

[SCANDAL IN ELECTRODYNAMICS, continued]

### MR. SAVARKAR AND THE CASE RINDLER

--with a 'close study' of the paper WORK AND MAKING RELATIVITY WORK

Attention to prior semblance of the Electrodynamic Torque Paradox has been brought forward by one Sadanand Savarkar, physics author and researcher. The work of Professor Wolfgang Rindler is cited in Mr. Savarkar's general NPA message attachment of 13 July 2006:

### **TORQUE PARADOX RESOLUTION**

The 'Electrodynamic Torque Paradox' claimed by Mr. S. I. Wells in a recent communication is actually widely known for long, and its plausible resolution is also discussed even in the text-books. In fact, a similar but absolutely elementary situation was treated in the Twenties, or perhaps even earlier, and is known as the 'Lewis and Tolman Lever'.

Both the situations appear paradoxical but admit plausibly resolutions in terms of the 'momentum densities' constituted by the energy currents.

The 'Torque Paradox' claimed by Mr. Wells appears in fact as the Problem No. 12 in Exercises-VI of the well known text-book 'Introduction to Special Relativity' by Wolfgang Rindler, Clarendon Press, Oxford, 1982, page No. 141. The Problem reads,

*Consider two opposite charges moving along parallel lines at the same constant velocity **but not abreast**. By both suggested methods determine the forces acting on these charges, and show that they do not act along the line joining the charges ( e.g. a non-conducting rod) but, instead, constitute a couple tending to turn that join into orthogonality with the line of motion.*

The situation is exactly the same as that described by Mr. Wells, although the forces depicted by Mr. Wells are not complete and correct so far as the special relativity theory is concerned. The forces acting on the rod through the charges are not normal to the line of motion. Therefore they do work, and this results in the existence of an energy current flowing along the rod. This energy current constitutes momentum-density. Since the distance of the rod from the Origin of the Stationary System is continually increasing the moving momentum-density carried by the rod results in the increase of the angular momentum of the rod at the same rate as what the couple would produce. Therefore, the angular momentum of the rod does increase because of the action of the couple although the rod itself does not turn, the action of the couple going into the increase of the moment of the momentum-density carried by the rod. This is the resolution of the Torque Paradox offered in the special relativity texts.

An absolutely simple and elementary paradoxical situation obtains in the ‘Lewis and Tolman Lever’. This is the Problem No. 9 in Exercises-VII at Page No. 164 of Rindler’s text-book. Consider Lewis and Tolman’s lever paradox. The pivot B of a right-angled lever ABC is fixed at the origin of  $S'$  while A and C lie on the positive x and y axes, respectively, and  $AB = BC = a$ . Two numerically equal forces of magnitude  $f$  act at A and C, in directions BC and BA respectively so that equilibrium obtains. [ The necessary forces are easily applied by connecting the ends A and C of the lever by a taut rope passing around a smooth pulley fixed at D ( $x = a$ ,  $y = a$ ) ]. Show that in the usual second frame S there is a clockwise couple  $f \cdot a \cdot v^2/c^2$  due to these forces. Why does the lever not turn ? Resolve the paradox along the lines of the previous exercise.

Rindler then gives a hint,

Note : *Intuitively* it is not difficult to see how the angular momentum  $\mathbf{h}$  of the lever continually increases in S in spite of its non-rotation. The force at C continually does positive work on the lever, while the reaction at B does equal negative work. The energy flows in at C and out at B. But an energy current corresponds to a momentum density. So there is a constant non-material momentum density along the limb BC towards B. It is the continual increase of **its** moment about the origin of S that causes  $d\mathbf{h}/dt$  to be non-zero.

Thus in principle the Paradox does admit of a Resolution in the Special Relativity Theory.

However, this resolution is only formal and tentative, as it involves introduction of new processes and concepts such as ‘non-material momentum density’ etc, about which little is empirically known. These new concepts can not be ruled out *per se* as they are not absurd or impossible. However, the validity of these new concepts is open to questioning, and could be questioned, as ‘Solid Mechanics’ of moving bodies has not been studied much and there is no experimental evidence.

