A Counter Example of Einstein's Covariance Principle

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Abstract

Recently, calculation of the deflection angle to the second order also shows gauge invariance in mathematics. Nevertheless, careful analysis shows that this calculation actually implies that the theory is intrinsically not gauge invariant since, for each gauge, the shortest distance r_0 from the sun center is different from that for another gauge. Some argued that r_0 is just a label, but not a physical quantity. This is directly in conflict with Einstein's calculation that the deflection angle is $4\kappa M/r_0$, where M is the total mass of the sun and $\kappa = G/c^2 = 7.425 \times 10^{-29} cmg^{-1}$. Thus, r_0 is not just a label. Hence, Einstein's covariance principle is intrinsically not valid in physics. Thus, logical maturity is currently a major problem in general relativity.

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1. Introduction

Einstein's "principle of covariance" [1, 2] stated, "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." This principle would be valid if all the equations are unrestricted tensor equations. However, Einstein's equivalence principle is not a tensor condition. Moreover, the gauge conditions such as the harmonic gauge are known to be not a tensor condition. Thus, validity of this principle needs to be tested and verified.

Since a physical quantity in a classical theory is uniquely determined by measurement, any physical quantity should be gauge invariant among those gauges of a given frame. Based on that both the Schwarzschild and the harmonic solution produced the same first order deflection of a light ray, Einstein [2] remarked, "It should be noted that this result, also, of the theory is not influenced by our arbitrary choice of a system of coordinates." Obviously, this gauge invariance should have been supported by all physical quantities in all orders of calculations [3].

Recently, it has been shown by Bodenner & Will [4] and Gérard & Piereaux [5] that the deflection angle is gauge invariant to the second order. Moreover, it is interesting that the deflection angle up to the second order is a function of the impact parameter of the light ray. Since such a parameter is gauge invariant, the notion of "genuinely measurable quantities" appears (see Appendix). It was argued that as long as one identifies genuinely measurable quantities, their values cannot depend on coordinates. In other words, the genuinely measurable quantities would serve as a new measuring system. Then, this new system would be gauge independent. However, it was not addressed what would guarantee the existence of such quantities.

Moreover, what would be used to represent the elements of the new system for measurement? If one has to use a gauge dependent representation for such a genuine measurable quantity, then the end result is still gauge dependent and thus such gauge independency could be only just an illusion (see Section 2). Note also that if a gauge is invalid in physics, it may not have physical outcomes. [1)] In this paper, it will be shown that this calculation of the deflection to the second order actually manifests intrinsic gauge non-invariance. Typically, some theorists would make errors in physics, mathematics, and logic (see Appendix). [2)]

2. Deflection of Light to Second Order and the "Covariance Principle"

In the papers of Bodenner & Will [4] and Gérard & Piereaux [5], their calculation is based on the assumption that the metric for a static, spherical coordinate system takes the form,

$$ds^2 = A^2(r)\, c^2dt^2 - B^2(r)dr^2 - D(r)^2\, r^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \quad (1)$$

In fact, the spherical coordinate system (r, θ, φ) is necessarily defined on the Euclidean-like structure [6, 7] of the frame of reference,[3)] and therefore this system is very special and is certainly not an arbitrary coordinate system.

From metric (1), for a weak field, the deflection angle is derived [4, 5] as

$$\alpha(b) = \frac{4GM}{c^2 b}\left[1 + \frac{15\pi}{16}\frac{GM}{c^2 b}\right] + O\left(\frac{m^3}{b^3}\right), \qquad \text{where} \quad b = \frac{D(r_0)}{A(r_0)} r_0 \qquad \text{and} \qquad m = \frac{GM}{c^2} \tag{2a}$$

Here, $\kappa = G/c^2 = 7.425 \times 10^{-29}\,\mathrm{cmg}^{-1}$, M is the total mass, and "r_0" is the closest (Euclidean-like) distance from the center, "b" as the impact parameter. For the first order, as Einstein obtained, one has

$$\alpha(b) \sim 4m/b \sim 4m/r_0 \tag{2b}$$

since $A(r) \sim D(r) \sim 1$. From (2a), it is clear that, A(r), B(r)dr, and D(r)r are gauge invariant. It follows that b is gauge invariant mathematically and so is $\alpha(b)$.

