Stellar Aberration, Relative Motion, and the Lorentz Factor

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Presented are the results of an in-depth investigative analysis conducted to determine the relationship between stellar aberration and the principle of relative motion. Introduced are two new principles of physics that were discovered during the process. The first of these principles involves the Lorentz factor and its inherent relationship to the principle of stellar aberration. The second involves the newly defined principle of distance perspective that integrates the principle of inverse stellar aberration with the principle of relative motion.

1. Introduction

After dispelling a possible misconception involving the standard principle of stellar aberration[1] a systematic analysis is conducted to determine the relationship of the standard effect to that of the inverse effect introduced in a previous work[2] by this investigator. In the process a connection is made between the standard effect and the Lorentz transformation factor[3] that seems to imply that relativistic effects are of a local nature and do not apply over great distances. The final discovery involves a new principle called distance perspective that defines the mathematical relationships between the principles of inverse stellar aberration and relative motion.

2. Displacement of Objects in Space as a Function of Distance

Given the observational evidence that the angle of stellar aberration is the same for all objects in the same locality of space it directly follows that the spatial displacement of those objects is a direct function of their distance to the observer. This behavior can be confusing depending on the perspective from which it is viewed. For example, in referring to Figure 1

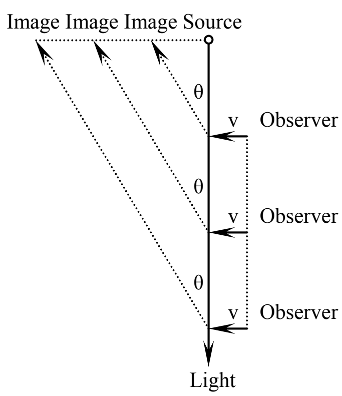

Fig. 1. Image Displacement for Sources at Different Distances

where the distance traveled at speed v and angle of observation θ is the same for each of the observers as they observe light propagating in a perpendicular direction to their path of motion, the distance by which the source's image is displaced to the left is clearly a function of the distance between the source and observer. To the casual observer this might seem to indicate that stars are seen in a scattered arrangement from their true positions in space as a function of their distance to the observer. That is not the case, however, as shown in Figure 2 where the relationships between the stars and observer are given from a different perspective.

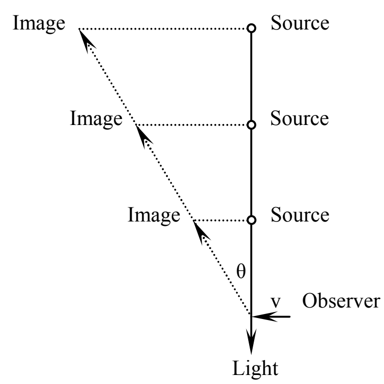

Fig. 2. Image Displacement for Sources at Different Distances

As can be seen in Figure 2 where speed v and the angle of observation θ are the same as in Figure 1, the farther the source is from the observer, the greater its image displacement must be in order to be seen at the same location in space as closer sources located in the same perpendicular direction from the point of observation. From this second perspective it is quite clear that the stars, regardless of their distance to the observer, will be observed in the same general arrangement relative to each other as in their true position but simply at a shifted location in space. It should also be understood that the amount of shift for a low orbital speed v, similar to that of Earth's orbit, is very small, too small, in fact, to illustrate faithfully. Subsequently, unless otherwise stated, such shifts are greatly exaggerated in this work for the sole purpose of illustrative practicality.

3. Stellar Aberration and the Lorentz Factor

In preparation for the process of extending the principle of stellar aberration to its inverse form involving its relationship to the principle of relative motion one final aspect of its behavior in the standard form must be considered. It involves the newly discovered direct relationship of stellar aberration to the Lorentz transformation factor that to the investigator's knowledge went undetected until this present research effort. Referring to Figure 3, the revised form of stellar aberration as given in the investigator's earlier work is illustrated whereby light speed c is assigned to the path the light travels in the moving frame of the Earth, and light speed L is assigned to the path light follows in the frame of the Sun. This is a reversal of the two paths as originally defined by James Bradley in order to bring the standard theory of stellar aberration into conformance with the evidence that supports relativistic theory.