S. S. Savarkar.

Except for the trivial detail of opposite charges instead of like charges at the ends of the rod (or ‘join’), the situation described by Rindler is indeed the same as that proposed in the Electrodynamical Torque Paradox. But here the correctness of Mr. Savarkar’s commentary ends:

First, the paradox is *not* known ‘for a long time’ previous to Well’s announcement in 2002. Rindler in fact sees no paradox at all (certainly no contradiction), but merely introduces it as a problem –a problem for which he does not actually provide a solution, but instead instructs students to prove (from other material provided in the book) the foregone conclusion that, although the forces do not act along the length of the rod, and thus constitute a ‘couple’, the rod nevertheless does not turn in any frame. Neither did Trouton and Noble, a full century prior to Wells notice any paradox in the conception of their famous experiment. Rather, Rindler appends the following remarkably incognizant comment of his own to the statement of the problem: “[Trouton and Noble, in a famous experiment in 1903, unsuccessfully tried to detect this couple on charges at rest in the laboratory, which they presumed to be flying through the ether. The fact that the rod’s reaction could also be not in line with it was unsuspected and the result seemed puzzling. However it contributed to the later acceptance of relativity.]” (!)

In fact just the opposite has been demonstrated. What Rindler has missed is that relativity requires the rod both to acquire angular momentum and not acquire angular momentum –which should have led to the later rejection of relativity.

Second, the Electrodynamic Torque Paradox is at most only superficially ‘similar’ to the Lewis and Tolman Lever. This latter is a strictly mechanical problem; it is distinguished from the former by the presence of a reaction mass (i.e. a rocket exhaust or a counter-rotating platform) against which the torque on the lever reacts, whereas in the former, the moving charged rod generates magnetic forces which must act upon itself. [Furthermore, neither problem is soluble through the agency of the phantom ‘momentum densities’: The former could be resolved within special relativity, but only through recognition and application of the ‘relativity of simultaneity’ (as will be demonstrated herein later); and the latter cannot be resolved, since the proposed ‘*action of the couple going into the increase of the moment of the momentum-density carried by the rod*’ occurs without producing any opposite angular momentum, and thus contradicts the third law of motion.]

Third, the forces depicted by Wells are in fact complete and correct so far as the special relativity theory is concerned. The electrostatic tension produced by the charges is strictly along the length of the rod in both frames, and thus these forces exactly cancel, leaving only the resultant moments of the magnetic forces (the torque) to act upon the rod when moving. These forces, by very definition of the Lorentz force (derivable from the relativity field transformations), must act in a direction exclusively normal to the direction of motion of the rod.

Finally, Mr. Savarkar wavers considerably from his initial position in his concluding remarks, where the resolution is described as ‘formal and tentative’, and whose validity is ‘open to questioning’, as though seeking another way out of the dilemma. The claim of Mr. Savarkar that Professor Rindler has in any way resolved the paradox within the structure of special relativity is in fact entirely fallacious.

Rather, the passage amply exemplifies the case Rindler as an otherwise brilliant academic (a mathematician) who has gained prominence in the wrong department (physics) –an occurrence noted in Wells’ 2006 NPA METHODOLOGY paper.

Later indeed, Mr. Savarkar, evidently not entirely satisfied with Rindler’s presentation, provides his own analysis of the situation in his 2007 NPA paper ‘WORK AND MAKING RELATIVITY WORK’; the relevant section of the paper is reproduced here below:

\* \* \*

## AN EXPERIMENT AND THE ANALYSIS

Let  $\Sigma (O, x, y, z, t)$  be a System at rest, and let  $\Sigma^* (O^*, x^*, y^*, z^*, t^*)$  be another System at rest, with their origins and axes coinciding. Let there be similar and similarly graduated platforms along the planes  $O$ - $X$ - $Y$  and  $O^*$ - $X^*$ - $Y^*$ . Since the axes of the two frames will be parallel at all times in our experiment, we shall designate the axes and the components of vectors in either frame by the symbols for the  $\Sigma$ -system.