To see the details, let us consider the harmonic, the isotropic and the Schwarzschild metrics explicitly as follows:

$$ds^2 = \frac{\rho - M\kappa}{\rho + M\kappa} c^2 dt^2 - \frac{\rho + M\kappa}{\rho - M\kappa} d\rho^2 - (\rho + M\kappa)^2 (d\theta^2 + \sin^2\theta d\varphi^2), \tag{3a}$$

where

$$\xi = \rho \sin\theta \cos\varphi, \qquad \zeta = \rho \sin\theta \sin\varphi, \qquad \eta = \rho \cos\theta, \qquad \text{and} \qquad \rho > M\kappa. \tag{3b}$$

$$ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\, c^2 dt^2 - (1 + M\kappa/2r)^4 (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2) \tag{4a}$$

where

$$x = r \sin\theta \cos\varphi, \qquad y = r \sin\theta \sin\varphi, \qquad z = r \cos\theta, \qquad \text{and} \qquad r > M\kappa/2. \tag{4b}$$

$$ds^2 = (1 - 2M\kappa/r')c^2 dt^2 - (1 - 2M\kappa/r')^{-1} dr'^2 - r'^2 d\theta^2 - r'^2 \sin^2\theta d\varphi^2, \tag{5a}$$

where

$$x' = r' \sin\theta \cos\varphi, \qquad y' = r' \sin\theta \sin\varphi, \qquad z' = r' \cos\theta, \qquad \text{and} \qquad r' > 2M\kappa \tag{5b}$$

Then, the diffeomorphism between metrics (5) and (3) is

$$r' = \rho + M\kappa, \qquad \text{for} \qquad r' > 2M\kappa. \tag{6a}$$

and the diffeomorphism between metrics (5) and (4) is

$$r' = r(1 + M\kappa/2r)^2 \qquad \text{for} \qquad r' > 2M\kappa. \tag{6b}$$

Equation (6) shows that the Euclidean-like structures are different for different gauges. Moreover, eq. (6) implies that the impact parameter b is gauge invariant. However, (6) itself means that the shortest distance r_0 from the center is <u>not</u> gauge invariant.

In conclusion, general relativity is <u>intrinsically</u> not gauge invariant, and this is consistent with Einstein's equivalence principle, which requires the physical gauge being unique for a given frame [1, 8]. Thus, Einstein's covariance principle is invalid because these physical quantities are not gauge invariant as Einstein requires.

Moreover, although eq. (6) implies that the first order would be gauge invariant in terms of measurements as shown by eq. (2b), gauge invariance of b and $\alpha(b)$ is only in mathematics. To be explicit, the deflection angle is respectively,

$$\alpha(b) = \frac{4m}{d_s}\left[1 + (\frac{15\pi}{16} - 1)\frac{m}{d_s}\right] + O\left(\frac{m^3}{{d_s}^3}\right), \qquad b = \frac{D(d_s)}{A(d_s)} d_s = \frac{d_s}{(1 - 2m/d_s)^{1/2}} \tag{7a}$$

$$\alpha(b)=\frac{4m}{d_h}\left[1+(\frac{15\pi}{16}-2)\frac{m}{d_h}\right]+O\left(\frac{m^3}{{d_h}^3}\right), \qquad b=\frac{D(d_h)}{A(d_h)}d_h=d_h\frac{(1+m/d_h)}{(1-m/d_h)} \tag{7b}$$

$$\alpha(b)=\frac{4m}{d_I}\left[1+(\frac{15\pi}{16}-2)\frac{m}{d_I}\right]+O\left(\frac{m^3}{{d_I}^3}\right), \qquad b=\frac{D(d_I)}{A(d_I)}d_I=d_I\frac{(1+m/2d_I)^3}{(1-m/2d_I)} \tag{7c}$$

where d_s, d_h, and d_I are respectively the closest distance (in their respective Euclidean-like structure) [4, 5]. From relation (7), it is even clearer that b and r_0 cannot be both gauge invariant.