With the understanding that light path L from the source is perpendicular to the orbital plane of the Earth, where v is Earth's orbital speed around the Sun, we have the Pythagorean triangle shown in the illustration.

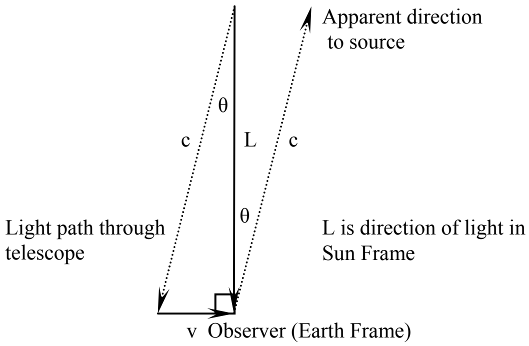

Fig. 3. Stellar Aberration - Revised

As previously shown[2], if θ is the angle of stellar aberration with regard to standard theory, then the standard formula for stellar aberration is given by

$$\theta = \operatorname{asin}\left(\frac{v}{c}\right) \tag{1}$$

where v is the speed of the observer and c is the speed of light. Using v = 29783 m/s for Earth's orbital speed, and c = 299792458 m/s for the speed of light we obtain θ = 0.00569207182 degrees, or θ = 20.4914586 arc seconds that is in full agreement with the evidence.

Alternatively, however, we could have solved for the angle of stellar aberration θ using the formula

$$\theta = \operatorname{acos}\left(\frac{L}{c}\right) \tag{2}$$

where L is the speed of light in the frame of the Sun (i.e., the stationary frame relative to which speed v is referenced) Since L is given by

$$L = \sqrt{c^2 - v^2} \tag{3}$$

we have by way of substitution into equation (2)

$$\theta = \operatorname{acos}\left(\frac{\sqrt{c^2 - v^2}}{c}\right) \tag{4}$$

where the resulting term

$$\frac{\sqrt{c^2 - v^2}}{c} \tag{5}$$

is the Millennium Relativity[4] form of the Lorentz transformation factor. This factor is simply another form of the more familiar version

$$\sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

used in Special Relativity[5]. In the interest of completeness, the gamma form of the factor is given by

$$\theta = \operatorname{asec}\left(\frac{c}{L}\right) \tag{7}$$

$$\theta = \operatorname{asec}\left(\frac{c}{\sqrt{c^2 - v^2}}\right) \tag{8}$$

or

$$\theta = \operatorname{asec}\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) \tag{9}$$

giving

$$\theta = \operatorname{asec}\gamma \tag{10}$$

for the final version using the γ gamma symbol in place of the gamma factor. Of course, all of these versions are valid only for the case where the light propagating at speed L is received in a direction that is perpendicular to the path of observer motion defined by speed v.

4. Inverse Stellar Aberration and the Light Clock

If the motion defined by speed v is assigned to the source instead of the observer and if the source's path of motion is local to the location of the observer and the received light travels a path that is perpendicular to the source's path of travel, we then have the simplest case of inverse stellar aberration as illustrated in Figure 4. In such case the ratio of distances D_v to D_c, is equal to the ratio of speeds v to c as given by

$$\frac{D_v}{D_c} = \frac{v}{c} \tag{11}$$

were D_v and D_c are the respective distances traveled at speeds v and c in the frame of the observer. The reason for this distinction will become clearer in Sections (5) and (6) when the equality just defined is broken. Referring now to Figure 4, the illustrated Pythagorean triangle is simply an upside-down version of the abbreviated form of the special relativity light clock used in the millennium theory of relativity. (By abbreviated it is simply meant that the second half of the light clock that gives the reflected path of the light is omitted on the basis of redundancy.)

