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MILNE'S KINEMATIC RELATIVITY
IDEAS OF COSMOLOGY: A PHILOSOPHER'S SYNTHESIS

Revised version (2011) of a paper printed in:
Duffy & Wegener, eds.: 'Recent Advances in Relativity Theory', vol.1
Hadronic Press 2000 (ISBN 1-57485-047-4)

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1. COSMOLOGY, A SCIENCE?

All science is cosmology! Sir Karl Popper [1958]

In our time, ***Cosmology*** is generally acknowledged to be *the science of the universe*. But what is the universe? Is it being, entity, or substance? Is it nature itself, ultimate reality? How do we overcome the desperate difficulties of speaking sensibly of everything at once? And in what sense can such an elusive subject be the object of anything like a real science? Can we avoid the danger of assuming either too little or too much even before we begin? Finally, the universe is one, or unique: how can it then give rise to a legitimate science at all? Such questions cry for their rational answers but, of course, it is easier to ask than to reply. Hoping for better progress later, we shall start by making our language a bit more precise.

Neglecting the traditional mythological and metaphysical implications of cosmology which are better left to the humanities for investigation, we prefer to concentrate on those senses of the term 'universe' which are more often brought up in relation to modern science; we shall therefore primarily use the term in the plural, to betoken scientific world-models. Nevertheless, it cannot be ignored that we all, for the purpose of preserving our lives in spite of an often hostile world, develop for ourselves what we might call: a practical metaphysics. We thus mostly accept that humans are mortal beings, that nature appear to be governed by general laws and, hence, that what happens may be explained as the effects of natural causes.
So we feel reasonably convinced that if we venture to jump into a vulcano (some like it hot!) we shall hardly avoid to burn up, and that if we kick a stone we shall probably hurt our foot. Thus we are tempted to assume the existence of a unique ordered entity: ***The Universe***.

Now, do such reasons entitle us to claim that *there is* an Ultimate Material Reality, that the Universe *must* possess a definite Formal Structure, that the Course of Nature *must* be ruled by Law, and that the Principle relating Cause and Effect *is* valid without exception? This question is not an issue of science, not even of common sense, but of philosophy, and to answer it in the affirmative would presuppose that our own practical metaphysics which is private, or particular, does in fact entail a theoretical metaphysics which is public, or universal. For people with an innate distaste for ontology this consequence may seem rather appalling: obeying some rules of behaviour clearly differs from adopting their casual conceptualization. Without committing myself to the entire critical philosophy of Kant, I must admit that his famous distinction between *reality-for-us* and *reality-in-itself* seems to be of relevance here, practical metaphysics relating to the first, and theoretical metaphysics treating of the latter. Armed with this distinction we are able to separate a *practical metaphysics* which is useful, even unavoidable, from a *theoretical ontology* which is redundant, and often odious.

But are we not, for the purposes of science, in need of a very abstract and general notion of the universe, one which might serve as an ultimate instance of reference and whose function it is to promote the final unification of the disparate elements of human experience? It seems that we are, and here again we can benefit from Kant's transcendental philosophy. According to Kant there are two types of concepts: *ideas* whose function is *regulative*, viz. to integrate our experience into a totality, and *categories* whose function is *constitutive*, viz. to differentiate the various kinds of our experience; with reference to the universe he speaks of cosmological ideas in the plural, but probably he refers to different aspects of the same. Now I do not want to let my own position depend on the right interpretation of Kant; however, I believe that I speak in accordance with the general tenor of his thinking when I propose the following *critical idea* of the universe, in contradistinction to the copious ideas of meagre value implied by the traditional profusion of pretentious metaphysical ontology.

The *critical idea of the universe* which I am here going to advocate is distinguishable from all ontological ideas of the same by the fact that it presents *the universe as an unknown X*, an unique, absolute and ultimate referent ("thing-in-itself") devoid of any specifiable properties. It serves the purpose of providing a minimum foundation for the stand of scientific realism, and in this way it seems *correlative to truth conceived as a regulative idea* in the sense of Popper. If a theory is to be true it must be true of an object which it represents and to which it refers, but as theories may be falsified, and never verified, we shall never be able to recognize truth; so it would be rash to hypostasize an ontology by ascribing definite properties to the universe. In the following I shall write *Universe* whenever I refer to some metaphysical idea, whereas I shall write *universe* in order to denote the idea of a totality devoid of intrinsic properties. With this convention I follow the proposal of Harrison (1981), the only difference being that he restricts the spelling with a capital U to the singular, saving the spelling without capitals for the plural, whereas I allow both spellings to be used both in the singular and in the plural.

My reason for this divergence is twofold: (1) The Universe of a metaphysical praxis needs no name as long as it is not deliberately conceptualized into some metaphysical theory. Now, although the proponent of a metaphysical theory will inevitably tend to believe that his Universe (capital U, singular) is the only true one, history knows innumerable examples of other Universes (capital U, plural) which are not merely different, but outright incompatible. So the plural is needed too. (2) Although it is reasonable to identify different world-models with different kinds of universes (no capitals, plural), we shall nevertheless refer to a unique instance whenever a specific model is falsified by confrontation with experience and we want to make clear that this model does not properly represent the universe (no capitals, singular), and it is precisely to this purpose that the critical idea of an empty referent, an *X*, is needed. The advantage of the present approach is that we can retain a critical realism although we are incessantly reminded that our best ideas and theories may turn out to be false.

I shall henceforth define ***cosmology***, *the science of the universe* (in singular), as the art of inventing, or constructing, universes (in plural), i.e. as *the art of designing world-models*. It is an ***art*** because its practitioner, the constructor of world-models, must exercise *phantasy*. It is a ***science*** because the free exercise of phantasy is kept within certain bonds stemming from *the obligation to obey the codes of scientific method* which are: (α) to pay due regard to formal *consistency*, and (ω) to respect the results of repeated *observation* and *experiment*. I shall further define a *world-model* as the material interpretation of a formal calculus which is taken to represent the structure of the universe. As we are forever unable to compare the *structure for-us*, viz. as *representation*, with the *structure in-itself*, viz as *reality*, except in so far as the representation may be tested pointwise against experience, it is consistent to say that *the structure of the universe in a sense has no real existence except as representation*. However, it must be admitted that *absolute identity is possible*, viz. as identity of structure; this fact opens the hypothetical possibility that our cosmological theory could be a hit in the sense that the structure of the model materializing our theory might, eventually, be identical to the "real" structure of the "actual" universe. It should nevertheless be clearly recognized that, as soon as we depart from the *hypothetical* way of expressing ourselves in favour of a *categorical* manner of speaking, when we hazard ascribing a final structure to the universe, we transgress the limits of science and give way to free speculation. So we have this scheme:

Metaphysics: explains reality-in-itself

The Universe: an imaginary totality │ *Other Universes: equally subjective*

Cosmology: describes reality-for-us

the universe: an unknown referent │ *other universes: scientific world-models*

It is a commonplace to distinguish *theoretical* cosmology from *empirical* cosmology. The difference is mainly considered as being a matter of orientation; however, in the present context it is more illuminating to take a clear stand by presenting it as a question of principle. In this way our presentation offers a comment to Bondi (1961) who confronts inductive or extrapolating cosmology with deductive cosmology, i.e., cosmology founded on principles. For the sake of distinctness we shall provisionally identify *empirical cosmology* as *inductive* and *theoretical cosmology* as *deductive*. By tradition, most cosmology has been inductive. The standard procedure has been to accept that the universe is governed strictly by so-called laws of nature and to build up models of the universe by generalizing those laws inductively, this procedure leading to questions about the relativistic invariance, or covariance, of laws.