Let  $A^*B^*$  be a rod of length  $\ell^*$  at rest in  $\Sigma^*$  on the graduated platform  $O^*$ - $X^*$ - $Y^*$ , with center at  $O^*$ , and inclined at an angle  $\theta^*$  to the  $x$ -axis, i.e.  $O^*A^*$  makes an angle  $\theta^*$  with the positive direction of the  $x$ -axis.

Let two identical horses pull at the two ends  $A^*$  and  $B^*$  with equal and opposite forces,  $\mathbf{F}^*$  and  $-\mathbf{F}^*$  respectively, with the force  $\mathbf{F}^*$  of the horse pulling at  $A^*$  making an angle  $\varphi^*$  with the positive direction of the  $x$ -axis.

Obviously the rod will be in translational equilibrium, and also in rotational equilibrium provided  $\varphi^* = \vartheta^*$ , meaning that the ropes by which the horses are harnessed to the rod lie along the line of the rod. The forces at the ends of the rod will not be doing any work.

Now let the System  $\Sigma^*$  be set into a uniform translation with velocity  $\alpha$  in the positive direction of the  $x$ -axis. The platform  $O^*$ - $X^*$ - $Y^*$ , the rod  $A^*B^*$ , and the horses pulling at its ends are now in a different inertial state.

Since the equivalence of inertial states is not a logical necessity, we could conceive that all the properties of all the bodies in  $\Sigma^*$ , as well as the actions amongst them, would suffer velocity-modifications.

The equivalence required by the Galilean theory is only a particular case of the general Gulliveresque proposition- viz. NO velocity-modifications, and we may analyse the situation in terms of the general Gulliveresque context from which the Galilean counterpart can always be obtained.

Suppose that because of the velocity-modifications the various entities are velocity-modified as follows:

- (1) The length  $\ell^* \rightarrow \ell$ , the inclination  $\vartheta^* \rightarrow \vartheta$ ;
- (2) the two horses were identical, but their orientations with respect to the motion are now different, and their forces could suffer different velocity modifications. Suppose the force exerted by the horse pulling at A\* is velocity-modified to  $F_A$  making an angle  $\varphi_A$  with the positive direction of the x-axis, and that of the other horse velocity-modified to  $F_B$  making an angle  $\varphi_B$  with the negative direction of the x-axis.

Here  $\ell$ ,  $\theta$ ,  $F_A$ ,  $\varphi_A$ ,  $F_B$ ,  $\varphi_B$  are the true measures that will be determined in the system  $\Sigma$  at rest. If relativity prevails, the observer in the moving  $\Sigma^*$  system would find no difference from the state of rest. The ends A\* and B\* will be coinciding with the same graduations on the platform O\*-X\*-Y\*, the rod making the same angle  $\theta^*$  with the x-axis, etc. The graduated platform, which is material, must also suffer velocity-modifications. This graduated platform is his instrument of observing locations, and since he is making observations with this distorted instrument he will obtain different incorrect measures. Similarly, all his observation instruments would be velocity-modified and distorted, and would not be yielding the true and correct measures. It is with these incorrect measurements obtained with distorted instruments that an observer in the moving system would be constructing his physical laws and theories.

As stated earlier, a Gulliveresque relativity would be the result of the coordinated conspiratorial concert of three factors: (1) the velocity-modifications suffered by the observed entities, (2) the distortions suffered by the moving observation instruments, and (3) the laws of nature. Little consideration has been bestowed upon the critical role of the second factor, and this neglect is at the root of many misconceptions and the malaise.

This has been discussed more at large from the analogy of the tachemetric surveys in the author's book *The Great Einstein-Sky-Ride*.

In the Rest- System  $\Sigma$  the rod AB will be moving uniformly in the direction of the x-axis with velocity  $\alpha$ .