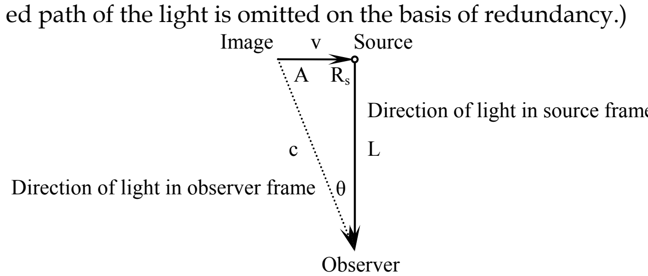

Fig. 4. Inverse Stellar Aberration (Local)

Since the triangle of Figure 4 has the same trigonometric relationships as the triangle of Figure 3 (discounting the reversal of the relationship between the source and observer) all of the formulas derived from Figure 3 also apply to Figure 4. Of special interest are equations (4) and (8) that establish beyond doubt the relationship of the Lorentz factor with the inverse nature of stellar aberration. The importance of this discovery will become clearer in Sections (5) and (6) when these factors take on a modified form as determined by the evidence.

5. The Principle of Distance Perspective

The first step in developing the mathematical relationships required to correctly integrate the principle of inverse stellar aberration with the principle of relative motion is that involving the apparently previously overlooked principle of distance perspective. Although the rationale for this principle is straightforward and easily understood, the mathematical derivation can be very confusing. Beginning with the rationale used, it is easily understood that all objects appear to grow smaller as they move off into the distance. If that is the case then all of the dimensions associated with such objects appear to shrink in direct comparison to identical objects in the vicinity of the observer. Stated another way, all far off distances appear shrunken in inverse proportion to their distance from the observer. To define this principle mathematically we need to simply, in the geometrical sense, place an object or dimension at a defined distance from an identical object oriented in an identical manner at the location of the observer. Or, otherwise stated, if the local distance traveled at speed v is proportional to the local distance traveled at the speed of light, then the remote distance traveled at speed v is the same as the local distance traveled at speed v, but the distance traveled at the speed of light is the increased distance between the light source and the observer. What we are doing then, is holding the distance traveled at speed v constant while extending the distance traveled by light at speed L to the distance between the source and the point of observation. Then, as with the local relationships between distances c, v, and L, the distance traveled by light at speed c remains a direct trigonometric function of the distances traveled at speeds v and L. The resulting triangular relationships will be given in the next Section.

6. The Principles of Relative Motion and Inverse Stellar Aberration

To integrate the principle of relative motion with the principle of inverse stellar aberration we need to simply invoke the principle of distance perspective discussed in the previous Section. For the simpler case where the distance defined by L is perpendicular to the distance traveled at speed v by the source we have the Pythagorean triangle relationships illustrated in Figure 5.

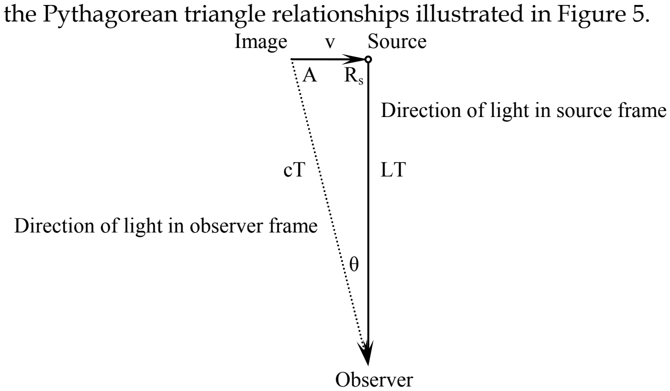

Fig. 5. Inverse Stellar Aberration

In this case the basic formula for the angle of inverse stellar aberration θ is given by the modified version of equation (1) in the form of

$$\theta = \operatorname{asin}\left(\frac{v}{cT}\right) \tag{12}$$

where v is the speed of the source, c is the speed of light in the frame of the observer and T is the time interval during which the light propagated from the source to the observer, at speed L in the frame of the source, and at speed c in the frame of the observer. Time interval T is normally given in seconds and is valid in the range

$$1 \le T \le \infty \tag{13}$$

as determined in the stationary frame of the observer. It is understood here that L is the variable speed of light in the frame of the observer and is subsequently valid for use with stationary frame interval T. This is a perfectly valid alternative to the method where light has a constant speed c in the moving frame of the source and subsequently requires time in the source's frame to vary relative to the time in the stationary frame as defined by the time transformation formula

$$t = T\frac{\sqrt{c^2 - v^2}}{c} \tag{14}$$

using the millennium relativity version of the Lorentz transformation factor.

