [18]$$

where the $\boldsymbol{\beta} << 1$ terms are dropped. When we plug [17]-[18] back into [14]-[15], we obtain:

$$\vec{\nabla} \times \widetilde{\mathbf{H}} - \frac{\partial}{\partial t}\left(\frac{\vec{\mathbf{v}}}{c^2} \times \widetilde{\mathbf{H}}\right) = \frac{\partial \vec{\mathbf{D}}}{\partial t} + \vec{\mathbf{J}} \qquad [19]$$

$$\vec{\nabla} \times \vec{\mathbf{E}} + \frac{\partial}{\partial t}\left(\frac{\vec{\mathbf{v}}}{c^2} \times \vec{\mathbf{E}}\right) = -\frac{\partial \widetilde{\mathbf{B}}}{\partial t} \qquad [20]$$

Again for non-relativistic **v**, the second terms on the left are dropped, resulting in the familiar reference-frame dependent forms. What's really fascinating is that to get to these equations, we have to throw away the relativistic terms. These terms, which arise naturally from the Hertzian formulation, require addition postulates like special relativity if we start from the Lorentzian. All the so-called relativistic solutions of electrodynamics are derivable from Hertzian total time derivatives, as will be shown in a subsequent paper.

Now what do the terms grad **v** and div **v** mean physically? How can we justify dropping them? By assuming "inertial frames". Everything everywhere is assumed to "move" wrt to this mythical inertial frame, whether determined by space itself or by some observer. If an "inertial frame" exists, **v** is the same for the fly and the driver no matter who sees them. If **v** is the same everywhere, then obviously grad **v** and div **v** are zero everywhere as well. On the other hand, if motion is determined wrt matter, then **v** is wrt to the earth for the fly and wrt to the car for the driver. **v** is not the same everywhere and neither are grad **v** and div **v**. The dyad grad **v** expresses how **v** changes topologically, especially noticeable at or near the surface of a moving solid object like a car or rocket ship. The div **v** term expresses how much **v** spreads out through space, and we should observe it in regions of vortex expansion or compression. We should notice it when rising from the stargazer to the satellite, which occupy different rotational frames or gauges in spite of the same orbit center and frequency. We should notice it in the Sagnac and Michelson-Gale experiments, where the mirrors do not all reside in the same gauge. These terms have already been used successfully in the context of fluid dynamics, solving problems of vortex and eddy type flows. Arising automatically from the Hertzian form of Maxwell's equations, they can account for many "anomalies" of rotation and explosive or implosive radiation in electrodynamics as well. The Lorenzian form, even with "corrections" for relativistic speeds completely miss terms involving variations in **v** because of the silly marriage to the concept of "inertial frame" and constant **v**.

So once again, what is **v**? It is the velocity of the observer wrt the proper frame at each point in space. The proper frame is NOT the same for all points in space, else **v** would be constant everywhere. The proper frame is based on matter, not space or observer. For the fly, the proper frame is the earth, while for the driver, the proper frame is the car. The proper frame may change wrt time as well wrt space, as conditions change. The values of $\mathcal{D}$, $\mathcal{H}$, $\mathcal{B}$ and $\mathcal{E}$ in the proper frame are the ones that satisfy the Hertzian version of Maxwell's equations [3-4]. Understanding and use of the paper frame is the way to separate facts of nature from mere observations. It is the way off Einstein's covariant train.

Back to the Universal Instant

Assuming the existence of the universal instant and thus the independence of time and space, we arrive at conundrum. How do we determine whether and how much something is moving? After all, this is the fundamental problem of dynamics, and ultimately all of physics. Should our choice of coordinate system or reference frame somehow determine how much something moves? Observer-based Einsteinian physics says yes; space-based absolute frame physics says yes; matter-based Machian physics says no. If the motion of an object is determined wrt matter, its energy is also. Per matter-based physics, the energy of an object does not depend on how we view it, but is invariant. Any physics giving different answers for a particle's energy depending on perspective doesn't satisfy this basic invariance requirement, despite Einstein's lifelong quest and no matter how much we warp space-time.

Can we determine the motion of an object based only on the snapshot of the universal instant? Or must we know something about the previous instant in order to give us a running start into the next instant? If space and time truly are independent, we must answer that motion is determined through either space or time separately. If we can determine the motion at a single point in space over time, then we must also be able to determine the motion within a single instant over space. The two results, obtained independently for any given point and time, must equal. That is exactly what Ampere's and Faraday's Laws state. In Ampere's Law, equation [3b], the right-hand term expresses translation $\mathcal{J} \equiv d\mathcal{D}/dt$ strictly in terms of time change. It is meaningful for a single point in space over time. On the right, the same translation is expressed in terms of space alone. The curl operates on $\mathcal{H}$ over all space with no time dependence whatever. Note that total current field $\mathcal{J}$ is reference frame independent precisely because it can be determined wrt the instant, which is automatically independent of reference frame, unlike Maxwell's displacement **J**. The same comments apply to reference-frame independent rotational term $\mathcal{W} \equiv -d\mathcal{B}/dt$. It is a fact of nature, independent of reference frame, and therefore meaningful in the instant. It can be described as the curl of reference-frame independent $\mathcal{E}$ for any given instant, independent of time.

Finally note that these amazing equations both express a correspondence between translation ($\mathcal{D}$ and $\mathcal{E}$) and rotation ($\mathcal{H}$ and $\mathcal{B}$), consistent with the closed-space idea that translation and rotation are two parts of a whole. All motion is both translational and rotational at once, and these equations show exactly how.

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1 Peter Erickson, *Absolute Time, Absolute Space*, (2007)
2 Greg Volk, "The Meaning of Maxwell's Equations"
3 Peter Woit, *Not Even Wrong*,
4 Heinrich Hertz,