An Ideal Inelastic Collision Model using Center-of-Mass Frames Shows Conservation of Kinetic Energy

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A frame transfer model introduced here shows that the kinetic energy is totally conserved and accounted for in the ideal inelastic collision as well as in the elastic collision. The kinetic energy transfer between colliding masses in the ideal inelastic collision case is found to be totally consistent with the law of conservation of energy which states that energy can be neither created nor annihilated. In all inelastic collisions, the two colliding masses move jointly at precisely their center-of-mass velocity, a velocity which is unchanging in a closed system of unbound masses. For this reason, a properly formulated energy transfer model is chosen to be one that goes from the initial frame of the moving mass to that of the center-of-mass frame. **Keywords:** invariance of velocity of center-of-mass, conservation of kinetic energy, closed system of masses.

1. Introduction

It has been traditionally taught in the physics classrooms that the kinetic energy is not conserved in the inelastic collisions. [1–5] This study finds that a frame transfer technique of kinetic energy from a moving mass into the center-of-mass frame of the combine masses at the inelastic collision event shows a conservation of kinetic energy for the inelastic collision case; a hitherto not covered topic in the textbooks. [2,4,5] The law of conservation of energy clearly states that energy can be neither destroyed nor created. In the rest frame, the kinetic energy is by definition that quantity of energy that is deliverable to a receiving mass in that frame. The kinetic energy of a moving mass is a function of its mass and the velocity of its mass referenced to the receiving mass. A moving mass can deliver the total content of its kinetic energy to a resting mass if and only if the moving mass of an initial velocity V were to come to a final velocity of V = 0. In order for this to occur, the resting mass must be tied to the rest frame so that it does not move. An occurrence of an ideal inelastic collision between a moving mass and a stationary mass that is free to move will always result in a motion of the joined masses at precisely their center-of-mass velocity (V_{CM}) of a closed system of unbound masses. This is confirmed by countless numbers of experimental observations. With this, it is clear that the total kinetic energy of a moving mass referenced to the rest frame cannot be transferred to the combined masses at the event of an inelastic collision with a mass that is initially at rest and is free to move. The proper frame transfer of kinetic energy must be referenced to the center-of-mass frame, not the rest frame, for the ideal inelastic collision. At collision, the resting mass gains a pulse of kinetic energy as it accelerates from its initial velocity $V = 0$ to the velocity V_{CM} while the moving mass loses kinetic energy as it decelerates from its initial velocity V to the velocity V_{CM}. This is an energy exchange bump; a frame transfer of a kinetic energy burst that is in consistency with the law of conservation of energy. The properly applied frame transfer model clearly shows that the kinetic energy is totally conserved and accounted for in the ideal inelastic collision case as well as in the elastic collision case.

2. Invariance of the Velocity V_{CM}

From countless experiments on the inelastic collisions, two colliding masses M_1 and M_2 move jointly at the velocity V_{CM} where

$$V_{CM} = \frac{M_1 V_1 + M_2 V_2}{M_1 + M_2} \tag{1}$$

in any closed system of unbound masses that are free to move. [2, 4]

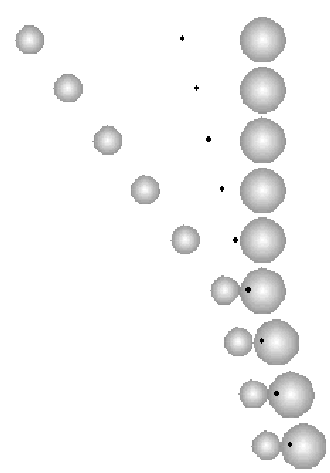

Fig. 1. Time Resolved Images illustrating the Invariance of the Velocity of the Center-of-Mass Points in a Closed System of Masses

In Figure 1, an inelastic collision process is illustrated, showing the instantaneous positions of two mass spheres and their center-of-mass point at snap shot instants in time. [6] The illustration clearly shows the linear motion of the center-of-mass point, demonstrating the invariance of the velocity V_{CM}. In the time resolve images, the center-of-mass point moves along a linear path and appears to be totally unaffected by any action taking place inside of the closed system of masses. In the elastic collision case, the two colliding masses move, for a brief instant in time, precisely at the velocity V_{CM} until they recoil. As a consequence of the invariance of V_{CM} given by (1) and the conservation of the kinetic energy laws, the following mathematical statement correctly describes the ideal inelastic case:

$$\tfrac{1}{2}M_1V_1^2 + \tfrac{1}{2}M_2V_2^2 - \tfrac{1}{2}M_1(V_1 - V_{CM})^2 - \tfrac{1}{2}M_2(V_2 - V_{CM})^2 = \tfrac{1}{2}(M_1 + M_2)V_{CM}^2 \tag{2}$$