This requires,  $F_A \cos \varphi_A = F_B \cos \varphi_B$ , and  $F_A \sin \varphi_A = F_B \sin \varphi_B$

It follows that,  $\varphi_A = \varphi_B = \varphi$ ; and  $F_A = F_B = F$

Thus, the forces acting at the ends A\* and B\* constitute a 'couple', and the torque G is given by,  $G = \ell F (\sin \varphi \cos \vartheta - \cos \varphi \sin \vartheta)$ . There is no logical necessity that  $\varphi$  must equal  $\theta$ . Therefore, G may not be zero, and this torque will affect the angular momentum as,  $dL/dt = G$ , and will try to rotate the rod.

Now the force acting at the end A\* has a component  $F \cos \varphi$  along the positive direction of x-axis, and the end A\* moves with a velocity  $\alpha$  in the same direction. Therefore the force exerted at A\* does work, and transmits energy to the rod at the rate,  $dW/dt = \alpha F \cos \varphi$ . Similarly, the force acting upon the end B\* in the negative direction of x-axis is  $F \cos \varphi$ , and the end B\* is moving with velocity  $\alpha$  in the positive direction of x-axis, i.e. opposite to the direction of the force. Therefore, the force exerted at the end B\* is doing 'negative' work, and withdrawing energy from the rod at the same equal rate, viz.  $dW/dt = -\alpha F \cos \varphi$ . Thus a stationary current of energy flows constantly from the end A\* to the end B\* through the rod.

This energy transmission must be by means of elastic actions.

Now, according to the law (6) as listed above, energy possesses inertial and dynamical properties, as expressed in Planck's Theorem (1908): "The momentum is proportional to the flux of energy.",  $\mathbf{p} = \kappa E \mathbf{v}$ , where  $\kappa$  is a universal constant, and  $\mathbf{v}$  the velocity.

Therefore, if the speed of elastic transmission along the rod is  $w$ , then the energy will take a time equal to  $\ell/w$  for the flow from A to B, and the constant energy content in the rod is given by,  $E = \kappa \alpha (F \ell / w) \cos \varphi$ . The flux of this energy in motion will have the momenta,  $p_x = (\alpha - w \cos \vartheta) E$ , and  $p_y = -w \sin \vartheta E$  distributed uniformly along the rod.

Now, the respective normal distances of the elements of the rod from the x-axis remain constant, and the moments of the momenta in the direction of the x-axis - both for the rod itself and the energy flowing through it -, also remain constant. The corresponding normal distances from the y-axis increase at the rate  $\alpha$  because of the motion of the system  $\Sigma^*$ . The rod itself has no momentum in the direction of the y-axis. Therefore, the angular momentum of the rod in the stationary  $\Sigma$  system will increase as,  $dL/dt = -\alpha w E \sin \vartheta$

Therefore, for rotational equilibrium we must have,

$$dL/dt = -\kappa \alpha^2 F \ell \cos \varphi \sin \vartheta = G = F \ell (\sin \varphi \cos \vartheta - \cos \varphi \sin \vartheta)$$

That is,  $\tan \varphi \cot \vartheta = (1 - \kappa \alpha^2)$

This is the necessary condition for the rotational stability of a uniformly moving rod acted upon by forces at its ends. Our analysis is absolutely basic and fully general, and does not essentially depend upon any physical theory except the six laws of mechanics listed earlier. Given those six laws, the same relationship must hold for rotational stability even if the forces were applied by means of springs,

screws, turn-buckles - or by attaching electrical charges at the ends. Also, obviously the same relationship must hold even if the forces were acting inwards – i.e. compressive forces at the ends.

Some momentous consequences follow as corollaries.

Any electrodynamics theory that accepts the six ‘laws’ of mechanics listed at the start, and aspires to incorporate relativity as a principle, cannot be based on a ‘central-force’ law for the force between two moving charges. Moreover, the force cannot be a ‘central-force’, along the line joining the two moving charges, even if the charges were moving with equal uniform velocities along parallel lines. This rules out all theories where the force between two moving charges is determined entirely by the relative velocities and accelerations.

