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BINARY STARS AND THE VELOCITY OF LIGHT

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Some scientists, such as Fox(1) and Moon and Spencer(2), have questioned the validity of some of the experimental evidence that is usually interpreted as supporting the second postulate of special relativity, the postulate that the velocity of light is independent of the velocity of its source. Moon and Spencer(2) have shown that the introduction of a certain Riemannian metric can explain observations of binary stars without requiring the second postulate. The purpose of the present note is to suggest a new metric which appears to be an improvement over the one suggested by Moon and Spencer(2), and which makes their argument more convincing.

Moon and Spencer(2) suggested that the usual distance r, between a binary star and the earth, be replaced by a Riemannian distance s, given by the formula

$$s = 2R \tan^{-1}(r/2R) \tag{1}$$

where R is a space constant, called the radius of curvature of the space. This new measure of distance was then used to alter the value of a characteristic constant Γ in an expression for the apparent angular position of the binary star, the value of Γ being altered in accordance with the formula

$$\frac{\Gamma_E}{\Gamma_R} = \frac{(r/2R)\,[1 + (r/2R)^2]}{\tan^{-1}(r/2R)} \tag{2}$$

where Γ_E is the Euclidean value of Γ, and Γ_R the value corresponding to the Riemannian metric (1). If the value of Γ were greater than unity, multiple images would be observed if the velocities of the source and the light were directly additive, while if Γ were much less than unity the motion would appear to be a simple rotation. By the use of equation (2) the value of Γ_R was made to be much less than unity, for all except a few peculiar spectroscopic binaries (see Table IV of (2)).

The modification that is now suggested is to replace the Moon and Spencer metric (1) by the following distance function:

$$d = P \tanh(r/P) \tag{3}$$

where P is a new space constant.

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How should the value of P be chosen, to make the new metric correspond in some way to the old one? One possibility would be to choose P so that the maximum value of d is equal to the maximum value of s (for $r = \infty$); this would mean that P should be taken equal to πR. If this value is chosen, it turns out that the short-distance discrepancy from the Euclidean distance is less, by a factor of $4/\pi^2$, than for the Moon and Spencer metric. If, on the other hand, we choose P to give the same short-distance accuracy as for the Moon and Spencer metric, we would take P equal to 2R. This is because the first two terms of the series for tanh x are the same as the first two terms of the series for $\tan^{-1} x$, namely $x - x^3/3$.

Using the new metric (3), the change in the quantity Γ is given by the expression

$$\frac{\Gamma_E}{\Gamma_P} = \frac{(r/P)\cosh^2(r/P)}{\tanh(r/P)} \tag{4}$$

where Γ_P is the value of Γ corresponding to the metric (3). If $P = 2R$, the value of the right side of (4) is approximately the same as the right side of (2) for small values of r, but (4) increases much more rapidly than (2) as r increases. For example, for $r = 6R$, the value of Γ_E/Γ_R is 24.02, whereas the value of Γ_E/Γ_P is 305.6 for the corresponding value $r = 3P$.

Using the new metric (3), and taking P to be 10 light-years to correspond to Moon and Spencer's value of R = 5 light-years, the maximum value of Γ_P, for any of the stars listed in Moon and Spencer's Table IV, would be less than 0.1. Such a small value would show little apparent difference from a pure rotation, and this metric therefore seems to support the thesis of Moon and Spencer[2] that observations on binary stars do not necessarily prove the truth of Einstein's second postulate.

It is interesting to examine the new metric a little further, and to observe that, if two Euclidean distances r_1 and r_2 are added (or subtracted) directly, the value of the new metric corresponding to the sum (or difference) is

$$P\tanh\left(\frac{r_1 \pm r_2}{P}\right) = \frac{d_1 \pm d_2}{1 \pm (d_1 d_2/P^2)} \tag{5}$$

where d_1 and d_2 are related to r_1 and r_2, respectively, by equation (3). The right side of equation (5) has an obvious similarity of form to the well-known expression for the composition of two velocities in special relativity. In terms of special relativity, it is possible to consider a measured velocity v as a measure of the form

$$v = c\tanh(u/c) \tag{6}$$

where c is the velocity of light, and u/c is the quantity that is sometimes called the rapidity[3]. If two velocities v_1 and v_2 are combined in accordance with the Lorentz transformation, the two corresponding rapidities are added (or subtracted) directly[3,4], and it is clear from equation (6) that the maximum possible value of v is c, the velocity of light.

It is hoped that the ideas presented here may lead to a reconsideration of Einstein's second postulate. If that postulate is indeed found to be invalid, perhaps the similarity between the new space metric (3) and the measure of velocity in special relativity (6) may suggest some possible extension or modifications of the theory.

References

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