On Understanding Negative Numbers

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This paper reviews the author's soon-to-be-published book, **The Nature of Negative Numbers**. In this book, the veritable number system is presented much more fully than it was in the earlier book, **Absolute Space, Absolute Time & Absolute Motion**,[1] or elsewhere, including the 2005 NPA Proceedings.[2] It is compared and contrasted to imaginary numbers and, especially, to the received real number system. The argument is that it is superior to the other two.

1. Introduction

Negativity is the theme of this new book, **The Nature of Negative Numbers** - not negativity in general, but in mathematics; not negativity in the whole of mathematics either, but in that part of it which pertains to negative numbers and their positive counterparts. This paper is based upon the first chapter of that book.

The *real number system* is a system of numbers which are continuous in principle and which has for its purpose the inclusion, along with zero, of all finite single numbers, whether rational or irrational, signed or unsigned.[3] It is the received system in mathematics.

There is a famous number which contradicts the real law of signs That number is $\sqrt{-1}$. In the real system, -1 cannot have a square root. There $-\cdot-=+$; also $-\cdot+=-$ and $+\cdot-=-$.

Some regard this contradiction to be a member of a tiny number system, while others hold that it is something which looks like a number but actually is not.[4] Its ontological status is established in the course of the book.

More importantly, another number system is presented. Based upon a different law of signs, it is fully consistent with negative square roots. In that system $\sqrt{-1}\cdot\sqrt{-1}=-1$. It is called the veritable number system.[5] It is not a replacement for the real system, but a complement to it. Arguably, however, it is better than the other system.

2. Historical

CHAPTER II introduces the subject from a historical standpoint and shows how the law of signs employed in the real number system is an adoption from the rules of multiplication used in arithmetic. The record shows that these rules were originated by the Alexandrian Greek, Diophantus. In a Diophantine equation, a subtracted number times another subtracted number is the same as an addition; a subtracted number times an added number or, *vice versa*, is a subtraction.

It is shown that these rules are independent of the real law of signs and could, theoretically, be used in a system contradictory to the real. This chapter also brings forth the fact that the square root of negative one is incompatible with the real law of signs.

The phrase ending the definition of the real number system given above, namely, "signed or unsigned," should be amended to read: "unsigned, or if signed, the signatures are patterned after the Diophantine rules of multiplication."

3. The Imaginary System

CHAPTER III presents the imaginary number system, the standard solution to the problem posed by $\sqrt{-1}$. It consists of two axes placed at right angles with respect to each other in a standard Cartesian format. One of the axes is real and the other is "imaginary." The third part is a moveable radius vector. All three are centered upon zero. Through a special form of multiplication, dubbed here as "rotational multiplication," it connects a real minus one as the "square" of an imaginary number (based somehow upon the square root of some kind of negative one). But this solution is unsatisfactory. It does not answer the question as to what imaginary numbers are in actuality.

4. The Veritable Number System

CHAPTER IV introduces the veritable number system, where not only does $\sqrt{-1}\cdot\sqrt{-1}=-1$, but an operation such as $+1\cdot-1=-1$ is impossible; neither can $+1/-1=-1$ exist. There can be no mixture of signs in that system.

The principle of reflectivity, according to which the product of two negative numbers doubles back and become positive is the hall mark of the real system. That is denied in the veritable system.

This chapter compares and contrasts the way each handles absolute numbers, the real making them effectively the same as positive numbers; the veritable, holding them to be strictly neutral to numbers of either sign.

There is also an important difference between the veritable and real number systems with respect to the handling ofzero. The difference is that the real system goes around zero when changing from the positive to the negative axis, or *vice versa*, while the veritable system goes through zero.

The choice of proper symbolization for the veritable system is essential; "&" represents addition and "~" represents subtraction in the veritable system, with "+" standing for the positive axis and "-" standing for the negative axis, (as it is with the real system).

5. The Imaginary System: A Hybrid

CHAPTER V shows the harmony which exists within the imaginary system where the two incompatible kinds of numbers are associated. The imaginary number is not a conjunction, but an association in which repetitions of the veritable number, $\sqrt{-1}$, supply the base and the real numbers provides the coefficients. An imaginary axis is a stack of ($\sqrt{-1}$)'s separated by a real zero. The count of the stack, whether plus or minus, is in real numbers. They are able to work together by being separate. It is the movement of the radius vector which supplies the unity.