Without denying the relevance of such issues it should suffice to recall the famous Bohr-Einstein controversy in order to confirm that the idea of deterministic laws dormant in nature is certainly not without problems; further, the idea of stochastic laws is not much worth after all, for how can we accept a "law" which is compatible with the most extraordinary deviations?

Moreover, most *empirical cosmology* is produced by a kind of "piecemeal engineering" that makes its structure resemble a patchwork of conflicting elements: thus, even when principles are introduced it is hard to check their independence, and their consistence remains hazy.

I shall here venture to advocate ***theoretical cosmology*** as *the* physics of *the* cosmos, i.e. as a deductive science which, exploiting a formalism based on definitions and principles, considers *the ultimate structural totality of the universe* to be its natural object of research. This stance is not meant to imply that there is no place for empirical cosmology: giving the lead to theory as the motor of research makes the need for effective brakes the more urgent. Hence *the proper function of empirical evidence* is to act as a needful brake on theoretical speculation in order that it shall not forfeit its foundations in our ordinary human experience. For that very reason there is an intimate relation between our *practical metaphysics* and our *empirical cosmology*, both giving important contributions to our so-called "world-picture". Now, what can be said of *theoretical cosmology*, if we are to abstain from principles of reason? Not much, I am afraid, but that is no reason to be silent - cf. my [1994] (this book, ch.7).

So let it be said: ***á priori it is a precept of natural science*** that its aim be threefold, viz., **(1)** ***to describe the present*** in order **(2)** ***to predict the future*** and **(3)** ***to explain the past***; therefore the triple division of time into past, present and future is indispensable to science. *It is simply a gross misunderstanding to believe that* ***TIME*** *- the basic feature of all existence - can be deduced from or explained by science, as it is a fundamental presupposition of science; but neither should we consent to those forms of science which are determined to ignore time. The least to be expected from a science which is faithful to experience is that its conceptual representation of nature remains compatible with the passage, hence also the direction, of time.* But precisely that is not the case with those models of the universe which represent reality as some multi-dimensional "continuum" existing timelessly, like a piece of crystalline mineral. That attempts to soften them up by introducing a ***temporal order*** have met with success may not obscure the fact that nothing less than the full acceptance of a ***temporal flux*** will do.

It is astonishing and one of the paradoxes of history that, faced with today's science, we shall have to insist on the factual evidence born out by everyone's own bodily senses.

2. MILNE'S KINEMATIC RELATIVITY

The theory with its recent developments now covers not only cosmology but a great part of theoretical physics, and the extent of its achievements is greatly to be admired.

Sir Hermann Bondi [1961[3]]

One cosmologist, probably the only one of his days, who recognized the fundamental importance of time to natural science was the British mathematician E.A. Milne of Oxford. His ambition was to present cosmology as a purely deductive system, without any loopholes.

As noticed by North [1965], at least some of Milne's more inflated claims were untenable, but he nevertheless clearly concedes that Milne "seems always to have hovered on the verge of a perfectly reasonable hypothetico-deductive account of scientific explanation" (p.303).

For example, Milne claimed to have promoted the law of gravitation from the status of an empirical result to that of a mathematical theorem. On account of the hypothetical elements which were latent in this deduction, his own collaborator McCrea (1953) denounced this claim.

Nevertheless, as he also admitted: "Milne *did* show that, starting from only a few general hypotheses about the contents of the universe as a whole, it is possible to infer properties that have something of the character of physical laws" (p.337). However, this is not very surprising, according to McCrea: "We know from other theories that we can start with physical laws and construct model-universes; if desired, we can reverse the mathematics" (p.338).

Milne constructed an extremely simple model of the universe based on his ***kinematic relativity*** which was developed as a challenge to the special and general theories of Einstein. He took his point of departure in the fact that *simple perception* will suffice to convince all conscious beings of *the passage of time*: time is a necessary concomittant of all existence. The basic ingredient of his world-model, then, is an abstract image of the human consciousness which we shall here call a *particle-observer*, or an *observer-particle*, or simply: a *monad*.

That Milne's kinematic cosmology can be viewed as a Leibnizian monadology translated into mathematics was pointed out by the French historian of ideas J. Merleau-Ponty [1965]; so here is a case where metaphysics may have proffered inspiration to scientific cosmology. The monads of the kinematic universe are primarily characterized by a capacity for continual communication: they are supposed to exchange information mediated by signals all the time.

In what follows I shall offer my own interpretation of Milne's ideas, trying to avoid those pitfalls which have most often evoked a censure from scientists of a more traditional stance. My presentation will, to begin with, concentrate on the two basic problems of time-keeping: (a) that for a pair of particle-observers, and (b) that for an infinite set of particle-observers.

Although I cannot recall any immediate reference to conventionalism, Milne is certainly a genuine conventionalist in the spirit of Poincaré. Thus, according to Milne, any mechanical device which associates events with a scale of ever increasing numbers may serve as a clock (a clock in this sense is just a local one-one relation between events and the set of real numbers).

The *first problem of time-keeping* is stated as thus: Granted that some particular observer *P* has set up an arbitrary clock for himself, is it then possible for another observer *Q* to set up a clock congruent to the first one in some specified sense irrespective of their relative motion? It should be clearly understood that the universe of discourse is here supposed to contain *P & Q* together with their clocks, and nothing else, so their motion is perfectly relative and symmetric.

The solution given by Milne assumes that the signals exchanged between observers are reflected immediately; they therefore constitute a single unbroken zig-zag signal. It further presupposes that all signals contain information of the clock-reading indicating the epoch of the latest reflection-event, so that observers are able to read off one another's clock indirectly.

Let us assume that a zig-zag signal corresponds to this series of clock-readings:

$$t_1^p \leq t_2^q < t_3^p < t_4^q < t_5^p < t_6^q$$

It is then possible for *Q* to plot his clock-readings against those of *P*, the graph giving rise to a so-called *signal-function* θ. In the same way it is possible for *P* to plot his clock-readings against those of *Q*, and this graph gives rise to another *signal-function* ϕ. We therefore have:

$$(1) \qquad t_{\nu+1}^q = \theta(t_\nu^p) \ .\Leftrightarrow.\ t_{\kappa+1}^p = \phi(t_\kappa^q)$$

According to Milne the two master-clocks, $\boldsymbol{T}^p$ of P and $\boldsymbol{T}^q$ of Q, can be made congruent, $\boldsymbol{T}^p \simeq \boldsymbol{T}^q$, if one of the observers, say Q, is able to *regraduate* (the readings of) his clock in a manner which makes the signal-function ϕ identical with the signal-function θ, i.e.: $\phi \equiv \theta$.

The problem is reducible to that of finding the square-root of the functional product $\phi\,\theta$, and Milne, assisted by his collaborator Whitrow, demonstrated that this can always be done. The significance hereof is that two observers are *equivalent* in the sense that their clocks are *congruent* if and only if the signal-functions connecting the observers are fully symmetric.

So far, our universe of discourse contains only two monads, or particle-observers, and their equipment, i.e. the master-clocks and the apparatus required for signal-communication; there is no trace of any forces or fields, neither do we presuppose the signals to have a velocity. What we dispose of to construct our world-model is up to now only an abstract calculus of t-numbers interpreted as the clock-readings of two observers connected by a zig-zag signal, and our only assumption is that the involved reflection-events do not occur simultaneously. This is an extremely meagre universe, of course, and in order that the comparison of our world model with experience shall make sense we need to refine its structure somewhat more.

We therefore proceed to the *second problem of time-keeping* which can be stated thus: Given that a primary observer *O* has constructed an arbitrary clock for himself, are two other observers *P & Q* then able to set up congruent clocks irrespective of their relative motions?