Moreover, the deflection of light to the second order is not really gauge invariant if the issue of measurement is included. When a number is obtained for the closest distance r_0 from a measurement, a question is which of (7a), (7b), or (7c) should be applied. Then, if r_0 represents d_i of the gauge, one has

$$b \approx m + r_0 , \tag{8a}$$

or

$$b \approx 2m + r_0 \tag{8b}$$

depending on what gauge is valid for reality. Thus, such gauge invariance has no practical meaning unless the first order approximation of the metric is known. Some incorrectly claimed that "r_0" is only an arbitrary label (see Appendix). However, the deflection of light has determined that $\alpha(r_0)$ is accurate up to the first order [1, 2], and thus r_0 is not arbitrary.[4)]

Bodenner & Will [4] argued that *b* is the ratio of angular momentum J to energy E for the photon (J = b E). However, to measure J similar problems would occur, and thus they do not have a valid argument [9]. Moreover, invalidity of the gauge invariance can be seen explicitly if the light speeds are calculated, one obtains

$$\frac{dr'}{dt}=c\left(1-\frac{2M\kappa}{r'}\right)=\frac{d\rho}{dt}=\frac{dr}{dt}\left(1-\frac{M\kappa}{2r}\right)\left(1+\frac{M\kappa}{2r}\right), \tag{9a}$$

$$\frac{r'd\theta}{dt}=c\left(1-\frac{2M\kappa}{r'}\right)^{\frac{1}{2}}=\frac{\rho d\theta}{dt}\left(1+\frac{M\kappa}{\rho}\right)=\frac{rd\theta}{dt}\left(1+\frac{M\kappa}{2r}\right)^2, \tag{9b}$$

$$\sin\theta\frac{r'd\varphi}{dt}=c\left(1-\frac{2M\kappa}{r'}\right)^{\frac{1}{2}}=\sin\theta\frac{\rho d\varphi}{dt}\left(1+\frac{M\kappa}{\rho}\right)=\sin\theta\frac{rd\varphi}{dt}\left(1+\frac{M\kappa}{2r}\right)^2 . \tag{9c}$$

Thus, the light speeds ($ds^2 = 0$), calculated from the Schwarzschild solution, the harmonic solution, and the isotropic solution, are explicitly not invariant although they are diffeomorphic solutions. (The light speeds are calculated according to the respective Euclidean-like structures.) Perhaps due to such a violation, some theorists [11] insist on the measured light speed in vacuum is always c although Einstein has remarked that the light speeds are no longer a constant since a light ray bends [1, 2].

4. Discussions and Conclusions

This analysis further confirms that general relativity is not a gauge invariant theory. This can be traced back to Einstein's equivalence principle [1-3]. 5) Moreover, this explicit calculation and analysis unequivocally rejects the "covariance principle". Based on Einstein's equivalence principle, it has been shown [6, 7] that the theoretical framework of general relativity has given a definite physical meaning to space-time coordinates. Thus, the criticisms of Whitehead (see Appendix) are actually toward Einstein's theory of measurement, but not general relativity. As Zhou [12] pointed out, physical quantities such as vectors and tensors are measurable and their components are expressed in terms of coordinates. Thus, the gauge does matter. The claim of arbitrariness of physical gauges (see Appendix), if put Zhou's [12] statement in a clear language, just make no sense in physics.