In the Lorentz theory, or the special relativity theory, the force between charges Q at A\* and B\* is given by,  $F_x = Q^2 \ell \cos \vartheta / N$ , and  $F_y = Q^2 \ell \sin \vartheta (1 - \alpha^2 / c^2) / N$ , where  $(1 - \alpha^2 / c^2) N = \ell^3 [\cos^2 \vartheta + (1 - \alpha^2 / c^2) \sin^2 \vartheta]^{3/2}$

Thus,  $\tan \phi = F_y / F_x = (1 - \alpha^2 / c^2) \tan \vartheta$ , and the necessary condition would be satisfied if  $\kappa = 1 / c^2$ . Therefore the Lorentz theory can possibly make relativity work for this particular system while accepting all the six laws of mechanics listed earlier.

Here we must note that c is a parameter determined from electro-magnetic experiments. The parameter  $\kappa$  is a mechanical parameter that could be determined from purely mechanical collision experiments. It is not a logical necessity that these two independently determined parameters should satisfy the required relationship. If they do not satisfy the necessary relationship, then we have a Contradiction, and the Gulliveresque Conspiracy being constructed here is not viable.

Any theory of electrodynamics in which this critical relationship is not valid could also possibly make relativity work in this particular situation by denying some one or more of the said Set of six laws of mechanics, and / or by hypothesizing some other actions – presumably by the ether. As stated earlier, these laws are not logical necessities, and a theory-maker can deny any one or more of them. But what is required is the construction of a fairly complete and consistent Conspiracy that makes relativity work in every conceivable situation. Further, the consequences deducible must not be in conflict with the physical experience.

\* \* \*

#### Analysis of Mr. Savarkar’s Analysis

Retaining Savarkar’s nomenclature, in the section ‘An Experiment and the Analysis’ (q.v.), the observation is first made in the example of the horses pulling at the ends of the rod on the graduated platform in the rest frame  $\Sigma^*$  that the rod will be in rotational equilibrium “*provided  $\phi^* = \vartheta^*$* ”. Later, with the statement “*There is no logical necessity that  $\phi$  must equal  $\vartheta$* ” (in the relatively moving frame  $\Sigma$ ), it is contended that a torque could apply, which “*will try to rotate the rod.*” (This would indeed be the case where velocity-dependent forces arise due to absolute motion of  $\Sigma^*$ .) But proceeding with the Gulliveresque relativity hypothesis, a “*conspiracy*” of velocity modifications is necessary to ensure that the rod does not turn as a result of the applied torque (or couple). In attempting to account for this phenomenon, Mr. Savarkar succumbs to introducing the same phantom quantities as did Prof. Rindler. In describing the situation, instead of the statement “*Therefore*

the force exerted at A\* does work, and transmits energy to the rod at the rate,  $dW/dt = \alpha F \cos \varphi$  .”, Mr. Savarkar would better have written “Therefore the force exerted at A\* strives to do work, and strives to transmit energy to the rod...” For the kind of work proposed in the given equation is translational, which is all cancelled by the action of the counter-force at B\*. Instead, the only work that could be done on the rod in system  $\Sigma$  would be rotational work. The root of the confusion is the attempted conflation of the actual torque,  $\vec{F} \times \vec{\ell}$ , with the work function,  $\vec{F} \cdot \vec{\ell}$ , --a confusion which is compounded by the oxymoronic descriptor “stationary current”, by which the purported “energy flows constantly from the end A\* to the end B\* through the rod.” The “elastic actions” by which this must be accomplished do not occur in system  $\Sigma^*$  --and in fact would not occur in  $\Sigma$  unless the applied forces were time-varying. The forces are constant and the stresses are entirely static in a non-rotating rod.

Therefore no energy flows along the line of the rod --and hence there is no ‘momentum density’. It is a misapplication of Planck’s theorem (1908) to imagine such a phenomenon. Besides, this Planck theorem presupposes the general correctness of the Einsteinian mass-energy relation, which has elsewhere been refuted (Wells 2007 NPA paper, MASS DISPARITIES FROM RELATIVISTIC DYNAMICS, in the section Contradiction from Electrodynamics).