Let the signal-function from *O* to *P* be θ_{op}, that from *O* to *Q* be θ_{oq}, that from *P* to *Q* be θ_{pq}, and let further the functional inverse of θ_{op} be θ^{-1}_{op}, that of θ_{oq} be θ^{-1}_{oq}, and that of θ_{pq} be θ^{-1}_{pq}. Assuming the observers to be equivalent, their clocks have to be congruent, which implies:

(2) $$\theta_{op} \equiv \theta_{po} \;\; . \;\; \theta_{oq} \equiv \theta_{qo} \;\; . \;\; \theta_{pq} \equiv \theta_{qp}$$

The simplest case to consider is that where the observers are collinear in fixed order;
this case Milne defined by the transitivity, in the given order, of the relevant signal-functions:

$\theta_{pq}\theta_{op} \equiv \theta_{oq} \; . \Leftrightarrow . \; \theta_{qo} \equiv \theta_{po}\theta_{qp}$ corresponding to the order *O,P,Q* ⇔ *Q,P,O*
$\theta_{qo}\theta_{pq} \equiv \theta_{po} \; . \Leftrightarrow . \; \theta_{op} \equiv \theta_{qp}\theta_{oq}$ corresponding to the order *P,Q,O* ⇔ *O,Q,P*
$\theta_{op}\theta_{qo} \equiv \theta_{qp} \; . \Leftrightarrow . \; \theta_{pq} \equiv \theta_{oq}\theta_{po}$ corresponding to the order *Q,O,P* ⇔ *P,O,Q*

It follows at once that congruence of collinear clocks implies that signal-functions commute:

(3) $$\theta_{pq}\theta_{op} \equiv \theta_{op}\theta_{pq} \;\; . \;\; \theta_{qo}\theta_{pq} \equiv \theta_{pq}\theta_{qo} \;\; . \;\; \theta_{op}\theta_{qo} \equiv \theta_{qo}\theta_{op}$$

The general commutative identity $\theta_{pq}\theta_{rs} \equiv \theta_{rs}\theta_{pq}$ was found to have two solutions:

(4) $$\theta_{pq}(t) = \omega\big(\omega^{-1}(t) + \lambda_{pq}\big) = \psi\, e^{\alpha_{pq}} \psi^{-1}(t)$$

These solutions were now taken to define the concept of a *linear equivalence* of observers.
It was also shown that, if two members of a linear equivalence ever coincide, they all coincide:

$$\theta_{qp}\theta_{pq}(t_1) = t_1 \; . \Leftrightarrow . \; \psi\, e^{2\alpha_{pq}} \psi^{-1}(t_1) = t_1 \; . \Leftrightarrow . \; e^{2\alpha_{pq}} \psi^{-1}(t_1) = \psi^{-1}(t_1)$$

If $P \neq Q$, then $e^{2\alpha_{pq}} > 1$, whence $\psi^{-1}(t_1) = 0$, and the conclusion follows by generalization.

The linear equivalence can be depicted as follows. Imagine a set of discrete particles separated by equidistant intervals along a string which is homogeneous and perfectly elastic. The distances of the particles will be proportional to the tension of the string, whence their relative motions will be dependent on how the string is slackened or tightened, and how fast. If the motion of a single particle be uniform, relative to a given observer-particle, that of all other particles will also be uniform, and their distances will be proportional to their velocity.

Now the structure of the model must be extended to 3-space; the linear equivalence must thus be generalized into a ***universal substratum*** which is an equivalence in 3 dimensions.

Intuitively it is not hard to see that, if the substratum is not a continous fluid but constituted by discrete particles, they cannot be equally distant from their neighbours in all directions. The consequence hereof is that, in 3-space, small perturbations should be expected to occur which might imitate the newly observed minute ripples in the cosmic background radiation.

Using standard coordinates, Milne could compare the clock-rates of different observers. With distances defined by *radar-signals* the rigid rod became redundant as a measuring device. He therefore interpreted the rate of radar-distance traversed to time spended as a proportionality factor, which thus constitutes an alternative to the light-principle of Einstein.

In retrospect this suggests that the quantum properties of "photons" be taken seriously: the visualization of "photons" as "something travelling in space" is probably very misleading.

To Milne, the step to a construction of transformations of coordinates was now small. Assuming the linear equivalence to be in uniform expansion as measured by so-called t-time, a time-scale which he later identified with that defined by the intrinsic oscillations of atoms, he was able to demonstrate the correct transformations to be those of Lorentz and Poincaré.

The form of signal-functions being $\theta_{pq}(t) = e^{\alpha_{pq}}(t)$, they can be derived thus $(c=1)$:

$$t_2^q \equiv t'-r' = e^{\alpha}(t-r) \equiv e^{\alpha} t_1^p$$
$$t_4^q \equiv t'+r' = e^{-\alpha}(t+r) \equiv e^{-\alpha} t_5^p$$

(5) $$\underline{t' = t\, cosh\alpha - r\, sinh\alpha}$$
(6) $$\underline{r' = r\, cosh\alpha - t\, sinh\alpha}$$

(7) $$e^{\alpha} = \sqrt{\tfrac{t+r}{t-r}} \underset{r'=0}{=} \sqrt{\tfrac{1+v}{1-v}} = \theta$$

The interpretation of t-time as atomic time, when multiplied with the universal constant c (the light speed), may be used to define the sizes of atoms as invariant according to the t-scale. The double solution to the commutativity problem indicates the possibility of a second scale of time related logarithmically to the first one according to the formula: $\tau = t_o\, log(t/t_o) + t_o$.

The τ-scale has the following peculiar property: If the clocks of all observer-particles constituting the universal substratum are instantaneously regraduated from t to τ according to this formula, they will find their relative distances to be constant with regard to τ-time. Further, if they have chosen their 3-space to be flat according to t-time, in agreement with the validity of the Lorentz transformations, they will find their 3-space to be hyperbolic with respect to τ-time.

Therefore the universal regraduation from t-time to τ-time of all clocks will transmute a uniformly expanding substratum in flat space into a stationary substratum in hyperbolic space. However, the structure of the substratum, and its internal proportions, must remain the same in spite of the regraduation. So all atoms must shrink continuously, according to the τ-scale.

We now introduce a crucial distinction in the model between two kinds of particles, viz. *fundamental particles* that belong to the substratum, and *accidental particles* that do not. Each fundamental particle is a universal center of spherical symmetry in the sense that its associated observer will see all the other particles of the substratum distributed isotropically. This crucial property is preserved irrespectively of whether the t-scale or the τ-scale is used.

What happens when an accidental particle is released in the presence of the substratum? We said that, if the substratum is mapped according to τ-time, it is perfectly stationary except for a steady decrease of the size of atoms (which we here ignore): no external forces are in play. So we conclude that the motion of an arbitrary accidental particle is perfectly inertial in τ-time.

Now it is intuitively obvious that the motion of a particle, which is inertial when mapped relative to the τ-scale, will no longer appear inertial when it is mapped relative to the t-scale. *The effect of passing from τ-time to t-time is that inertial motion is replaced by accelerated.*

What Milne demonstrated was in fact that, when the motion of a test-particle is analysed mathematically with respect to the t-scale, the result is the emergence of an apparent "force" giving the particle a spontaneous acceleration which depends on the degree of its asymmetry. *His idea was to explain the nature of "forces" by local deviations from global symmetry*.

This brilliant program he proposed to implement by an exceedingly ingenious procedure. Having analysed the motion of a single test-particle in the substratum, he next examined the effects of asymmetry on a whole infinite set of test-particles superposed upon the substratum. To this purpose he inverted the famous Boltzmann equation of statistical mechanics in order to derive accelerations from a position-velocity distribution instead of the other way round.