Similar to the treatment used for standard stellar aberration, an alternative formula for the angle of inverse stellar aberration is available in the form of a modified version of equation (2) given as

$$\theta = \operatorname{acos}\left(\frac{LT}{cT}\right) \tag{15}$$

where T is the stationary frame time during which the light traveled from the source to the observer. Although it might seem that this is actually the same equation as equation (2) since the variable T cancels out of the fraction, such is not actually the case. The reason is simple. Variable L has a different definition in this latter version of the equation from that defined by equation (3) used in the original equation (2). That is, in the present case L is defined by

$$LT = \sqrt{(cT)^2 - v^2} \tag{16}$$

$$L = \frac{\sqrt{(cT)^2 - v^2}}{T} \tag{17}$$

$$L = \sqrt{\frac{c^2T^2 - v^2}{T^2}} \tag{18}$$

$$L = \sqrt{c^2 - \left(\frac{v}{T}\right)^2} \tag{19}$$

that when substituted into equation (15) (after the cancelation of the T variables) gives

$$\theta = \operatorname{acos}\left(\frac{\sqrt{c^2 - \left(\frac{v}{T}\right)^2}}{c}\right) \tag{20}$$

for the complete version of the alternate formula for inverse stellar aberration. Of great interest is the fact that the factor included in the brackets of equation (20) is a modified form of the millennium relativity transformation factor arrived at earlier in equation (4). Simplification of equation (20) in the manner of

$$\theta = \operatorname{acos}\left(\sqrt{\frac{c^2 - \left(\frac{v}{T}\right)^2}{c^2}}\right) \tag{21}$$

$$\theta = \operatorname{acos}\left(\sqrt{1 - \frac{\frac{v^2}{T^2}}{c^2}}\right) \tag{22}$$

$$\theta = \operatorname{acos}\left(\sqrt{1 - \frac{v^2}{c^2 T^2}}\right) \tag{23}$$

then gives

$$\theta = \operatorname{acos}\left(\sqrt{1 - \left(\frac{v}{cT}\right)^2}\right) \tag{24}$$

for the final version based on a modified form of the Lorentz transformation factor. The importance of this finding will be discussed later in Section 12.

7. The Complete Formulas for Inverse Stellar Aberration

Now that the principles have been established involving the true nature of stellar aberration we can derive the general formulas for inverse stellar aberration at all angles of observation from 0° to 180°. We begin by referring to Figure 6 where the trigonometric relationships of an approaching and a receding source are illustrated.

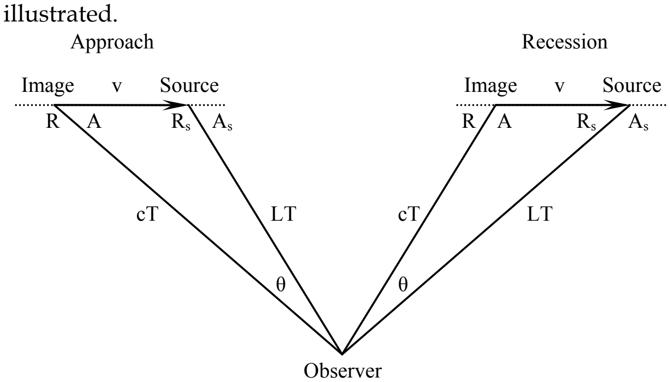

Fig. 6. Inverse Stellar Aberration

In referring to Figure 6 the following definitions apply:

- c – Constant Speed of Light in the Stationary Frame of the Observer
- v – Speed of Source in the Stationary Frame of the Observer
- L – Variable Speed of Light in the Stationary Frame of the Observer
- T – Stationary Frame Time Interval during which Distances Were Traveled
- R – Angle of Recession (Image – What is actually seen)
- A – Angle of Approach (Image – What is actually seen)
- R_s – Angle of Recession (Source – The actual physical object)
- A_s – Angle of Approach (Source – The actual physical object)
- θ – Angle of Separation between Image and Source – Inverse Stellar Aberration