CHAPTER VI performs the act of synthesis which corresponds to the analysis of the previous chapter. It reconstructs the imaginary configuration from the ground up and shows how the two number systems can be associated to form a third kind of number, *i*, without acting upon each other. Some interesting and far reaching peculiarities of the construction are also identified.

APPENDIX A, which appears after the final chapter, shows how the imaginary system can be improved by being made completely circular.

6. Veritable Versus Real

CHAPTER VII compares and contrasts the veritable with the real number system. It begins with a discussion of the number line as an indispensable prerequisite for understanding numbers. It continues with a discussion of how the two signed number systems are modifications of the absolute number system, which is simply numbers without signs. It emphasizes the different ways that the two signed number systems handle the concept of direction; how the negative sign, "-" in the real system is used to mean both subtraction and direction, whereas in the veritable system that sign is used to mean direction and nothing but direction. This chapter shows two important meanings of zero. The first is 0 as a point of neutrality in the number line; it is an infinitesimal and, therefore, has some being; the second is utter zero, symbolized by **0**, and means the total absence of being. The discrimination between the two is essential to the veritable but not the real. By failing to be fully cognizant of the two kinds of zero, the real system divides into the wrong zero.

CHAPTER VIII discusses negative roots and powers. The veritable system can handle negative roots for rectilinear as well as rotary uses. Like the real, it uses " $\sqrt{\ }$ " for the symbolization of roots. Like the other system, it can also represent it with a fractional exponent. The powers of a number are symbolized the same way as in the real number system. The differences show up in the symbolization of veritable negative powers, which are handled in the manner exclusively reserved for positive exponents in the other number system. While the status of negative exponents is fraught with controversy in the real system, its handling is straightforward in the veritable system. Inverses in the veritable system are symbolized by "9". The veritable inverse is more precise than the real; and circumstances exist where that is an advantage.

While a number to the zero power, (A^0), equals 1 in the real system, ($\pm A^0$) can equal either +1 or -1 in the veritable system, depending on the sign of A.

CHAPTER IX shows how the veritable system handles logarithms. This explanation is needed because the range between +1 and -1 in ($\pm A^0$) = ±1 cannot be treated in the veritable system exactly the same way it is in the real system. The answer is that subtraction can take place within an axis without changing signs, i.e. without crossing the 0 of neutrality. This is called "the veritable deficit." The chapter also introduces a third use of the symbol "0", that of a place-setter, such as is found in the decimal system.

CHAPTER X explains how the veritable understanding of deficit negates any superiority which might be claimed for the real system, owing to the latter's recourse to the principle of reflectivity. This chapter also shows how the real number system confutes both subtraction and direction, on the one hand, and utter zero and the zero of neutrality, on the other. This chapter also introduces the veritable surplus, the opposite of its deficit.

CHAPTER XI shows that the veritable system handles fractions with mixed signs like +3 / -4 far better than does the real system. The real number answer that the quotient must be negative actually runs against the standard definition of a fraction as a part of a whole and replaces it with a variant tinged with a positivistic meaning. It also subtly makes division an appendage to multiplication.

CHAPTER XII shows how the veritable system handles Cartesian coordinates better than does its real rival. The signature of the first quadrant is clearly distinguished from that of the third; the second from that of the fourth -which is not the case with the standard real presentation. A symbolization appropriate to veritable Cartesian coordinates is introduced.

CHAPTER XIII shows how easily the veritable system handles polar coordinates. It also tells why curvilinear signs cannot be reduced to rectilinear signs, that the symbols for the former, " ⊕ " and " ⊙ ", can never be equated with + and -. Tabulations of the two sets of signs are given for tangents, cotangents, secants, and cosecants.

CHAPTER XIV shows how the veritable systems can handle certain types of calculus derivatives which the real system must skip. It also solves the apparent difficulty raised by trigonometric derivatives like $d\cos 0 = -\sin 0$, derivatives which have a positive expression on side of an equation and a negative on the other. The solution is the veritable deficit.

CHAPTER XV shows that the real system cannot change its heading; the veritable system must. The principle of reflectivity cannot function in the shift from counterclockwise to clockwise. It is in this chapter that the different ways that the two systems handle negativity is brought to a state of maximum clarity.