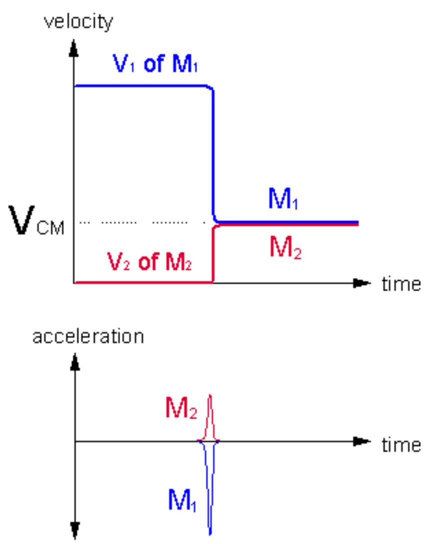

Fig. 2. Frame Transfer of Kinetic Energy during Inelastic Acceleration Bump

3. The Transformation of Energy from Frame to Frame

The quantities $\tfrac{1}{2}M_1V_1^2$ and $\tfrac{1}{2}M_2V_2^2$ are the initial kinetic energies of the masses M_1 and M_2, respectively, as would be noted from the rest frame. These are the quantities of energy that are available to be deposited into the rest frame if the two masses go from their initial velocities, V_1 and V_2, respectively, to a complete stop, $V_1 = 0$ and $V_2 = 0$. For instance, a mass M_1=1 Kg moving with an initial velocity of $V_{1i} = 12$ m/s collides with a resting non movable wall, bringing the mass to full stop, hence, to a final velocity of $V_{1f} = 0$, will deposit at the wall the kinetic energy of

$$\tfrac{1}{2}M_1\left(V_{1i} - V_{1f}\right)^2 = \tfrac{1}{2}M_1(12 - 0)^2 = 72J \tag{3}$$

This is the frame transfer of the kinetic energy. To write down $\tfrac{1}{2}M_1(12)^2 - \tfrac{1}{2}M_1(0)^2$ would be a very serious error that is made too often by qualified physicists. The kinetic energy of mass M_1 moving with an initial velocity $V_{1i} = V_1$ that would be deposited into the center-of-mass frame coming to a final velocity $V_{1f} = V_{CM}$ is

$$\tfrac{1}{2}M_1\left(V_{1i} - V_{1f}\right)^2 = \tfrac{1}{2}M_1(V_1 - V_{CM})^2 \tag{4}$$

An observer in the center-of-mass frame of two colliding masses, M_1 and M_2, would note that the mass M_1 approaches with the velocity $V_1 - V_{CM}$ while the mass M_2 approaches with velocity $V_2 - V_{CM}$. It is important to note that the observer's frame of reference has nothing at all to do with the physics of the problem. The laws of physics are independent of the frame of reference. The quantity $\tfrac{1}{2}M_1(V_1 - V_{CM})^2$ is the kinetic energy that the mass M_1 transfers from its frame to the center-of-mass frame as it goes from an initial velocity V_1 to a final velocity V_{CM}. This is the quantity of kinetic energy that M_1 transfers to the center-of-mass frame at collision. Similarly, the quantity $\tfrac{1}{2}M_2(V_2 - V_{CM})^2$ is the kinetic energy that the mass M_2 transfers to the center-of-mass frame as it goes from an initial velocity V_2 to a final velocity V_{CM}.

In Figure 2, a frame transfer of kinetic energy is illustrated. The energy quantities $-\tfrac{1}{2}M_1(V_1 - V_{CM})^2$ and $-\tfrac{1}{2}M_2(V_2 - V_{CM})^2$ represent the total kinetic energy that is extracted from the initial kinetic energies, $\tfrac{1}{2}M_1V_1^2$ and $\tfrac{1}{2}M_2V_2^2$, during a transfer of kinetic energy between the two mass spheres of different frames of reference. This is the pulse of kinetic energy transferred during the acceleration bump, bringing the lumped masses M_1 and M_2 into the center-of-mass frame. This is an energy transfer that takes place according to the law of conservation of energy at the ideal inelastic collision event. During the inelastic collision event, M_1 decelerates from velocity V_1 to velocity V_{CM} while M_2 accelerates from velocity V_2 to velocity V_{CM}. From this we can see that for any ideal inelastic collision case, all kinetic energy quantities are totally accounted for. It is very important to note that the experimentally observed invariance of V_{CM} should have been a tip off that the kinetic energy in a well designed mass collision experiment has to be conserved in the inelastic collisions as well as in the elastic collisions. The mass collision experiment can now be well designed so as to limit mechanical deformations, energy dissipations due to sparks or thermal shock emissions, acoustical vibrations and sound emissions.