Next we apply the Law of Cosines to the propagation triangles of Figure 6 to obtain

$$(LT)^2 = (cT)^2 + v^2 - 2cTv\cos A \tag{25}$$

where the constant and variables have the definitions just given. From equation (25) we then derive

$$LT = \sqrt{(cT)^2 + v^2 - 2cTv\cos A} \tag{26}$$

for light propagation distance LT. This is the actual distance between the source and observer at the instant of observation. For our next step we apply the Law of Sines to the same propagation triangles to obtain

$$\frac{\sin\theta}{v} = \frac{\sin A}{LT} \tag{27}$$

that through the process of substitution with equation (26) gives

$$\frac{\sin\theta}{v} = \frac{\sin A}{\sqrt{(cT)^2 + v^2 - 2cTv\cos A}} \tag{28}$$

that in turn yields

$$\sin\theta = \frac{v\sin A}{\sqrt{(cT)^2 + v^2 - 2cTv\cos A}} \tag{29}$$

for the sine of θ. This then gives

$$\theta = \operatorname{asin}\left(\frac{v\sin A}{\sqrt{(cT)^2 + v^2 - 2cTv\cos A}}\right) \tag{30}$$

where θ is the angle of inverse stellar aberration. In applying the above formula in relation to the propagation triangles of Figure 6 it is understood that the relationship between supplementary angles A and R is given by

$$A = 180° - R \qquad \text{and} \qquad R = 180° - A \tag{31}$$

where A is the angle approach and R is the angle of recession with respect to the point of observation.

8. Confirming the Principle of Relative Motion

In comparing the trigonometric relationships of Figure 6 that equation (30) is based on to the trigonometric relationships of Figure 7 that follows, it is easily seen that, except for the variable T used in Figure 6, the two relationships are that of relative motion. That is, if speed v of the sources of Figure 6 is attributed instead to the Earth in Figure 7 the relationships are unchanged.

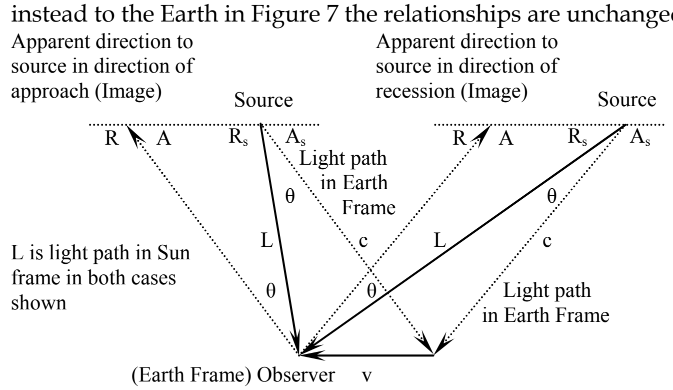

Fig. 7. Stellar Aberration - Revised

This is equivalent to assigning the value of unity to time interval T in Figure 6 and subsequently means that the principle of distance perspective is a factor in relative motion as already established in the previous Sections. If we next, reverse the direction of motion of the Earth in Figure 7 as shown in Figure 8 we can make a direct comparison with the trigonometric relationships given previously in Figure 3 that equation (1) is based on. Then, setting light speed L in Figure 8 to the value given by equation (3) we have the Pythagorean triangle of Figure 3 that equation (4) is based on. In that case equation (1) also applies to the trigonometric relationships of Figure 8.

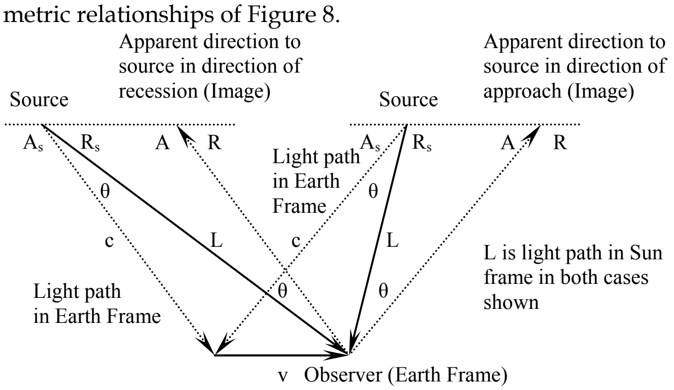

Fig. 8. Stellar Aberration - Revised

There is, however, a problem with equation (1) that has never been properly addressed before. Equation (1) is based on an angle that can only be arrived at indirectly since it is not directly discernable through visible means. This shortcoming will be discussed in greater detail in the next Section.