But suppose we were to grant that Mr. Savarkar’s reasoning so far were somehow correct: The situation then consists of an increasing moment of momentum flow along the rod (instead of an actual rotation) in system  $\Sigma$ , which must be balanced by a counter-torque applied to the horse-carrying platform. Thus the platform must acquire some rotation here, which does not occur in  $\Sigma^*$ . Then, when the horses quit pulling, the accumulated angular momentum of the platform is no longer balanced by the now vanishing moment of momentum flow in the rod --a recurrent violation of the third law of motion. To attempt a rescue of the argument with the claim that the torque on the platform must also produce an energy current (which also evaporates when the action quits) is merely to introduce another apparition into the picture --more ‘phantom physics’. Since no such energy flow is detectable in either  $\Sigma^*$  or  $\Sigma$ , we must conclude that in this case,  $\varphi$  must equal  $\theta$  (where  $\varphi^* = \vartheta^*$ ) of logical necessity, whereby the forces act strictly along the line of the rod in both frames, otherwise the rod (and platform) would rotate in  $\Sigma$  but not in  $\Sigma^*$ . Thus when the forces act obliquely in  $\Sigma^*$  (where  $\varphi^* \neq \vartheta^*$ ), they act obliquely in  $\Sigma$  ( $\varphi \neq \vartheta$ ), producing rod rotation and platform counter-rotation in both frames. Only in the pathological case, where the relative velocity  $\alpha \rightarrow c$ , where the  $x$  dimension of the rod would contract to zero, could we find the moment of the applied force to vanish in  $\Sigma$ .

In the case of the Electrodynamic Torque Paradox itself, arranged “by attaching electrical charges at the ends”, an important distinction must be recalled: removal of the massive platform from the picture as a recipient of the expected counter-torque leaves the charged rod exerting a torque upon itself! --which is part of the content of the original paradox. Only by ignoring the simple physics of the situation (i.e., the third law of motion), and substituting imaginary quantities (e.g., the phantom energy flow), does Mr. Savarkar manage to arrive at the “momentous” conclusion:

“Any electrodynamics theory that accepts the six ‘laws’ of mechanics listed at the start, and aspires to incorporate relativity as a principle, cannot be based on a ‘central force’ law for the force between two moving charges.”

In fact, just the opposite conclusion has been demonstrated: Any relativistic electrodynamics can only be based on a ‘central force’ law for the force between two moving charges. (The Trouton-Noble experiment should have been recognized as proof of just such a law.) This would be true even if we were to accept the erroneously construed ‘sixth law’ of mechanics (based on the tacit acceptance of the mass-energy equivalence); this ‘sixth law’ does not apply here, since no energy flow is possible along the charged rod when in mechanical equilibrium. And even if there were such flow, the rod would acquire angular momentum in  $\Sigma$  (from the ever-increasing  $x$  moment of the  $y$  component of the purported associated momentum of the ‘elastic actions’ along the rod) for which no counter-angular momentum is anywhere generated. Again the third law of motion is violated --and a contradiction between the previously declared ‘fifth’ and ‘sixth’ laws of mechanics is now obvious. This contradiction occurs prior to the entry of any factor ‘ $K$ ’ into the discussion.

Elsewhere [General NPA communication, attachment ‘Electrodynamic Force’ dated 23 July, 2006], Mr. Savarkar persists in attempting to discredit the original ‘central force’ formulation of the magnetic interaction of the form proposed by Ampere and Weber, with the statement:

**As a matter of fact, Ampere's law is NOT the correct special relativistic law for the force between two steady current elements. The correct special relativistic law is that given by Grassmann, based on the Biot-Savart law, and is of a quite different form.**

But there is no justification for dismissing this original law in favor of the later Biot-Savart/Grassmann (or Lorentz) formulation –except for the purely mathematical convenience that it supplies. In fact, only such central force law could be compatible with any relativistic construction of electromagnetism, as should be evident from the content of the analysis provided herein. Mr. Savarkar and the cited W.G.V. Rosser are both incorrect on this point by insisting upon the Lorentz version of the force law. Indeed, under the former the paradox does not occur at all, since here the forces never possess moments in either reference frame. The latter is certainly much simpler in form –but the appearance of force moments in one frame and not the other leads to contradictions that are contrary to the foundations of physics.