Let us assume that mass $M_1 = 1$ kg moving with velocity $V_1 = 12$ m/s and collides ideally with a resting mass $M_2 = 2$ kg. From equation (1), we can determine that the velocity $V_{CM} = 4$ m/s. We find that the initial kinetic energy of mass M_1 is 72 Joules (J). This is the available kinetic energy that is transferable to the rest frame. We know that all of this kinetic energy cannot be transferred into to the center-of-mass frame, as the velocity V_1 of M_1 does not go to zero, namely, $V_1 = 0$, or to a complete stop which is a requirement for the total dissipation of the available kinetic energy of mass M_1. The initial kinetic energy for the resting mass M_2 is zero. We find that from (4) the transferable kinetic energy needed to bring M_1 into the center-of-mass frame is $\frac{1}{2}M_1\left(V_1 - V_{CM}\right)^2 = \frac{1}{2}M_1\left(12-4\right)^2 = 32J$ and, similarly, the transferable kinetic energy needed to bring M_2 into the center-of-mass frame is 16 J. The remaining energy left from these energy transfers is 72J - 32J - 16J = 24J. The resulting lumped mass kinetic energy from the right hand side of equation (2) is found to be exactly 24J. From this, we can see that the kinetic energy in this inelastic collision is totally accounted for. From equation (2), an expansion, simplification and collection of the energy terms arrive at the kinetic energy equation (5) which leads directly to a valid conservation of momentum equation (6) and (7).

$$\frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 - \frac{1}{2}M_1V_1^2 + M_1V_1V_{CM} - \frac{1}{2}M_1V_{CM}^2 - \frac{1}{2}M_2V_2^2 + M_2V_2V_{CM} - \frac{1}{2}M_2V_{CM}^2 = \frac{1}{2}\left(M_1 + M_2\right)V_{CM}^2 \tag{5}$$

$$M_1V_1V_{CM} + M_2V_2V_{CM} = \left(M_1 + M_2\right)V_{CM}^2 \tag{6}$$

$$M_1V_1 + M_2V_2 = \left(M_1 + M_2\right)V_{CM} \tag{7}$$

This may be considered to be a mathematical proof that the energy is conserved and totally accounted for in all well designed ideal inelastic collision experiments as in the elastic collision experiments. One can see that the conservation of energy law is clearly at work here where one moving mass sphere appears to exchanges its energetic action, passing it along to be received by another mass sphere in a closed system of masses that are free to move. [6] It is clear from experiment that the center-of-mass velocity V_{CM} and the linear motion of the center-of-mass point remains totally unaffected by all collision processes.

4. The Dissipation of Heat Energy during an Inelastic Collision: the Experimental Results

We shall apply the well known Newton's Law of cooling to examine the heat retention of the stainless steel mass spheres used in this inelastic collision experiment. One of the spheres, each with mass 111.5 grams and diameter 3.750 cm, was selected and deliberately heated to 36°C, a temperature of 14°C above the temperature T_{env} of the laboratory environment, recorded to be 22°C during the measurement. The heated sphere was suspended in the lab on 3 sharp pointed pins for thermal isolation and allowed to cool. The temperature $T\left(t\right)$ was recorded using a digital thermometer that was electrically connected to a miniature thermistor probe. The probe was coupled to the heated sphere using a heat conducting silicon grease. The cooling rate was recorded and summarized in Table 1. The Newton's Law of Cooling, equation (8), states that the cooling rate of a heated mass body is directly proportional to the difference between the temperature of the environment T_{env} and the time varying temperature $T\left(t\right)$ of the cooling mass body.

$$\frac{dT}{dt} = -k\left(T\left(t\right) - T_{\text{env}}\right) \tag{8}$$

A solution to the differential equation (8) has the form given by equation (9)

time (sec)	T(t) (°C) Measured	T(t) (°C) Fitted
0.0	36.000	36.356
60.0	36.800	35.846
180.0	35.000	34.873
314.0	34.000	33.858
360.0	33.800	33.526
390.0	33.400	33.314
440.0	33.000	32.967
480.0	32.500	32.697
540.0	32.400	32.301
600.0	32.000	31.918
750.0	31.000	31.013
920.0	30.000	30.070
1163.0	29.000	28.862
1333.0	28.000	28.104
1560.0	27.000	27.194
1860.0	26.000	26.148
2265.0	25.000	24.975
2940.0	23.500	23.502
3300.0	23.000	22.908
3900.0	22.200	22.138
4200.0	22.000	22.032

