Alternative Interpretation of Special Relativity Formulae

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In this paper we show that with the use of hyperbolic functions calculus the Einstein formula for velocity addition and the Lorentz transform formulae can be both derived from the Minkowski space-time formula. This simply means that the formulae are fully consistent, although it says nothing about the physical meaning of the symbols used. We claim that two different versions of physical interpretation of the formulae are possible. In the Special Relativity Theory moving objects are considered in two different inertial frames of reference. Except for the Minkowski proper time, other physical quantities are considered as relative. It is believed that even the simultaneity is relative. We propose something quite different, a notion in which we have adopted: the Minkowski formulae as the definition of a local time, proper time as the universal time, relative distance as the absolute distance, and relative time as the local time. In the Minkowski space-time (one frame of reference only) we consider the following: two observers A and B (moving or stationary), their distances from the origin of coordinates and resulting local times. When the distance remains unchanged, i.e. the object or the observer do not move, the difference between the indication of local time and the indication of universal time is constant. With the change of distance (the object or the person moves) the local time depends on an absolute velocity of that movement. In the theory of local time there is no relativity of simultaneity. When comparing the two possible versions of interpretation it is evident that the theory of local time is at least as believable as the Special Relativity Theory.

1. Introduction

In the earlier paper [1] we attempted to compare SRT with a local time theory (LTT) for the solar planetary system. At that time we knew nothing about the relationship between SRT and the hyperbolic functions theory. Here we do not propose to replace SRT by LTT, but we show that the meaning of physical symbols used in SRT is not the only one. In other words we propose an alternative interpretation of the symbols without changing the fundamental formulae.

Special Relativity Theory (SRT) in fact deals with two different physical phenomena. In the first group we consider moving objects in two different inertial frames of reference: stationary and non-stationary. In the second group we consider changes of rods and clocks caused by their movement. For small velocities the first group formulae reduce to Galilean transform formulae. That is not the case with the second group formulae as they have nothing to reduce to.

We are going to show that in the case of the first group, the Minkowski formula forces the use of hyperbolic functions. The reason for doing so is obvious. It guarantees that the velocity of any object cannot exceed the speed of light. In section 2 we show that the use of hyperbolic functions makes formulae of the first group clearer and shows their mutual consistency. In section 3 we show that the formulae for changes of moving clocks and rods do not fit that in the first group. In section 4 we present the alternate meaning of the symbols used in the formulae of the first group. The numerical example showing this new interpretation of physical symbols can be found in section 5. Conclusions are presented in section 6.

The assumptions used in the paper are as follows:

A. In a physical equation which can be presented as a mathematical equation of hyperbolae, the physical symbols are hyperbolic functions.
B. In any inertial frame of reference (stationary or not) except for the observer located in the origin of the coordinates system, there can also be other observers that are not fixed to the origin of coordinates (A and B in this paper).
C. For a given interpretation a physical symbol (e.g. primed velocity) means the same throughout all formulae of the group.
D. It is all right to consider the simplest case of moving objects and observers.

2. Hyperbolic Functions in SRT

In the Minkowski formula

$$t^2 - (x/c)^2 = s^2 = (t')^2 - (x'/c)^2 \tag{1}$$

by substituting

$$t/s = ch\theta \qquad x/cs = sh\theta \tag{2, 3}$$

and

$$t'/s = ch(\theta - \eta) \qquad x'/cs = sh(\theta - \eta) \tag{4, 5}$$

the following parametric equation of hyperbolae can be obtained

$$ch^2\theta - sh^2\theta = 1 = ch^2(\theta - \eta) - sh^2(\theta - \eta) \tag{6}$$

Then by exploiting the relationships between hyperbolic functions we obtain the formulae below:

$$th\theta = \frac{sh\theta}{ch\theta} = \frac{x}{ct} = \frac{v}{c} \tag{7}$$

$$ch\theta = \frac{1}{\sqrt{1-th^2\theta}} = \frac{1}{\sqrt{1-(v/c)^2}} = \gamma_v \tag{8}$$

Comparing equations (2) and (4), using formulae for cosinus hyperbolic of a difference, defining velocity u and coefficient γ_u

$$\frac{u}{c} = th\eta \tag{9}$$

$$\gamma_u = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-th^2\eta}} = ch\eta \tag{10}$$

the following is obtained

$$\begin{aligned} t' &= t\frac{ch(\theta-\eta)}{ch\theta} = \frac{t}{ch\theta}(ch\theta \cdot ch\eta - sh\theta \cdot sh\eta) \\ &= t \cdot ch\eta \cdot \left[1 - th\theta \cdot th\eta\right] = t \cdot \gamma_u \left[1 - uv/c^2\right] \end{aligned} \tag{11}$$