His conclusion was that any test-particle, when exposed to the influence of the substratum as well as that of particles not belonging to the substratum, will receive an acceleration which is the resultant of two different contributions: (a) one directed towards the provisional center of the universe as perceived by the test-particle, viz. that fundamental particle with regard to which it is momentarily at rest, and (b) another directed towards to local center of the set of accidental particles superposed on the substratum. The formula for the induced acceleration follows below:

(8) $$d\boldsymbol{v}/dt = -(\boldsymbol{r}-\boldsymbol{v}t)(Y/X)\big(1 + C/\psi(\xi)(\xi-1)^{3/2}\big)$$

C is a constant, ψ is an indefinite function, and $\boldsymbol{r}$ & $\boldsymbol{v}$ are vectors of position & velocity; further:

$$\xi \equiv Z^2/XY \quad . \quad X \equiv t^2 - \boldsymbol{r}^2/c^2 \ . \quad Y \equiv 1 - \boldsymbol{v}^2/c^2 \ . \ Z \equiv t - \boldsymbol{rv}/c^2$$

For small $\boldsymbol{r}$ & $\boldsymbol{v}$, with $\boldsymbol{l} \equiv \boldsymbol{r} - \boldsymbol{v}t$, $M \equiv M_o \frac{C}{\psi(1)} \equiv [\frac{4}{3}\pi(ct)^3\varDelta]\frac{C}{\psi(1)}$ and $\gamma \equiv c^3t/M_o$, we derive:

(9) $$d\boldsymbol{v}/dt \simeq -\boldsymbol{l}/t^2 - C\frac{c^3t}{\psi(1)}(\boldsymbol{l}/|\boldsymbol{l}|^3) = -\boldsymbol{l}/t^2 - \gamma M\big(\boldsymbol{l}/|\boldsymbol{l}|^3\big)$$

By means of so-called "superpotentials" Milne also constructed a new electrodynamics. He likewise considered the nature of cosmic rays, the structure of galaxies, and that of atoms. He further proposed the use of the formulae discovered by W. Voigt for the transformation of coordinates between fundamental and accidental particles when the τ-scale is referred to.

In his last years he got problems as regards the correct relation between a t-time mapping and a τ-time mapping, especially when it comes to the representation of optical phenomena. He did not live long enough to solve these problems and - his ideas not finding the followers they deserved (with one exception, see §3) - his cosmology remains partially incomplete.

3. WALKER'S ANALYSIS OF MILNE'S IDEAS

As Eddington has remarked: *The theory of the* '*expanding universe*' *might also be called the theory of the* '*shrinking atom*'. Quoted from Whitrow [1961, p.248; 1980, p.293]

Despite the obvious importance of Walker's contributions to relativistic cosmology they suffered the bad luck of being overshadowed right from the beginning by the slightly earlier work of Robertson, and only very few of the experts in the field appear to be acquainted with his original papers. It therefore seems pertinent here to outline some of their contents.

In a paper on Milne's kinematic theory [1936], Walker stressed that his choice of the Lorentz formulae to connect the observational data in t-time involves an unnecessary restriction. He therefore set himself the task of investigating the possibility of applying Milne's kinematic method to models which do *not* presuppose the validity of the Lorentz formulae.

In order to perform this task he assumed the following two principles to hold good: (1) the c*osmological principle **(CP)***, in the formulation by Milne, implying that the totality of observations that a fundamental observer can make is "similar" (involving identity of structure) to that of any other fundamental observer; and (2) the *principle of symmetry*, implying that each fundamental observer sees himself to be at a centre of spherical symmetry.

As he later [1944] presented a proof demonstrating that the first principle, which is often interpreted as prescribing *universal homogeneity*, is contained in the second one, which entails *universal isotropy*, we shall treat the two principles as if they were one. Since the principle of isotropy is the only cosmological principle relevant to the present paper, I shall feel at liberty to ignore world-models incompatible with that principle such as the rotating one of Gödel.

The principle of cosmic isotropy is a precondition for the definability of cosmic time which is the most fundamental feature of the synthesis of ideas discussed in this book.

The first of Walker's results was a quadratic invariant implying a Riemannian geometry. The only difference to the general theory of relativity was its scope of greater generality with regard to the path descriptions of free particles: these were not necessarily given by geodesics. The technique used was that of continuous groups of transformations which, combined with the principle of isotropy, yielded a relation of congruence needed for the conservation of angles. The two principles together implied the free paths to obey a variational principle of least action.

In this paper he deduced what became famous as the Robertson-Walker Metric (***RWM***):

(10) $$\underline{dT^2 = d\tau^2 - R^2(\tau)\left(\tfrac{d\rho^2}{1-\kappa\rho^2} + \rho^2(d\vartheta^2 + sin^2\vartheta\, d\varphi^2)\right)}$$

For $R(\tau) \equiv \tau$, $\kappa \equiv -1$, $\rho \equiv sinh\alpha$, this is reducible to a metric of uniform expansion:

(11) $$\underline{dT^2 = d\tau^2 - \tau^2\left(d\alpha^2 + sinh^2\alpha(d\vartheta^2 + sin^2\vartheta\, d\varphi^2)\right)}$$

Translating into standard coordinates, using $c \equiv unity$, $t \equiv \tau\, cosh\alpha$, $r \equiv \tau\, sinh\alpha$, he obtained:

(12) $$\underline{dT^2 = dt^2 - dr^2 - r^2\left(d\vartheta^2 + sin^2\vartheta\, d\varphi^2\right)}$$

Here *kinematic frame time* t is connected with *private flat space*, *dynamic proper time* τ being connected with *public curved space*. Of these two time-scales only τ is truly cosmic.

Metric (12), of course, is the quadratic invariance of the differential Lorentz equations. These equations are supposed to involve a reciprocal retardation of clocks in inertial motion.

But one thing is to admit that the master-clock of a fundamental observer will appear to be retarded relative to the slave-clocks belonging to the frame of another fundamental observer. Quite another thing is to claim that the master-clock of one fundamental observer is retarded relative to the master-clock of another fundamental observer.

There is no problem in the first case due to the fact that slave-clocks are not fundamental. However, there *is* a problem with *the second case*, since it *involves a formal contradiction*! My point is that, if the master-clocks of two fundamental observers do not agree, they are not congruent, therefore at least one of the two fundamental observers cannot be fundamental.

That I am right, and a very widespread interpretation of relativity is wrong, is stressed by the fact that the RWM clearly not merely involves, but presupposes, a Cosmic Time.

That this is indeed the case is evidenced by Walker's remarks in §3 of his first paper:

"*Each observer can (thus) provide himself with a clock and can choose a coordinate system so that for any two observers, their clocks are (congruent) .. in the sense (of) Milne. .. It may be assumed, therefore, that (all) clocks are .. synchronized (to) record the time τ .. Since there is a four-fold infinity of events, we may set up a (1,1) correspondence between events and points of a four-dimensional manifold M_4. Each fundamental particle occupies a single infinity of events during its history and corresponds to a world-line in M_4.*

There is a world-line corresponding to each particle, and, since each event is occupied by one particle, there is just one fundamental world-line passing through each point of M_4. The points of each world-line may be specified by the times recorded by the clock associated with the corresponding particle, hence τ may be regarded as a parameter for each world-line. This parameter defines a family of three-dimensional surfaces S_τ ... Thus τ, which may be regarded as a ***cosmic time****, serves as a (public) coordinate of M_4.*"

Later in the same paper (§16) he made these remarks, at once illuminating and intriguing: "*In §3 the clock recording time τ was any clock constructed by one observer. However, we already possess a clock recording time t which we may call "physical" (atomic? MW) time, and all our time measurements are made with this clock. It is therefore desirable to refer to time t and, since the clock recording time τ was arbitrary, we may now assume that $\tau \equiv t$.*"

In that same paper Walker also verified Milne's expression for the density-distribution of a substratum, and his equation for the motion of test-particles in the substratum as well as that for a statistical ensemble superposed hereon, both relating to Milne's uniform expansion model.