Table 1. Newton's Law of Cooling applied to the 111.5 gram mass Stainless Steel Spheres

$$T\left(t\right) = T_{\text{env}} + \left(T\left(0\right) - T_{\text{env}}\right)e^{-kt} \tag{9}$$

where T_{env} is the temperature of the laboratory environment and $T\left(0\right)$ is the initial temperature of the heated mass sphere at time t=0. The constant k is the cooling rate that describes the radiation of the mass sphere according the Newton's law of cooling. The measurements made on the cooling rate of the mass spheres are important as the time rate of change in $T\left(t\right) - T_{\text{env}}$ is needed to gauge the validity of the mathematical model used for the ideal inelastic collision case as opposed to the conventional understanding of the inelastic collision as published in the literature. The recorded cooling rate for the stainless steel spheres was carefully done using a simple and easily repeatable experiment. An exponential fit was made to the data using a KaleidaGraph data analysis and graphic program, version 3.52. The mathematical fit to the data is presented in equation (10).

$$T\left(t\right) = 20.085 + \left(36.356 - 20.085\right)e^{-0.0005307t} \tag{10}$$

This experimental result finds that after repeated inelastic collisions between the mass spheres. the measurements could not account for sufficient heat loss to support conventional understanding of the inelastic collision cases. After a careful monitoring of the temperature of the spheres after many collisions, the expected accumulation of heat was not sufficient to support any of the inelastic collision models as published in the literature. The cooling rate was experimentally found to be k=0.00055152°C /sec. The countless number of inelastic collision impacts shows an expected lack of heat buildup in the mass spheres, confirming the correctness of the ideal inelastic collision model presented here. The metal spheres of mass 111.5 grams were slung at the velocity of 666.6 cm/sec, delivering a kinetic energy of 2.47E07 dynes or 2.47 Joules for each shot from spring loaded launcher. The velocity of the slung sphere was determined using two micro switches at positions x_1 and x_2 that were tripped as the sphere flew by, causing timing pulses to occur at times t_1 and t_2. The pulses were recorded with a Hewlett Packard 54510A digitizing oscilloscope. The moving sphere was determined to have the equivalent of 2.47 Watt-sec of energy deliverable at each shot. This would be ca. 25 Watt-sec of deliverable energy accumulated for every 10 shots or 250 Watt-sec of deliverable energy for every 100 shots of the mass spheres. In a controlled experiment, an accumulation of heat should be easily measurable after repeated inelastic impacts if any of this energy were to be converted into heat. However, the experimental result shows this has not been the case. We have already seen in the previous section that, from the invariance of the velocity V_{CM} and the law of conservation of energy, the kinetic energy in the ideal inelastic collision is totally accounted for. The conservation of kinetic energy is found to apply directly to the ideal inelastic collisions.

A time rate of change in $T(t)-T_{\text{env}}$ is found to be far too slow for the conventional understanding of the inelastic collision processes as published in the literature. The measured cooling rate of the 111.5 gram mass stainless steel spheres used in this experiment confirms that the repeated collisions, injecting energy at the rate of 2.5 Watt-sec per collision, a measurable heat loss between inelastic collision events is counter to the inelastic collision models published in the literature.

5. Conclusion

The invariance of the velocity V_{CM} and the law of conservation of energy are found to apply directly to the ideal inelastic collision cases. It has been historically taught that the kinetic energy is not conserved in the inelastic collision case. This teaching has prevailed in the classrooms of physics for nearly a century now. A destruction or annihilation of energy, still taught in all too many physics lectures, is contrary to the law of conservation of energy. This study finds that the kinetic energy equations presented in the textbooks and lectures for the inelastic collision case totally omit the energy transfer terms, $-\frac{1}{2}M_1\left(V_1-V_{CM}\right)^2$ and $-\frac{1}{2}M_2\left(V_2-V_{CM}\right)^2$; a clear misrepresentation of the ideal inelastic collision process. In the rest frame, the kinetic energy is by definition that quantity of energy that is deliverable to the rest frame if and only if the moving mass of initial velocity V were to reach the final velocity V_{CM}. In the ideal inelastic collision case, it is clearly understood that the final velocity after an ideal inelastic collision is precisely that of the velocity V_{CM}. With this, it is clearly seen from the frame transfer model presented here that the energy transferred to the combined masses at collision is exactly equal to the combined initial kinetic energy of the colliding masses before collision less the frame transfer of kinetic energy that takes place between the masses of different frames at collision as dictated by the law of conservation of energy. This center-of-mass technique clearly shows that the kinetic energy is totally conserved and accounted for in the ideal inelastic collision case as well as in the elastic collision case.

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