#### Analysis of the 'Tolman Lever Paradox'

The first appearance of this paradox seems to have been published in 1909 (Lewis and Tolman, Philosophical Magazine, 18, 1909, page 510) as cited in the lengthy analysis of one Harry H. Ricker: [www.wbabin.net/science/ricker12.pdf](http://www.wbabin.net/science/ricker12.pdf)

In the previous reference to the 'Lewis and Tolman Lever' (erroneously compared to the Electrodynamical Torque Paradox), Mr. Savarkar makes the statement:

*Both the situations appear paradoxical but admit plausibly resolutions in terms of the 'momentum densities' constituted by the energy currents.*

The Tolman Lever Paradox might admit a plausible resolution within special relativity, though not in terms of the 'momentum densities' proposed by both Savarkar and Rindler. In fact Mr. Savarkar seems later to 'change horses in mid-stream' in the general NPA communication of 3 July 2007 (TORQUE PARADOX RESOLUTION) with the concluding comment:

*I will send shortly a Note describing the Tolman's Lever Paradox, which is much more blatant than the one claimed by you.*

This seems to indicate that Mr. Savarkar now believes that the Tolman Lever does constitute a paradox –although this position is not entirely clear, since the Note has not yet been communicated.

But a careful point-by-point analysis of Rindler's presentation will demonstrate the unphysical nature of the argument, several flaws in its development, as well as a fundamental defect in reasoning from the known laws of physics.

First, Prof. Rindler describes the pivot B of the lever 'fixed' at the origin of  $S'$  but does not indicate any mass  $M$  to which it is fixed. Co-ordinate systems, being imaginary constructs, carry no mass, and cannot provide any such fixture. As soon as we do consider the pivot anchored to some massive framework, we must also acknowledge the gain in kinetic energy of the system when the described forces are applied. The resultant force vector will have a component along the  $x$  axis, which will produce the energy differential  $Mvdv$ . In order to eliminate this term, counter-forces would have to be applied near the pivot point, thus preserving translational equilibrium, duly eliminating the need for the ballast mass  $M$ . [Mr. Savarkar's parenthetical introduction of a pulley system in the exercise only complicates the situation, since the pulley must also be stabilized by an appropriate array of forces.]



As previously stated, the introduction of ‘momentum densities’ to accommodate the applied forces’ tendency to produce angular momentum without rotation is faulty –because again the accumulated counter angular momentum of the massive platform or rocket exhaust will not be balanced by the evaporating moment of the ‘momentum densities’ when the forces quit. The only possible course for resolution of the paradox within special relativity (through observation of the ‘relativity of simultaneity’) would proceed as follows:

Let the problem be simplified by the ‘welding’ of another identical lever, rotated through 180 degrees, to the pivot point B of the original lever. We now have a perfectly symmetrical rigid crossbar, ACED, with the point B at the center. Next, we reduce the applied ‘forces’ to a series of instantaneous impulses produced by successive launches of identical individual particles, perpendicular to the respective ends of the crossbar, simultaneous in  $S'$ , where no net torque is exerted through the equal lengths AB, CB, EB, DB. In  $S$ , however, it will be observed that not only is the crossbar length-contracted along CBD, but that the times of the four launches are not simultaneous. Thus the progress of the action in  $S$  will be: first, the launch (and reactive impulse) at D will occur at time  $t = 0 - \gamma(v\Delta x / c^2)$ , when the point D is found at a negative distance  $\gamma\Delta x'$  from the origins of  $S$  and  $S'$  when they coincide at time  $t = t' = 0$ . Then, the launches from A and E will occur simultaneously when  $t = t' = 0$ , at  $x = x' = 0$ . Finally, the impulse at C will occur at time  $t = 0 + \gamma(v\Delta x / c^2)$  when C is also at a positive distance  $\gamma\Delta x'$  from the common origin.