In three later papers on relativistic mechanics [1940 ff.], also inspired by Milne's ideas, he studied their relevance to a range of models comparable to the scope of general relativity.

According to Walker, "the great disadvantage of general relativity" is that it allows the form of space to depend on the distribution of matter, albeit the problem of finding that form is too difficult except for the simplest systems. A more serious objection is that no system can exist in isolation from the universe, so the problem of world-structure should be given primacy. The outstanding merit of Milne is, precisely, to have called attention to this crucial point.

Again Walker took the principle of isotropy as his starting point, this time setting himself the task of framing a general scheme differing from that of Milne by laying even more stress on a correspondence with classical mechanics. His result was that systems similar to those of classical mechanics are definable relative to the comoving frames of all fundamental observers. Further, since any accidental observer can describe his position and motion by reference to that fundamental particle relative to which he is momentarily at rest, their frames are also covered.

He then proved the theorem that all physical objects can be described by 3-space tensors transforming between fundamental particles. His general conclusion is as follows: for each of the two time-scales there is a 3-space representation analogous to that of classical mechanics, and in both cases this 3-space representation is simpler than a 4-space representation.

4. NON-STANDARD COORDINATES OF TIME

In other words there are universally acknowledged to be in heaven and earth things not dreamt of in Einstein's philosophy; yet Einstein's philosophy is allowed to block the simplest ways of categorizing and describing those things ... Thus it is taken for granted that quantum mechanics has to be "relativized", i.e., changed to conform to the Lorentz group, whereas it might more rationally be assumed that relativity theory and its invariance group have to be changed to conform to the existence in nature of nonlocal actions ...

T.E. Phipps: *Heretical Verities* .. [1986]

It is a dogma of the special theory of relativity (***SR***) that clocks are retarded and that rods are contracted, both as an effect of inertial motion. What are we to think of that dogma? Laws of nature being distinguished by invariance, how can the dilatation of time ever be a law? What reasons are adduced to substantiate it? Primarily a reference to the Lorentz-formulae!

Well, the Lorentz transformations are inevitable if we stick to the standard definition of coordinates proposed by Einstein and repeated ever since. But definitions are conventions. How can such conventions be necessary? We are then referred to the so-called light-principle: If the velocity of light is a universal constant, how can the standard definitions be avoided?

A simple straigth-forward answer to this question is: by a little intuition and imagination! Further, how can a state of motion, supposed to be unaltered, ever be the cause of anything? Moreover, contractions are assumed to be reciprocal, retardations not; why this difference? How - disregarding bifurcations - can asymmetry ever be a consequence of symmetry?

In order to evade this impasse and make progress possible, a distinction must first be introduced, viz. that between the *one-way velocity* of light and the *two-way velocity* of light (to which we may add a generalization of the second concept: the round-trip velocity of light).

Today it is generally agreed that ***SR*** offers no experimental way of deciding the value of the one-way velocity of light which does not itself depend on a conventional definition of simultaneity at a distance implicitly taking the one-way light-velocity to be a universel constant. However, the procedures devised to test the two-way - or round-trip - velocity of light do not involve such definitional circularity. Hence the light-principle should be interpreted as referring not to the one-way light-velocity, but to the two-way - or round-trip - velocity of light instead.

This insight opens up for possibilities ignored by almost all experts on relativity theory, although the general theory of relativity ***(GR)*** similarly introduces a variable velocity of light (the problem here is that Riemannian geometry and tensors are assumed to be indispensable). But if the one-way velocity of light is finally recognized as a theoretical variable, the search for a rational alternative to the Einsteinian time-coordinate needs no longer be blocked.

The second point to be realized is that the substratum functions as a *compass of inertia*. The term, invented by Weyl, was used by Gödel to denote a reality distinct from the substratum in order to make sense of a model where the substratum "rotates" relative to "something else"; but this empty conception is clearly unsatisfactory from the point of view of epistemology. The only rational procedure is to identify the two, and the natural precondition of doing so is that no spatial direction is privileged, as perceived by observers belonging to the substratum.

Thus, if two of its particles recede from each other with velocities depending on distance the whole substratum must expand in all directions according to the same function of time. Naturally, this can be the case only if the same temporal parameter holds for the entire universe.

An ideal substratum characterized by perfect symmetry can be treated as a cosmic "fluid" subject to the laws of hydrodynamics. However, our universe is not continuous, but discrete. How great are the irregularities it can absorb, and yet still retain an approximate symmetry? This is a deep question allowing no easy answer, and we shall put it aside at present.

Let us instead agree to utilize *the idea of the substratum*, subject to perfect continuity, as a kinematic background - a neutral "sheet" of tempo-spatial geometry - on which we can plot irregularities in order to disclose the hidden pattern, or structure, of the observed universe - i.e., let us think of the substratum in the same way as the geometers think of coordinate systems. When interpreted thus, a substratum ***S*** is nothing but a reference frame in uniform expansion.

The ***CP***, requiring the conservation of angles, implies motion in a substratum to be radial: this holds for the relative motions of fundamental particles, whereas accidental particles deviate. Now, clearly, the comoving rest-frames of either particles have no real function of their own; this indicates that the standard time-coordinates referring to such rest-frames are redundant.

So we are free to start our search for alternatives to the standard conventions of Einstein. As mentioned above, experts today generally agree that the one-way light-speed (1-*wls*) may vary in accordance with the particular definition of simultaneity adopted as long as the average, or two-way, light-speed (2-*wls*) remains a universal constant. Writing $c_{\leftarrow}$ and $c_{\rightarrow}$ for the 1-*wls* to and fro (out and home), respectively, the 2-*wls* must fulfil the Light Principle ***(LP)***:

$$\tfrac{1}{c_{\leftarrow}} \equiv 1+\zeta \ . \ \tfrac{1}{c_{\rightarrow}} \equiv 1-\zeta \ . \ \tfrac{1}{2}(\tfrac{1}{c_{\leftarrow}}+\tfrac{1}{c_{\rightarrow}}) \equiv \tfrac{1}{c} \equiv 1$$

This suggest a redefinition of Einstein's coordinates, $t \equiv \frac{1}{2}(\tau_3+\tau_1)\, , r \equiv \frac{1}{2}(\tau_3-\tau_1)$:

(13a) $\qquad t + r = \tau_3 \equiv \tau + r/c_{\leftarrow} = (\tau+r\zeta) + r$

(10b) $\qquad t - r = \tau_1 \equiv \tau - r/c_{\rightarrow} = (\tau+r\zeta) - r$

Recalling the invariant time $\tau \equiv \sqrt{t^2-r^2}$ of Törnebohm (1963), we recover the deviation ζ by:

(14) $\qquad \zeta = (t-\tau)/r = \{t - \sqrt{t^2-r^2}\}/r$

$$v \rightarrow r/t \ . \Rightarrow : \ \zeta \rightarrow \{1-\sqrt{1-v^2}\}/v \ \Rightarrow \ \tau \rightarrow t/\sqrt{1-v^2}$$

This suggests a radical re-interpretation of ***SR***, call it ***AR***, based on the idea that the deviation of 1-*wls* from 2-*wls* discloses a real (probably unobservable) difference: ***SR*** $\neq$ ***AR***. We start by accepting the Lorentz Transformations (***LT***) for fundamental observers $F, F' \in$ ***S***:

$$t' = \ t\, cosh\omega - x\, sinh\omega$$
$$x' = \ x\, cosh\omega - t\, sinh\omega$$
$$x = 0 \ . \ t = \tau \ \Rightarrow \ x' = -\tau\, sinh\omega$$
$$x' = 0 \ . \ t' = \tau \ \Rightarrow \ x = \tau\, sinh\omega$$

Proper velocity has no upper limit. So why not put the master-clocks of F & F' on a par? This we do by relating fundamental observers pairwise *via* the frame of their ***midway particle***. Applying to x' & t' the non-standard definitions $t^o \equiv t - x\, tanh\frac{\omega}{2},\ x^o \equiv x - t\, tanh\frac{\omega}{2}$, we get:

$$x' = x - t^o sinh\omega \ . \ t' = t - x^o sinh\omega$$

The coordinates t^o & x^o refer to the comoving frame of a particle M midway between F & F'. But x^o is of no avail: $x^o = 0 \ \Rightarrow . \ t = t' = t^o cosh^2\frac{\omega}{2} \ . \ x-x' = t^o sinh\omega = 2t\, tanh\frac{\omega}{2}$.