Likewise, comparing equations (3) and (5), using formulae for sinus hyperbolic of a difference and equation (9), the following can be obtained:

$$\begin{aligned} \frac{x'}{c} &= \frac{x}{c}\frac{sh(\theta-\eta)}{sh\theta} = \frac{x/c}{sh\theta}(sh\theta \cdot ch\eta - ch\theta \cdot sh\eta) \\ &= ch\eta \cdot \left[\frac{x}{c} - \frac{x}{c}\frac{th\eta}{th\theta}\right] = \gamma_u\left[\frac{x}{c} - \frac{u}{c}t\right] \end{aligned} \tag{12}$$

Thus the Lorentz transform formulae can be derived from the formula of Minkowski. Finally, using equations (4) and (5) and formulae for hyperbolic tangent of a difference, the Einstein formula for velocity addition is obtained:

$$\begin{aligned} &\frac{v'}{c} = \frac{x'}{ct} = \frac{sh(\theta-\eta)}{ch(\theta-\eta)} = th(\theta-\eta) = \\ &\frac{th\theta - th\eta}{1 - th\theta \cdot th\eta} = \frac{v/c - u/c}{1 - uv/c^2} \end{aligned} \tag{13}$$

Please note that the Minkowski formula is sometimes erroneously written as

$$t^2 - (x/c)^2 = 0 = (t')^2 - (x'/c)^2 \tag{14}$$

which fits the second Einstein postulate but destroys the consistency shown above. The left hand side and the right hand side of eq. (14) are different from null, which can be easily proved (see the numerical example).

A correct version of Minkowski formula can be found in the work by Roger Penrose. [2] The existence of the hyperbolic functions in SRT has also been mentioned by W. A. Ugarow. [3]

3. Length Contraction and Time Elongation Do Not Fit Formulae of SRT

In the previous section we have proved that the three: Einstein, Lorentz and Minkowski's formulae are fully consistent. However, they say nothing about length contraction and time elongation as such phenomena are described by the following [4]

$$x' = \gamma_u x \qquad t' = t/\gamma_u \tag{15, 16}$$

At first it seems that the formula (15) can be obtained if in equation (12) time equal to zero is substituted. It is not so, however, because according to eq. (2), $t = 0$ implies $s = 0$ (cosinus hyperbolic is equal to or greater than 1). Then according to equation (3), $s = 0$ implies $x = 0$.

Similarly, if equation (16) is to be obtained from equation (11), $u = v$ is required, which according to equation (13) results in $v' = 0$ and $x' = 0$. It is evident that formulae (15) and (16) do not fit basic formulae of SRT. Kruusing [5] is of a similar opinion. Osborne and Pope [6] present yet another point of view on the matter.

4. The Meaning of Symbols in SRT Formulae

So far we have not mentioned the meaning of symbols used in the considered formulae. Let us do it now. In SRT we usually consider a moving object in two different frames of reference: stationary in which an observer does not move and non-stationary in which the observer does move in the same direction as the moving object. In the case of the latter, the distance x, time t and velocity v are primed, whereas in the former they are not. Minkowski proper time s is velocity-independent but other times and distances are velocity-dependent. The difference between a stationary and non-stationary frame of reference is exemplified in Wikipedia. A car moving with velocity v is considered as a moving object. A stationary observer is fixed on the road. A non-stationary observer travels in the other car with velocity u. Please note that the formulae considered in the Wikipedia example say nothing about length contraction or time elongation.

We have already shown the fundamental meaning of the Minkowski formula in SRT. In the alternative interpretation the Minkowski formula is considered as a definition of local time for two different observers A and B in the same stationary frame of reference. We consider proper time s as the universal time and other times as the local ones, dependent on the universal time and the distance from the place in which the clocks of both times (universal and local) indicate the same. If the observer does not move, the difference between local time and the universal time is constant. Otherwise it is not. Below, the meanings of the eight symbols for the SRT and the alternative versions are presented. Arguments θ, η are called rapidity (adopted from Wikipedia).