In fact, logical competency is a problem in general relativity starting at least from Einstein's covariance principle [3]. First of all, a coordinate system in physics is related to physical measurements [13], and experimentally α (r_0) is accurate to the first order [1, 2]. *In Einstein's theory, coordinates are always more than just a label, since arbitrary labels are insufficient to obtain the spherical symmetry of a metric* [1-3].

Invalidity of the covariance principle would imply that Einstein's theory is incomplete. Fortunately, it has been proven, *independent of the Einstein equation,* that the Maxwell-Newton Approximation is valid for the first order approximation of a physical gauge for gravity induced by massive sources [14] such that the binary pulsar experiment can be explained satisfactorily [15, 16]. This implies that eq. (8b) is valid, and thus the second order of the deflection angle can be obtained from measuring the shortest distance r_0 from the sun center. Clearly the claim of r_0 being an arbitrary label (see Appendix) is proven invalid. Although the Royal Society of London is the first finding out inconsistency in Einstein's theory [17, 18], they stood on the wrong side [3, 9]. The covariance principle was a major obstacle that separates general relativity from the rest of physics [3, 19], and thus prevents unification. The experiment on the local light speeds would verify the Maxwell-Newton Approximation [6].

Fundamental concepts in a great theory are often difficult to grasp [20]. To mention a few, this happened to Newton, Maxwell, Planck, Schőrdinger, and C. N. Yang [21]. Einstein is simply not an exception. Unlike Newton, Einstein did not have adequate background in mathematics, and this affects the logical structure of his theory. He believed the solutions with different gauges as equally valid [2], but did not see that his covariance principle is inconsistent with his notion of weak gravity [17]. Nevertheless, Einstein is a great theorist since the implications of general relativity such as the need for unification have been discovered and verified [22, 23]. Einstein's accurate predictions created a faith on his theory. However, theoretical developments [23, 24] and NASA's discovery of the Pioneer anomaly imply that Einstein's theory is clear inadequate [25, 26].

Moreover, Einstein's conceptual errors are becoming obstacles to progresses. To mention a few, theoretically anything related to the covariance principle is glossed over. The Stanford experiment Gravity Probe-B on precessions ignores the problem of covariance. Prominent theorists such as Straumann [27] Wald [28], and Will [29] failed responding to the inconsistence [17] discovered since 1959. [6)] Einstein's equivalence principle and its consequences are essentially ignored [8, 16]. Thus the need of unification being a natural consequence of general relativity was overlooked since 1916 [30, 31]. Zhou's experiment [32] on local light speeds has been practically abandoned. It is hope that this paper would facilitate making further progresses.

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Appendix: Invalidity of Einstein's Principle of Covariance and its Supporting Arguments.

Based on Einstein's "principle of covariance", some argued that diffeomorphic solutions (i.e., they are equivalent in mathematics) should be physically equivalent. In Einstein's "principle of covariance", what has been missing is the relation between coordinates and measurements. A difference between physics and mathematics is that a coordinate system in physics is related to measurement and its method, whereas a coordinate system in mathematics need not be related to these.

For the covariance principle, Einstein's supporting arguments [1] are as follows:

> "That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences."

Note that the meaning of measurements is crucially omitted. In order to predict events, one must be able to relate events of different locations in a definite manner [3]. However, the physical meaning of the coordinates would implicitly depend on the gauge, which may or may not be physically realizable [9]. As shown by relation (6), the Schwarzschild and the isotropic solutions cannot be both physically realizable. *A basic error is assuming that physical results can be obtained from any gauge.*

Einstein's theory of measurement, which is invalid in physics [19], is based on invalid applications of special relativity [1] such that the notion of local distance in Riemannian geometry could be justified. Whitehead [33, p. 83], strongly objected,

> "By identifying the potential mass impetus of a kinematic element with a spatio-temporal measurement Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion, when it is confronted with the actual conditions of our perceptual knowledge. The potential impetus shares in the contingency of appearances. It therefore follows that measurement on his theory lacks systematic uniformity and requires a knowledge of the actual contingent physical field before it is possible."