Ignoring x^o and the standard frame times t & t', we focus on t^o which we interpret as the common proper time τ displayed by the master-clocks of observers F, F', and that of M:

(15) $\underline{x' = x\,cosh\omega - t\,sinh\omega = x - \tau\,sinh\omega}$

(16) $\underline{\tau\; = \; t - x\,tanh\tfrac{1}{2}\omega = t' - x' tanh\tfrac{1}{2}\omega' = \tau'}$

Evidently, the *midway particles* between pairs of observers belonging to a substratum $\boldsymbol{S}$ are themselves members of that substratum, and the time shown by the master-clocks of all substratum members is invariant. This may be generalized to objects moving in the substratum. *All coordinates being communicated among members of* $\boldsymbol{S}$*, any pair of observers* $F, F' \in \boldsymbol{S}$ *may describe an object* O *by reference to substratum time* τ*, disregarding the frame times* t, t'. The observers F, F' are now able to determine the motional state of O in the following way:

While O's distance relative to F, resp. F', at instant τ is equal to that of particle $P_1 \in \boldsymbol{S}$ with which O momentarily coincides, O's velocity relative to F, resp.F', at the same instant τ is equal to that of particle $P_2 \in \boldsymbol{S}$ with respect to which O is momentarily at rest:

$$\vec{r}'_o = \vec{r}_{F'O}(\tau) = \vec{r}_{F'P_1}(\tau)\ .\ \vec{\dot{r}}'_o = \vec{\dot{r}}_{F'O}(\tau) = \vec{\dot{r}}_{F'P_2}(\tau)$$

Now, from Walker's metric for the Milne universe it nevertheless follows that:

$$dT = \{d\tau^2 - \tau^2 d\sigma^2\}^{\frac{1}{2}} = \{dt^2 - dr^2\}^{\frac{1}{2}} = invar.$$

For motion relative to the substratum ($d\sigma \neq 0$), ***AR*** still predicts a retardation of clocks. Such *clock retardation*, however, is not just an effect of inertial motion, as generally assumed. In Milne's universe, it can always be interpreted as *a gravitational effect* of the substratum $\boldsymbol{S}$.

In any case, $d\sigma \to 0 \;\Rightarrow d\tau \to dT$ shows that the deviation of τ from true time T depends on the deviation of the observed object from fundamentality, i.e. on its motional anisotropy.

5. TOWARDS A NEW STEADY STATE THEORY

In any theory which contemplates a universe (evolving with time), explicit and implicit assumptions must be made about the interactions between distant matter and local laws .... These are necessarily of a highly arbitrary nature, and progress on such a basis can only be indefinite and uncertain. It may .... be questioned whether such speculation is required. If the universe (is an a steady state) none of these difficulties arises.

Sir Hermann Bondi [1961[3]]

The idea that the universe originated in a dramatic explosion, the "big bang", appears to be supported by two important discoveries: (i) that of a systematic redshift in the light from distant galaxies [Hubble 1929], and (ii) that of a faint cosmic background radiation displaying a highly isotropic character [Penzias & Wilson, and Dicke, 1965]. It has since become one of the most cherished dogmas of modern science.

But the redshift in itself does not imply a singularity at $t = 0$. Further, other explanations of the $3K$ ***CBR*** than a "big bang" are open, cf. Surdin [2002]. Narlikar [1980] reckons several alternatives. Finally, the "big bang" idea has recently run into severe difficulties. As pointed out by Lerner [1991], certain huge galactic structures seem to be as old as a hundred billion yrs, and new reports of very distant, but fully mature, galaxies abound, cf. e.g. Kennicutt [1996].

During the seventies the "big bang" hypothesis gradually superseded the once popular "steady state" theory of Bondi, Gold & Hoyle; cf. North [1965], Harrison [1981], Kragh [1996]. However, it would be wrong to suppose that the "big bang" hypothesis and the "steady state" hypothesis are mutually exclusive. In fact, it is easy to construct world-models which originate as a point-singularity, or even as a quantum-state of infinite density, and which then later by a temporal evolution, violent or gentle, progressively approximate towards a stationary state.

If approximation is allowed to take place, at least two interesting possibilities are open: (1) a universe which starts as a singularity and expands in hyperbolic space according to a hyperbolic sine function of time, and (2) a universe which begins like a cosmic "egg" and then develops in hyperbolic space in agreement with a hyperbolic cosine function of time.

In his first book Milne proposed a distinction between two important concepts relating to any rational model of the universe: (1) its *world-map*, describing the universe as a totality of structure that exists independently of observation; and (2) its *world-view*, depicting the universe in perspective as perceived by an observer. Whereas the world-map is a structural "snapshot" presupposing a definition of universal simultaneity, the world-view combines spherical "layers" of varying age, their age increasing with their radial distance from the observer, due to the fact that we usually take the one-way velocity of light to be finite. A genuine "steady state" model is characterized by the structural identity of the two, whereas for "big bang" models they differ. Most cosmologists ignore the difference, being unable to make sense of the concept of world-map, but this invalidates their interpretation of data. Indeed, one could say that their refusal to transcend a pure world-view excludes them from developing cosmology as a theory.

Let us consider the original model of Bondi & Gold which is based on the metric:

$$(17) \qquad dT^2 = \mathrm{dt}^2 - \mathrm{t}_o^2\, e^{2\mathrm{t}/\mathrm{t}_o}\left(d\rho^2 + \rho^2(d\vartheta^2 + sin^2\vartheta\, d\varphi^2)\right)$$

This simple model with comoving coordinate ρ and cosmic time t is invariant to time-zero shift, but other properties are less agreeable: $dT = d\vartheta = d\varphi = 0$ yields $\pm\, d\rho = e^{-\mathrm{t}/\mathrm{t}_o}\mathrm{dt}/\mathrm{t}_o$, whence:

$$(18) \qquad \rho = \int_{\mathrm{t}_1}^{0} e^{-\mathrm{t}/\mathrm{t}_o}\mathrm{dt} = \int_0^{\mathrm{t}_3} e^{-\mathrm{t}/\mathrm{t}_o}\mathrm{dt} = \mathit{const.}$$

$$(19) \qquad \rho = 1 - e^{-\mathrm{t}_1/\mathrm{t}_o} = e^{-\mathrm{t}_3/\mathrm{t}_o} - 1 \geq 0$$

These relations imply: (a) the proper distance $e^{\mathrm{t}/\mathrm{t}_o}\rho$ has a horizon at $\mathrm{t}_3 \simeq \infty \Leftrightarrow \mathrm{t}_1 = -\mathrm{t}_o\, ln2$; ($b$) all particles receding from a finite distance leave our part of the universe after a finite time; (c) only a finite number of particles are at any time visible within the horizon, whereas an actual infinity of particles are existing in an imaginary space beyond the horizon. Now (c) reduces the visible universe to a finite drop in an infinite ocean of "reality". This is highly problematic.