9. Reinterpreting the Standard Formula for Stellar Aberration

It appears that most explanations of stellar aberration are based on formulas such as equation (1) in the present work that are in turn based on a Pythagorean triangle. With that being the case, the angle given by the formula is valid only for a right triangle such as that shown in Figure 3 and repeated below in Figure 9 with angles A and R_s added.

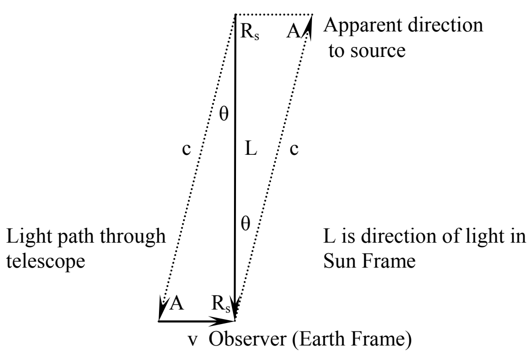

Fig. 9. Stellar Aberration - Revised

It is understood then in referring to Figure 9 that R_s is a 90° angle. With that being the case, if the angle at which the source is viewed can be independently established to be θ = 20.4914586 arc seconds in advance of the direction of the Earth's orbital motion, as discussed in Section 3, then the physical source must actually be located directly above the observer in a direction perpendicular to the Earth's motion at that instant. It's really that simple. The problem, however, is that formulas such as equation (1) are only valid for the right triangle relationships just discussed. For all other triangular relationships a more general formula such as equation (30) is required. And now that we understand the principle of distance perspective it is clear that equation (30) reduces to

$$\theta = \operatorname{asin}\left(\frac{v \sin A}{\sqrt{c^2 + v^2 - 2cv\cos A}}\right) \tag{32}$$

for use in determining stellar aberration angle θ at any angle A. This, of course, follows from the fact that time interval T disappears at the location of the observer because it takes on the value of unity. It is important to remember here that we only see the image of the source and not the source itself. Or, stated otherwise, angle A and its supplementary angle R, are the only angles that are directly associated with the observed image along its path of motion relative to the observer. (Note: Equation (32) verified in previous work[6].)

10. Summarizing the Principle of Distance Perspective

There is no doubt that the concept of distance perspective can be very difficult to completely comprehend. That is most likely the reason why it was not discovered earlier. Yet, with a little thought it is readily seen that the principle of distance perspective is absolutely valid and can no longer be ignored in the application of the combined sciences of physics, mathematics, and cosmology. With that objective in mind, we can summarize the principle of distance perspective as follows: According to the principles of Newtonian mechanics we observe an object to move a certain distance during a certain interval of time. When considering this proposition it is quite obvious that such distance to time relationship does not change simply because the object is farther away in one case as opposed to another. More specifically, it is quite obvious that the distance an object travels at a given speed v during a specific time interval T is independent of the distance to the observer. Thus, we can determine the distance an object travels at speed v during time interval T locally and place that same distance as far away as necessary to evaluate its geometrical relationship relative to its distance from the point of observation. Or simply stated, the distance traveled at speed v remains constant while the distance to the observer increases to whatever distance necessary to define the actual distance of observation.

Referring to Figure 10 we can visualize what was just discussed in a more graphical sense.

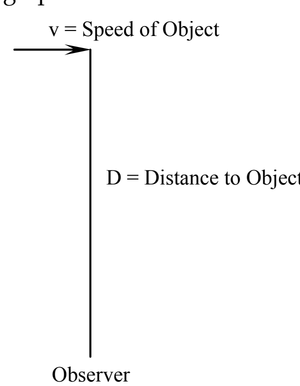

Fig. 10. Distance Perspective

In viewing the illustration it should be quite apparent that the distance traveled by the moving object at speed v is dependent only on the time interval during which the motion takes place and is not affected by the distance to the observer. Hence for an observer at twice the distance D illustrated, the distance traveled at speed v during the same interval T is unchanged. Subsequently, a triangle constructed from the two distances illustrated becomes narrower in inverse proportion to the distance to the observer as shown previously in this work. It should also be obvious that the angle at which the motion of the object is observed also has no affect on the distance traveled. Again, only the distance between the object and the observer matters and the farther away the object, the narrower the resulting triangle.