Einstein's theory of measurement is also inconsistent with the observed light bending [7, 34, 35].

In a way, Einstein's equivalence principle is an ingenious answer to the question of measurement for a physical Riemannian space. For a physical space, from the invariant ds, his principle provides a measurement of the space contractions and the time dilation [6, 7]. Einstein's error is that he overlooked that his equivalence principle provides only a *dynamic* method of measurement for space contractions since the measurements are done in a movable local space resting but under the influence of gravitational acceleration, but are not done in the frame of reference with a *static* method of attachment.

Moreover, a physically realizable Euclidean-like structure is operationally defined in terms of spatial measurements essentially the same as Einstein defined the frame of reference for special relativity [36]. If the measuring rods are attached to the frame of reference, since the measuring rods and the coordinates being measured are under the same influence of gravity, a Euclidean-like structure emerges as if gravity did not exist [6, 7]. Hence, independent of the space-time metric, a physical space must have a frame of reference with a realizable Euclidean-like structure. A good example is the metric of a rotating disk [18]. Moreover, since a gauge may or may not be physically realizable, the issue is really whether a gauge is valid in physics.

Some theorists argued,"… deflection of light bending is described by solving equations of motion in some background curved spacetime. The result must be independent of any coordinates since the light curve is a geometric object. The physical observation should also be possible to measure the angles which are independent of coordinates. Coordinates are just representations of geometric objects. I think that any physical argument can be expressed in a very clear way especially through mathematics." However, this argument actually involves some implicit assumptions.

Physical coordinates are representations of geometric objects since such coordinates are based on real measurements. However, the coordinates for a gauge is based on assumed measurements, which may or may not be realizable. Thus, the coordinates for a gauge need not be representing geometric objects. A crucial criterion is that physics involves actual measurement, whereas mathematics involved only assumed measurements or no measurements at all.

To justify the "covariance principle", it was also argued [37],

> "If one makes the mistake of attributing incorrect physical properties to one's coordinates -- for example, assuming that because two objects in different coordinate systems are both labeled r, they must measure the same distance, or

assuming that any coordinate labeled *r* must measure proper radial distance -- then one can mistakenly conclude that computations in different coordinate systems disagree. But as long as one identifies genuinely measurable quantities, their values cannot depend on coordinates."

However, it has been shown, for the case of the precession formulas, such "genuinely measurable" quantities do not exist at all. Moreover, for quantities such as b, invariance could be only a illusion since it is not clearly supported by measurements.

Some theorists simply cannot tell the difference between mathematics and physics, which is based on measurements. They discuss about physics without addressing the issue of measurement, and make errors without being aware of them [3, 19, 8-16]. [2)] An interesting example is a "Board Member Comments" of the Royal Society [38] as follows:

"… The conclusion that "the shortest distance r_0 from the center of the sun is not gauge invariant, on the other hand, is silly. The value of a coordinate is, of course, not "gauge invariant" if one chooses to use the same letter to denote two different coordinates. But the value of a coordinate is not a "distance," either; it is an arbitrary, human-made label, and nothing more. [7)] … The impact parameter b, on the other hand, is a physical observable, a proper distance, [7)] and does not depend on one's arbitrary choice of labels. This is the object one actually measures when one measures a distance. The fundamental error here is what philosophers call reification -- the treatment of a hypothetical construct as if it were a concrete physical entity. The space around the sun does not come equipped with little labels giving the coordinates of points. A coordinate system is, rather, an arbitrary human-made set of labels, that can be chosen in any way one desires. [7)] To compute a real physical quantity, one must relate one coordinate-independent object to another. Coordinates are useful intermediate quantities, but they disappear at the end. "

To justify the invalid invariance, this board member acts irrationally. He claimed the shortest distance r_0, is just a label. Moreover, according to his understanding of coordinates, r_0 should be considered as a "hypothetical construct". This is absurd since r_0 is a physical quantity as shown in Einstein's calculations [1, 2]. Experimentally, $\alpha(r_0)$ has been verified as accurate to the first order. If the coordinates were just arbitrary labels, where the physical meaning of the impact parameter b comes from?