There are two main reasons to search for an alternative to this model of the universe. The first reason is theoretical: it is gross speculation to assume almost the entire universe to exist beyond the horizon of all possible observation as a hardly intelligible kind of reality. The second reason is empirical: it has turned out to be outright impossible to reconcile the observed number-distance statistics for galaxies with the statistics predicted by the model.

In contrast to that, the uniform expansion model of Milne is unbounded by any horizon, and hence it contains *a potential infinity of visible objects* inside its imaginary circumference. It is therefore natural to ask whether these properties of Milne's "big bang" model could not be transferred to a new "steady state" model. We shall address this question in §6.

Until then, it seems mandatory that we discuss some important preliminary difficulties. Unfortunately, we are forced to leave the most important one unsolved, viz. that concerning the degree of tolerance against irregularities and deviations from perfect universal symmetry. However it is intuitively clear that, as our new model of the "steady state" universe will be constructed so as to leave no real (i.e. existing) light source invisible or unobservable in principle, it will also be able to accomodate much vaster irregularities and differences of development. Hence patterns as that of the "great wall" do not in themselves make the model improbable.

Another difficulty, related to the first one, is that the concept of a linear equivalence cannot be applied to a "steady state" model which must be invariant to arbitrary time-zero shift. We shall therefore have to define the equivalence of observers in a manner which makes it independent of the form of signal-functions and their commutativity for collinear observers. This we do by postulating clock-congruence to be transitive for all fundamental observers:

$$\forall P, Q, R: \;\; \boldsymbol{T}^p \simeq \boldsymbol{T}^q \;\&\; \boldsymbol{T}^q \simeq \boldsymbol{T}^r \;.\Rightarrow\; \boldsymbol{T}^p \simeq \boldsymbol{T}^r$$

So we define a substratum in this new sense as an infinite set of fundamental particles which are equivalent by virtue of the fact that their master-clocks remain mutually congruent. Our definition corresponds exactly to a tacit assumption of orthodox cosmology: namely, that *atoms of the same type oscillate with the same frequency when unaffected by local forces*. This assumption is clearly indispensable to all natural science, especially cosmology. Our only addition to it is that such atoms rule the clocks of a universal class of equivalent observers.

The advantage of this interpretation is that it changes the status of fundamentality from something absolute into a matter of degree: the less the atoms regulating the master-clock of a particle-observer deviate from the ideal standard of cosmic time, the more fundamental he is. By the same token the deviation of a master-clock from universal time becomes a measure, or an index, of the influence upon it caused by local gravity, or other kinds of force.

Hence cosmic time is an ideal, just like the perfect vacuum, the perfect inertial motion, or the absolute temperature zero; but natural science, of course, is in need of such ideals. Bondi sees it as a merit of his own "perfect cosmological principle" that it entails a maximum of uniformity in nature; only on this condition, says he, can nature be subject to strict laws. But, as already hinted at, it may be doubted whether nature is really governed by strict laws.

Probably it should be reckoned as a merit of a world-model if it allows a maximum of non-uniformity in nature without surrendering its structural totality and fundamental integrity. Let it be admitted that cosmic time, as represented by atomic clocks, is a statistical concept; but, as an index for deviations from universality it is undoubtedly also a crucial regulative idea, and it is a legitimate task to construct world-models in conformity with this regulative idea.

The "steady state" model of Bondi, Gold and Hoyle, implying the continued creation of "matter" in "space", has often been the target of heavy criticism for its presumed violation of one of the two most imperative principles in science, viz. that of the conservation of energy. This vital principle of physics is frequently assumed to be derivable from the ancient maxim *ex nihilo nihil fit* without recognition of the fact that the maxim is a piece of metaphysics.

Moreover, the participants in this often emotional debate repeatedly forget that, in order to state the energy principle properly, mention must be made of a specific volume of space wherein a certain amount of energy is supposed to exist: How is this volume to be defined? How can a volume be specified without presupposing a definition of distant simultaneity?

Let us yet suppose that a certain mass is definable as the product of density by volume, and that a volume is definable by its spatial extension as measured by radar signals; even with the assumption of an absolute simultaneity as a cut "at right angles" to the universal time axis, the question as to the time-scale to be used is also ambiguous: what time should be counted? Let us then settle for the scale of atomic time, and still one question is left for us to decide: do we want to measure spatial volume by means of proper distance or coordinate distance?

If the question is stated that precise, the problem of energy becomes one of hypothesis, and the difference between "big bang" models and "steady state" models can be specified thus: In a pure "big bang" model energy conservation is defined relative to constant coordinate space, whereas it is defined with respect to constant proper space in a pure "steady state" model. But it is hard to see why coordinate space should be given priority over proper space.

In fact, proper distance defined as radar-distance with respect to atomic time appears to be closer to our ordinary intuition of "reality" than any convention of coordinate distance. The only conundrum to our intuition is that energy conservation as defined for an expanding substratum relative to proper space leads to the consequence that the apparent loss of matter caused by the recession of particles has to be compensated by the creation of other particles.

This puzzle finds a solution when it is realized that the model of continued creation here presented allows no particle to cross its imaginary border: there is nothing beyond the horizon. This universe can be described as a pseudosphere in flat space, of finite radius and finite mass, containing an infinite (yet apparently ever growing) number of atoms which apparently shrink with their distance from an observer, reaching zero value precisely as they pass its periphery. Neither light nor matter leaves the universe which thus appears to be the perfect "black hole".

Moreover it should be realized that "big bang" models and "steady state" models have a different status with respect to the presumed "constants" of nature. While "big bang" models are open to the possibility of a secular variation of their values and their numerical proportions, the invariant structure of "steady state" models urges the need for a theoretical explanation. The idea of Eddington that a theory of cosmic numbers may be feasible is indeed fascinating; cf. Whittaker [1943], who for that reason described Eddington as "a modern Archimedes".

Finally, according to Milne's theory, gravity does not hamper the universal expansion. Now what could be the mechanism to cause the acceleration needed in a "steady state" model? In any model of non-zero density there is a certain pressure which could cause an expansion. If the stuff of the universe can be viewed as a gas, the internal pressure of this gas might explain universal dissipation, hence also gravity, leaving the universe itself as the only "black hole".

6. A PROGRAM FOR SYNTHESIS

In physics everything depends on the insight with which the ideas are handled before they reach the mathematical stage. A.S. Eddington [1939 ch.4].

The program of *Einstein* aimed at *a geometrization of physics*, as he often repeated. That of *Milne*, by contrast, aimed at *an arithmetization of physics*, to use a phrase of Whitrow. These characterizations reflect the fact that, whereas Einstein buried the classical forces in the geometric structure of space (indeed a strange bearer of properties), Milne set them free by deriving them as the arithmetic consequence of local perturbances in the global expansion.

Gravity in his theory does not act as a brake on the expansion which remains uniform. Thus, in a way, Milne confirms the famous conjecture of the atomists: viz. that real objects, as well as all forces operating on them, stem from the free fall of atoms through empty space. Another characterization of that part of their programs, due to myself (1990), runs as follows: *Whereas the aim of **Einstein** was to reduce **inertia to gravitation**, in accordance with Mach's principle, that of **Milne** was to reduce **gravitation to inertia**, in opposition to Mach's principle.* Only the latter procedure can free physics of one of its greatest mysteries: that of gravity.

From a philosophical point of view there is no doubt, to my mind, that Milne has offered not only the best but, indeed, the only reasonable explanation of the mechanism of gravitation. What he has presented us with is a world-model which allows a derivation of all so-called "laws of nature" from first principles by superposing layers of accidental particles upon a universal substratum of fundamental particles supposed to be in uniform dispersion.