When the principle of distance perspective becomes part of our normal thought process the following characteristics become evident:

1. We can hold distance D constant and increase or decrease distance v because speed v increased or decreased.
2. We can hold distance v constant and increase or decrease distance D because the distance to the observer increased or decreased.
3. We can increase or decrease both distance v and distance D because both speed v and the distance to the observer increased or decreased.
4. We can increase or decrease distance v while doing the opposite to distance D because speed v increased or decreased while the distance to the observer did the opposite. And so on.

11. Summarizing Stellar Aberration, Inverse Stellar Aberration, and Lorentz, Effects

Figure 11 was added as a final step to aid in the assimilation process involving all of the principles introduced in this latest research effort by this investigator.

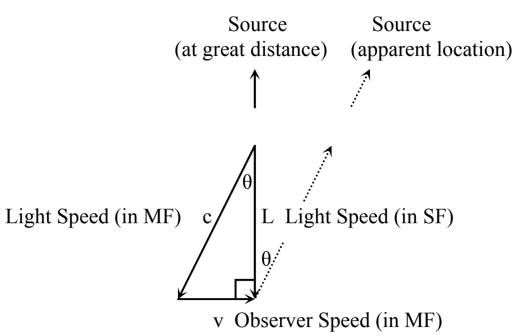

Fig. 11A. Local (Stellar Aberration)

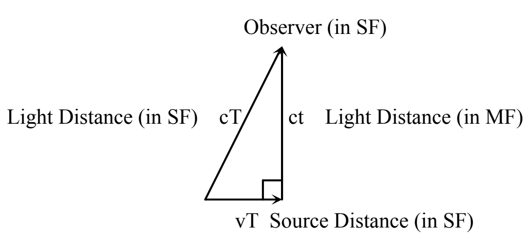

Fig. 11B. Local (Lorentz Effect)

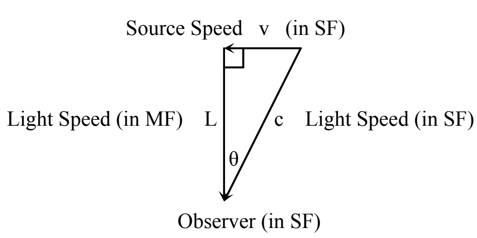

Fig. 11C. Local (Inverse Stellar Aberration)

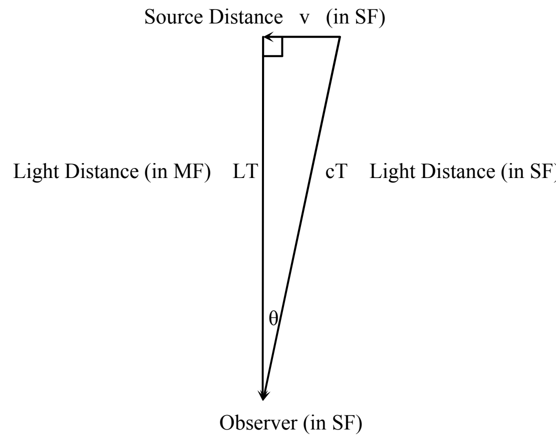

Fig. 11D. Remote (Inverse Stellar Aberration)

Fig. 11. Stellar Aberration – Light Clock – Inverse Stellar Aberration

Reviewing each of the four included illustrations should be helpful with regard to visualizing how the different effects relate to each other. And whereas, similar to the case with distance perspective, the manner in which all of these effects are integrated together can be very difficult to comprehend, the need to do so is of ultimate importance if we wish to advance our understanding of the principles that appear to govern the universe. This is because from an almost paradoxical point of view what has been shown in this work is so fundamental as to be obvious in terms of validity. Let us now discuss one of the more obvious repercussions of this new point of view.