1. Proper time s *universal time.*
2. Normalized distance interval of a moving object in a stationary frame of reference $x/c = s \cdot sh\theta$. *Normalized absolute distance interval of the observer A.*
3. Time interval of a moving object in a stationary frame of reference $t = s \cdot ch\theta$. *Local time at the location of the observer A for a given universal time.*
4. Normalized object velocity in a stationary frame of reference $v/c = th\,\theta$. *Normalized local velocity of the observer A.*
5. Normalized relative velocity of a non-stationary frame of reference with respect to the stationary one $u/c = th\eta$. *Rapidity showing the difference between velocities of the observer A and the observer B.*
6. Normalized distance interval of an object moving in a non-stationary frame of reference $x'/c = s \cdot sh(\theta-\eta)$. *Normalized absolute distance of the observer B.*
7. Time interval of a moving object in a non-stationary frame of reference $t' = s \cdot ch(\theta-\eta)$. *Local time at the location of the observer B for a given universal time.*

8. Normalized object velocity in a non-stationary frame of reference $v'/c = th(\theta-\eta)$. *Normalized local velocity of the observer B.*

5. Numerical Example of Calculations

Let us consider the example in which both observers A and B start from the origin of coordinates at the same moment and travel during the same universal time interval equal to 10 hours.

Local velocity of the observer A has been obtained assuming rapidity θ; local velocity of observer B assuming rapidity $\theta-\eta$.

$$\theta = 55^0 \qquad \eta = 29.79^0 \qquad \theta-\eta = 25.21^0$$

$$\frac{v}{c} = th\theta = 0.74428 \quad \frac{v'}{c} = th(\theta-\eta) = 0.41364$$

The same applies to absolute velocities w/c and w′/c defined below.

$$\frac{w}{c} = sh\theta = 1.1144 \quad \frac{w'}{c} = sh(\theta-\eta) = 0.45434$$

In order to calculate the absolute distances x/c and x′/c

$$\frac{x}{c} = s\cdot sh\theta = 11.144 \quad \frac{x'}{c} = s\cdot sh(\theta-\eta) = 4.5434$$

as well as local times *t* and *t′*

$$t = s\cdot ch\theta = 14.9729 \quad t' = s\cdot ch(\theta-\eta) = 10.9837$$

the information that universal time s =10h is required.

The bottom row of Table I gives the results for universal time equal to ten hours. The remaining rows of Table I present the results for two, four, six and eight hours of universal time. Columns two and three show the distance and a local time for the observer A; columns four and five the distance and the time for the observer B. Distance is measured from the place in which local time equals the universal time.

s	x_A/c	t_A	x_B/c	t_B
0	0	0	0	0
2	2.23	2.49	0.91	2.20
4	4.46	5.99	1.82	4.39
6	6.69	8.98	2.73	6.59
8	8.92	11.98	3.63	8.79
10	11.1	14.97	4.54	10.98

Table 1. Local time for two observers in the Minkowski space-time

Let us explain the meaning of velocity *w*. Product of this velocity and proper time *s* equals (by definition) the product of the velocity *v* and local time t. Both products result in an identical value of the absolute distance *x*. The same applies to the primed symbols. That velocity we consider as the absolute one.

In terms of the SRT the interpretation of the results in Table 1 would be significantly different. The distances and times for the observer A would be considered as describing an object moving in a stationary frame of reference. The distances and times for the observer B would be considered as describing the same object in a non-stationary frame of reference.

Please note that the proper time *s* equals 9.99995 and 9.99996 when calculated using non-primed distance and time and primed distance and time respectively.

Conclusions

Concluding, the following can be formulated:

1. The basis of SRT constitutes the formula of Minkowski. It forces the use of hyperbolic functions and it is the starting point for derivation of the Lorentz transform formulae and the formula for velocity addition. Consequently these formulae are fully consistent and any change introduced to any one of them will result in existing consistency being destroyed.
2. The meaning of symbols used in the considered formulae is not necessarily the only one. Here we have presented an alternative interpretation of the meaning of the symbols. In the SRT interpretation a moving object is considered in two different inertial frames of reference i.e. stationary and non- stationary. In the alternative interpretation we consider two moving observers in one stationary frame of reference.
3. The formulae for length contraction and time elongation do not fit the basic formulae of SRT presented in section 2. Whatever the meaning the symbols have, they do not fit from the mathematical point of view.
4. Except for the usual meaning of the physical symbols used in SRT, there exists an alternative meaning presented in this paper and considered as a theory of local time. Being mathematically correct, the theory however (as opposed to the local time theory used on Earth) has nothing in common with physical reality. Perhaps, this might also be the case with the original SRT. It secures no more than the notion that the velocity of an object cannot exceed the speed of light.
5. The Minkowski formula forced the connection between a hyperbolic functions calculus and the formulae of SRT. In particular it imposed a new way for velocity addition, which is in disagreement with the rules of vector calculus and the principle of momentum conservation. It would be useful to test the formula within the realm of Quantum Mechanics.

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