This kinematic technique was since generalized by Walker who succesfully applied it to a whole range of world-models (universes) with different expansion-functions and geometries, and who demonstrated the value of *multiple time-recording*, though neither the kinematic scale nor the dynamic scale should of necessity be associated with the intrinsic rhythm of atoms. The point is that the substratum is represented as being stationary according to one scale and uniformly expanding according to the other; but this has no implication for atomic rhythms.

Now, if the kinematic technique is to be applied at all to a "steady state" universe of continued creation, that model must display some analogy to the ideas of Milne and Walker. The standard metric of the universe of Milne, as well as that of Törnebohm & Prokhovnik, is:

$$d\mathcal{T}^2 = \ dt^2 - dr^2 - r^2(d\vartheta^2 + sin^2\vartheta \, d\varphi^2)$$

I now suggest that we take over the same metric, only adapted to hyperbolic 3-space:

(20) $$\underline{d\mathcal{T}^2 = \ dt^2 - dr^2 - sinh^2 r \, (d\vartheta^2 + sin^2\vartheta \, d\varphi^2)}$$

to characterise a new world-model, or universe, representing the continued creation of matter**.** The diffusion of fundamental particles in this universe is described by the following differential equations which, replacing the linear equations $r = t \, tanh\alpha = \tau \, sinh\alpha$ of Milne's model, must hold good for a "steady state" model, both according to *world map* and according to *world view*:

$$v \equiv dr/dt \equiv tanh(r/r_o) \propto r \ \text{ for } \ r \ll r_o$$
$$V \equiv dr/d\tau \equiv sinh(r/r_o) \propto r \ \text{ for } \ r \ll r_o$$

These equations immediately yield the following fundamental expression so distinctive of ***SR***:

(21) $$\gamma_r \equiv dt \, / d\tau = cosh(r/r_o) = \sqrt{1+w^2} = 1/\sqrt{1-v^2}$$

Following Milne, the Doppler-shift (*DS*) is the derivative of the signalfunction θ, cf. (7):

(22) $$1+z(t) = \{(dt+dr)/(dt-dr)\}^{1/2} = e^{r(t)}$$

We shall later (ch.5) see that, in order to calculate the cosmological redshift, we have to incorporate the effect of shifting our distant time coordinate from frame time t to proper time τ. A natural choice of distance-unit r_o is where the cosmological redshift takes on the value e:

(23) $$1+z(t) = e^{r/r_o} = e \quad \Leftrightarrow \quad r = r_o \equiv unity$$

According to Prokhovnik [22 §6 & app.] the expression $1{+}z = e^r$ matches the observed number-redshift relation that was utilized by astronomers to disprove the "steady state" model of Bondi; but he ignores the difference between *world-map* and *world-view*, redundant to a "steady state" model, but essential to his own "big bang" model which is identical to Milne's. Thus, rather than supporting the Milne-Prokhovnik model, it supports the model proposed here. Indeed, recent observations corroborate accelerated dispersion in hyperbolic space, cf. [36].

Our basic assumption is that the motion of fundamental particles can be described by:

(24) $$\underline{dr = dt\,tanh\,r = d\tau\,sinh\,r}\ \ .\ \ \underline{d\mathcal{R} = \mathcal{R}d\mathcal{T}}$$

This is easy to integrate these equations, t representing standard relativistic coordinate time and τ representing standard relativistic proper time, hence also the intrinsic rates of atoms; we get:

(25) $$\rho \equiv e^{-t}\,sinh\,r \equiv e^{-\tau}\,2\,tanh\tfrac{1}{2}r\ \ .\ \ \rho = const.\ \Leftrightarrow\ \tau = \mathcal{T}$$
$$d\rho = e^{-t}\{cosh\,r\,dr - dt\,sinh\,r\} = e^{-\tau}\{dr - d\tau sinh\,r)\}\,cosh^{-2}\tfrac{1}{2}r$$

Here ρ is a fixed comoving coordinate characterizing an arbitrary fundamental particle. Our eq.s, valid for all fundamental particles in agreement with the principle of cosmic isotropy, yield the "steady state" proportionalities $w \propto v \propto r$ locally and $\mathcal{V} \equiv \frac{d\mathcal{R}}{d\mathcal{T}} = \mathcal{R}$ globally, and their corrollary $e^{t-\tau} = cosh^2\tfrac{1}{2}r = 1/(1-\tfrac{1}{4}\mathcal{R}^2)$ prove them independent of arbitrary time zero shift. Whenever we make the choice $\tau = 0$, which is always possible, we find: $\rho = 2\,tanh\,log\sqrt{1{+}z}$. The bond $0 < \rho < 2\,e^{-\tau}$ does not preclude $\tau \to \infty$, only ρ is not defined for $\tau \geq log(2/\rho)$; for τ later than $log(2/\rho)$ we shall say that the particle has left our universe - it no longer exists!

A third time-scale, standing in a well defined relationship to t and τ, is t defined by:

(26) $$e^{\mathrm{t}} \equiv e^t/cosh\,r\ \ \Rightarrow .\ \ e^{\mathrm{t}}d\rho = dr - dt\,tanh\,r\ .\ \mathrm{dt} = dt - dr\,tanh\,r$$

Applying $\{\mathrm{dt}{+}\,e^{\mathrm{t}}d\rho\}\{\mathrm{dt}{-}\,e^{\mathrm{t}}d\rho\} = \{(dt{+}\,dr)(1{-}tanh\,r)\}\{(dt{-}\,dr)(1{+}\,tanh\,r)\}$ we obtain:

$$d\mathcal{T}^2\ \equiv\ dt^2{-}dr^2 = \{\mathrm{dt}^2{-}e^{2\mathrm{t}}d\rho^2\}\,cosh^2 r$$

So, if $d\mathcal{T}^2$ is rewritten with t & ρ, the recession of fundamental particles seems exponential in t. This fact strikingly illuminates the analogy between the model of Bondi and that here described:

(27) $$d\mathcal{T}^2\ =\ \left\{\mathrm{dt}^2 - e^{2\mathrm{t}}\big((d\rho^2 + \rho^2(d\vartheta^2{+}\,sin^2\vartheta\,d\varphi^2)\big)\right\}/\left(1{-}\rho^2e^{2\mathrm{t}}\right)$$

(NB: on account of (25)&(26), eq.(20) with hyperbolic space is equal to eq.(27) with flat space). The significance of the t-scale is that it maps our universe as infinite and ever expanding, and yet as enclosed within *a finite pseudo-sphere*, confer the beautiful *Circle Limit 4* of M.C. Escher: *the contents of the sphere reaching actual infinity at its periphery, there is nothing beyond.*

A static model with shrinking atoms is found by the regraduation $\mathrm{T} \equiv 1{-}e^{-\mathrm{t}} = e^{-t}/\gamma_r$:

(28) $$d\mathcal{T}^2\ =\ \left\{\mathrm{dT}^2 - \left(d\rho^2 + \rho^2(d\vartheta^2{+}\,sin^2\vartheta\,d\varphi^2)\right)\right\}/\left((1{-}\mathrm{T})^2{-}\rho^2\right)$$

So we have constructed a steady state model free of arbitrary constants and horizons. Milne explicitly considered such property as distinctive of a sound model of the universe.

From a philosophical point of view, the metrics in t & T make it clear that our universe constitutes *a perfect synthesis of the opposite positions of Parmenides and Heraclitus.*

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ACKNOWLEDGMENTS

The author expresses his gratitude to Søren Halse for several very helpful discussions, and to Alexei Nesteruk for having checked the math. Their kindness towards the author does not oblige them to assume any responsibility for his views, nor for any possible mistakes.

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