Thus, contrary to the original supposition, the moments of the forces applied to C, D in  $S$  are greater, not lesser, than the moments in  $S'$ . (The traveling mechanical stresses from these impulses will meet at B simultaneously in both frames, but this ‘elastic flow’ is really immaterial to the substance of the discussion.) To balance the torques, we are required to find the ‘transverse’ and ‘longitudinal’ momenta of the particles to be in exact inverse proportion to the respective transverse and longitudinal moments. Viz.,

With the respective moments along CD and AE produced by the series of events thus found in the ratio  $\frac{\ell_x}{\ell_y} = \gamma \frac{\ell_x'}{\ell_y'}$

(where  $\ell_y' = \ell_x'$ ), we apply the standard relativistic velocity composition formulae,  $u_x = \frac{u' + v}{1 + u'v/c^2}$ ;

$$u_y = u' / \gamma$$

(where  $u$  and  $u'$  are the respective particle velocities in  $S$  and  $S'$ ) to write the necessary equality  $\ell_x(m_y u_y) = \ell_y(m_x u_x)$ .

This satisfies the requirement that the respective angular momenta of the launched particles are equal and opposite. Thereby the moments of momentum of each pair of particles A,E and C,D would be exactly equal and opposite in  $S$  as they are in  $S'$ : hence the net reactive torques would also be equal and opposite in  $S$  as they are in  $S'$ .

Although this is the expected result within the requirements of special relativity, the ‘solution’ does not at all vindicate the theory generally. For re-arranging this latter equation yields the rather unusual expression:

$$\frac{m_x}{m_y} = \frac{\ell_y u_x}{\ell_x u_y} = \frac{1 + v/u'}{1 + u'v/c^2}$$

for the ratio of the ‘longitudinal’ to ‘transversal’ masses. Outside the context of the customary values given for these masses, the lever paradox could thus be resolved; however, the otherwise necessary requirement, elsewhere in relativity theory, that

the masses also be in the ratio  $m_x / m_y = \gamma^2$ , merely signifies yet another contradiction in the assignation of relativistic inertia, in addition to the contradictions brought forward in the MASS DISPARITIES paper of Wells op. cit.

### Concluding Remarks

In the foregoing exposition, we have been introduced to the extremes that some theoreticians will go in order to 'save the phenomenon' necessary to make relativity work. Both Rindler and Savarkar have insisted upon a mysterious 'energy flow' which occurs along the rod in one reference frame (but not in another), whose 'momentum density' accommodates the applied torque in that frame. We have seen how, when the torque is mechanically applied, and then the forces quit, the accumulated angular momentum against the massive platform counterweight is no longer balanced by the then evaporating 'moment of momentum' in the rod. Conversely, when the forces are electrodynamic in origin, angular momentum, which purportedly develops in the rod from an energy flow along its length, is not conserved by a reaction in any present massive body. (The same misconception figures into their incorrect resolutions both of the Electrodynamical Torque and the Mechanical Lever Paradoxes.) Thus the Savarkar-Rindler argument, accomplished as it is through an egregious neglect of the Third Law of Motion, is refuted by 'close study' of physical reality, and the mysterious 'moment of momentum density', vainly imagined as a convenient *'deus ex machina'* to make the theory work, is exposed for the fissiparous phantom that it always was.

Mr. Savarkar has provided much valuable background information in his paper 'WORK AND MAKING RELATIVITY WORK', but the arguments introduced to support the principal thesis are physically unsound. They in fact exemplify Phipps' acute observation regarding theoretical difficulties, whereby [faulty] "physics comes to the rescue of foundering logic". It is therefore rather disappointing that Mr. Savarkar in particular, generally quite astute in his analytical explorations elsewhere, in this case falters in so elementary a fashion.