However, physics requires that there is only one measured length for the shortest distance r_0 from the sun center just as only one length for the impact parameter b. This board member also tried to justify logical errors with an incorrect assumption. However, there are physical principles such as the principle of causality that must be satisfied [15, 23]. Note that b is independent of only some gauge choices, which are not *arbitrary*. The meaning of coordinates is implicitly included in all physical quantities. Thus, the right question should be what the correct physical gauge is.

This board member claimed, "To compute a real physical quantity, one must relate one coordinate-independent object to another" after reciting Einstein's erroneous view that has never been put into practice [1-3, 6, 19]. Apparently, this board member does not really know what this means. If the deflection angle must be calculated in term of the coordinate-independent object such as the impact parameter b, what should be used to calculate b, and so on …. As expected, this board member has a problem in logical thinking because he claimed r_0 arbitrary although $\alpha(r_0)$ has been verified as accurate up to the first order. A basic problem is however, that he absurdly believed that any gauge non-invariant quantity were not a concrete physical entity.

Also, unlike Zhou [33] this board member [8)] failed to appreciate Einstein's equivalence principle that implies the covariance principle invalid [19]. This board member failed to see that a measurement of b necessarily involves r_0. Thus, an actual measurement of "b" requires a choice of gauge between (8a) and (8b). He also tries to justify his errors with invalid assumptions. Although nobody knows what the physical gauge is, [9)] $\alpha(r_0)$ has been verified as accurate to the first order [1, 2, 39]. Moreover, to obtain r_0 to the first order, it is sufficient to know whether the first order approximation of a metric is valid.

Apparently, this board member and his colleagues did not know that the Maxwell-Newton Approximation has been proven as the valid first order approximation [14] because of the need to explain observations of the binary pulsars [15, 16]. Consequently, to claim r_0 as arbitrary is now clearly nonsense. Thus, their over confidence is due their logical errors and collectively ill-informed. Nevertheless, had they acted carefully according to logic demands, they could have avoided such errors. However, since the covariance principle is a product of inadequacy in logic, its followers inevitably have the same kind of problem.

ENDNOTES

1) A theory in physics must be supported by experiments, in addition to being logically self-consistent.

2) In order to defend an error, it would inevitably create more errors, logical errors in particular.

3) A Euclidean-like structure is a mathematical notion that the Pythagorean Theorem is satisfied. This notion is independent of the space-time metric, and thus can be different from the Euclidean space, which is based on physical measurements.

4) Zhou [31, 39] has proposed that the harmonic gauge condition to be necessary for an asymptotically flat metric, and thus rejected the Schwarzschild solution. He also correctly argued that the "covariance principle" is not valid in physics.

5) A satisfaction of Einstein's equivalence principle requires that a time-like geodesic must represent a physical free fall. The Einstein-Minkowski condition [2] requires that the co-moving local space must be locally Minkowski. Thus, the difference between Einstein's equivalence principle and Pauli's version is not just a matter of philosophy as some speculated [15].

6) The editorial of Royal Society [18] still stands by the invalid criticism of Bondi et al [17] on Einstein's weak gravity.

7) This board member considers, "the value of a coordinate is not a 'distance,' either; it is an arbitrary, human-made label, and nothing more". On the other hand, he considers b as "a proper distance", which is a distance in terms of a Euclidean-like structure, but not a distance that Einstein defined in terms of the metric. Thus, it is not clear what he meant as a distance.

8) This board member of the Royal Society has problems in at least three areas, namely: physics, mathematics, and logic.

9) For an unphysical gauge, since the coordinate system is not compatible with measurements, r_0 is not a physical distance.

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