CHAPTER XVI show the mistake made by Sir William Rowan Hamilton as he tried to raise vectors upon the basis of real numbers. His system, however, may be adaptable to veritable numbers.

Appendix B presents an earlier attempt Sir William Hamilton to explain $\sqrt{-1}$ without leaving the realm of real numbers. His device was a number couple, which does not answer to the requirement that it be a single number. It is shown to rest upon a radicalization of the Kantian argument. This appendix follows Appendix A.

CHAPTER XVII shows the essential meaning of the veritable system. Direction is a fundamental attribute of space. In the veritable system, the concept of direction is used in its purity, unblemished by subtraction. The real system cannot have a real negative square root, because it has no substance apart from the positive; in the real system, the negative is really a glorified minus, a lesser form of the positive, not an independent axis connected to the positive through a neutral point. The veritable system is not dialectical. If the real system can be compared to the paying of a debt, the total difference between the two axes in the veritable system can be likened unto morality. Knowledge of the veritable system corrects some of the unreality critics have found in modem mathematics. The gateway is thrown open to some possible advancements of mathematical and physical science.

Conclusions

The veritable system can do things which the real system cannot. It can have negative square roots. It is more precise in the determinations of quadrants in coordinate geometry. It can handle paths and, therefore, circuits, with better reason than can its rival. It can do other operations discussed in the course of the book. Yet, it can handle logarithms and the infinitesimal calculus as well as the real system. Knowledge of it reveals the character of imaginary numbers, a character which is still a mystery to people working solely within the real system.

The real system can be viewed as a locomotive. Staring from the positive axis: If it backs-up sufficiently, it will subtract enough from the positive to reach zero. Should it back-up more, it will be in negative territory; the zero point could have been located a little bit differently. With the veritable system, on the other hand, once it has backed up next to zero, the locomotive must change its heading in order to traverse negative territory. To do this, it would have to pivot at zero (or in more practical terms, enter into a circuit at that point). Aside from a change of heading, the veritable system should be used when the negative and the positive are of different quality or substance; with the real system, the negative is only a lessening of the positive.

The mere fact that it can handle negative square roots in a straightforward manner, which has long been a barrier to mathematics, should afford territory for the enterprising physical scientist to explore. After all, mathematics is the primary language of modem physical science, and the latter has difficulty traversing regions clouded over with intellectual imprecision. When a change in direction is involved, ask the question, "Is there one heading or two?" For example, magnetism involves a change of heading, divided by a zone of neutrality, does it not? What other phenomena are like that?

References

[1] Peter F. Erickson, **Absolute Space, Absolute Time, & Absolute Motion**, (Philadelphia, P A: Xlibris, 2006), 267 pp.

[2] Peter F. Erickson, "The Solution to the Mystery of $\sqrt{-1}$," Proceedings of the NPA, Vol. 2, No.1, (Arlington, MA: Space Time Analyses, Ltd., 2005), pp. 25-29.

[3] Other definitions have been offered. For example, according to **The Universal Encyclopedia of Mathematics**, "real numbers are those numbers which can be represented by decimal numbers with finite or infinite number of places (periodic or non-periodic)." [**The Universal Encyclopedia of Mathematics**, With A foreword by James R. Newman, tr. from the German by George Allen and Unwin, Ltd., (New York: Mentor Books, 1964), p. 362.] This definition is unsatisfactory. An "infinite number of places" is a theory which this writer does not accept. Furthermore, the use of the decimal system as differentiam ignores the fractional mode of presentation, which is not only serviceable, but more accurate An example is the rational fraction 1/3, which the digital can only approximate; incalculably more is this the case for common irrational numbers like $\sqrt{2}$. Even worse is the definition sometimes offered is that a real number is one which fits into the equation, a + bi. That makes the meaning of a known depend upon a mystery. The definition offered above of the real number system pertains to what is discussed in this book and is also open to amendment.

[4] W. W. Sawyer, **Mathematician's Delight**, (Middlesex, England: Penguin Books, 1959), p. 219.

[5] Peter Erickson, "Introduction To The Veritable Number System The Solution To The Mystery of $\sqrt{-1}$ ", Certificate of Registration, United States Patent Office (Effective Date 5/21/99).