12. A Fundamental Change in Our View of the Universe

Based on our previous understanding of the universe, that did not include the principle of distance perspective, we erroneously concluded that the objects we see at great distances in space are no longer at the locations in which we see them. That is, due to the expansion of the universe they have continued to move away from us during the time it took the light they emitted to reach us. Thus, we are seeing them as they appeared in the past when the light was emitted and at the locations they were in at that time. From what was shown here, however, due to the principle of distance perspective, the farther away an object is the less effect its motion has on where it is seen by the distant observer. Subsequently, when we view objects far out in space, even those at the edge of the visible universe that are traveling away from us at near the speed of light, we are seeing them where they are and not where they were so long ago. In summary, light does take on the speed of the source as has been shown throughout this work. It is simply because of the principle of inverse stellar aberration that we see the light source at a previous location and observe the speed of its light at the constant value of c. However, as has also been shown here, the effects of stellar aberration and inverse stellar aberration are local effects due to the principle of distance perspective. Subsequently, we observe far off objects right where they are at the instant the light from them is received. While it is true that we see such objects as they appeared in the past when the light from them was emitted, we see them where they are at the time of observation. This conclusion can be arrived at mathematically as will be shown next.

For the purpose of emphasis, let us take the worse case example of an object moving at light speed c in a transverse direction to the point of observation. Using equation (24) it is found that the angle of inverse stellar aberration goes to zero even if taken to 17 decimal places when the distance between the source and observer increases to 4.255 light-years. More specifically, when T is increased to a value of $T = 10^{8.128}$ seconds in equation (24), that equation will produce a result of zero up to 17 decimal places. Since the number of seconds in a year is given by

$$y_s = 60^2 \bullet 24 \bullet 365.256 = 3.156 \bullet 10^7 \text{ seconds} \tag{33}$$

where y_s is the number of seconds in a year, we can divide the value given previously for interval T by y_s to obtain

$$y = \frac{T}{y_s} = \frac{10^{8.128}}{3.156 \bullet 10^7} = 4.255 \text{ years} \tag{34}$$

where y is the time interval in years.

In referring to Figure 11-D that equations (15) through (24) represent it is seen that if the angle $\theta = 0°$ then $L = c$. With that being the case the distance D between the source and observer is given by

$$D = LT = cT \qquad \text{where} \qquad T = 4.255 \text{ years} \tag{35}$$

that equates to traveling at light speed c for 4.255 years, or a distance of 4.255 light-years. Since the nearest star to our Sun is Proxima Centauri at a distance of 4.243 light-years, this means that the 0° result for θ applies to virtually the entire universe with the exception of the local vicinity of our solar system and its immediate location within the Milky Way galaxy. Of course, similar would be true at any other location in the universe. That is, the standard laws of physics are valid only in the immediate vicinity of that location and dissipate entirely within a distance of approximately 4.255 light-years. By way of reference, Proxima Centauri is the companion star of the binary system Alpha Centauri AB that contains the next nearest stars to our Sun at a distance of 4.37 light-years.

As a final point of interest it should be noted that when $LT = cT$ as given in equation (35), the fraction in equation (15) reduces to 1 thereby giving a result of $\theta = 0°$ as already discussed.

13. Conclusion

It has long been apparent to this investigator that experimental and observational evidence founded upon local conditions can be misleading and even invalid when applied to great distances. This does not necessarily mean that the evidence is invalid but rather that it can be incorrectly applied to great distances because of misconceptions derived from its local application. A striking example of such misconceptions is the speed of light. At 300 million meters per second it is the fastest thing known to mankind, yet it takes thousands, millions, and even several billions of years to reach us from relatively nearby stars. Due to locally induced biases we attribute this extraordinarily long travel time to the great distances involved. Yet, we could have just as validly concluded that light only seems fast because it is observed locally. There are endless examples of biases and misconceptions such as this founded on the principle that our knowledge is primarily derived from local experiences. In fact such biases form the very basis of our mathematical and physical sciences since such scientific disciplines were developed primarily to advance our understanding of local experiences and permit beneficial returns. If we wish now to further our understanding of the universe we must break free from these biases and commence with the development of a new branch of physics and mathematics that deals with the opposite extremes associated with the great distances involved. This just completed research by this investigator is but a first step in that direction.

References

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[4] Joseph A. Rybczyk, A series of works, including the present work, beginning with the Millennium Theory of Relativity, (2001), in which the concept of spherical reference frames was introduced. All of the works are available at http://www.mrelativity.net.

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