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Applying the Uncertainty Principle to Single-Particle Interactions

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Abstract

Existing derivations of uncertainty for single-particle interactions violate complementarity because they require that the photon exhibit wave and particle behavior simultaneously, and thus singularly. In order to restore physical consistency to quantum theory, a model of the photon is proposed with spatial extension equal to the wavelength. As a result, uncertainty and indeterminacy must be assigned independent meanings. Uncertainty describes the limit in measurability for instantaneous exchanges of momentum and is due to a photon's particle properties. It is causal in origin and provides a model for the kinematics of quantum theory. Indeterminacy, however, is statistical in nature and is attributed to the exchange of momentum by time-averaged fields. The need for two limitations on measurability is seen as an extension of the duality principle to measurement theory.

Key words: uncertainty, indeterminacy, complementarity, Heisenberg's microscope experiment, spatial extension, photon, absolute space, kinematics

1. INTRODUCTION

The uncertainty principle is in many ways the starting point in discussions of quantum mechanics because it provides a link between the classical and quantum worlds with respect to measurement theory. Several methods of deriving uncertainty have been used since there is a wide variety of natural phenomena to which it applies. Perhaps the most familiar methods are the microscope experiment,(1) a consideration of the properties of wave-packets,(2) and in terms of the wave-function.(3)

The last method is the most accurate and also the one with the most precise theoretical basis. It is obtained from Schrödinger's wave equation and the subsequent introduction of the concept of "wave-function." The wave-function must be interpreted as a probability statement since it refers to the motion of a particle in a large number of identical, nonoverlapping regions of space, or to a large number of independent repetitions in the same region of space with distinct time origins.(4) When uncertainty is derived in terms of the wave-function, we obtain

$$\Delta x \Delta p \geq \hbar / 2, \tag{1}$$

where Δx is defined as the root-mean-square deviation from the expectation value of a position measurement and Δp is the root-mean-square deviation from the expectation value of a momentum measurement. Because the derivation places emphasis upon the *object* of measurement, "indeterminacy" is preferred as a means of referring to it. Feynman describes the situation appropriately in more practical terms:(5) "Why can we only predict the probability that a given experiment will lead to a definite result? From what does the uncertainty arise? Almost without doubt it arises from the need to amplify the effects of single atomic events to such a level that they may be readily observed by large systems." The uncertainty described by (1) applies to particle ensembles assembled by integrating over space or time. The logical clarity of its derivation together with the fact that it can be experimentally verified has caused the ensemble view to be referred to as the most natural interpretation of the physical meaning of uncertainty.(6)

2. THE UNCERTAINTY OF SINGLE-PARTICLE INTERACTIONS

2.1 Heisenberg's Microscope Experiment

Even though Heisenberg's formulation of uncertainty by means of an ideal microscope preceded the statistical interpretation, their logical order must be reversed. This is because it is necessary to extrapolate from the observable ensemble derivation to an unobservable particle derivation. To derive the uncertainty of a particle's position, therefore, we *imagine* the scattering of a γ-ray off an electron and its subsequent passage through a microscope to determine its momentum direction. In order to observe the scattering direction as well as possible, the γ-ray beam is focused by passing it through a lens. According to classical optics and the finite aperture effect, the lens diffracts the beam. Its direction is thereby made imprecise so that the electron's position is imprecise as well. An analysis of the imprecision results in uncertainties of position and momentum given by

$$\Delta x \Delta p \geq h. \tag{2}$$

Stating the above thought experiment as a sequence of physical events, we see that the photon first exchanges momentum with an electron by colliding with it, and then it enters the lens as a wave to establish the electron's position. It is assumed that the microscope's aperture is big enough to admit a photon, but smaller than the wavelength. For the photon to enter the aperture and then be recorded it must be localized as a particle and immediately thereafter focused as a wave. However, the requirement that the photon be localized and diffracted is the same as requiring it to change *instantaneously* from particle to wave. This violates the principle of complementarity prohibiting the simultaneous manifestation of wave and particle behavior.

A two-slit interference experiment serves to illustrate the requirement of wave-particle complementarity. Any attempt to determine which slit the photons pass through will cause the interference pattern to disappear. In fact, it is impossible to conceive of an experimental arrangement in which localized photons demonstrate wave behavior. Although it is not essential that thought experiments be confirmed by actual experiment, they must at least conform to the known physical laws. Heisenberg's thought experiment fails this minimum requirement because it does not obey the very principle, complementarity, that it is meant to define. Thus a microscope, even if it is ideal, cannot be used to determine the position of a point particle because it is a classical instrument that functions by means of the diffraction of waves.

2.2 Bohr's Wave-Packet Interpretation

The wave-packet provides a more elegant method of deriving the uncertainty relations by using Fourier expansions. The time interval during which the bulk of the wave-packet passes some fixed point is defined as Δt, the frequency interval in which the bulk of the participating frequencies lie is $\Delta \nu$, and Δx is the spatial extension of the packet. They are used to obtain the classical relations for the resolving power of a microscope and are then substituted into the quantum-mechanical relations for particles — $E = h\nu$ and $p = h/\lambda$ — to obtain the uncertainty relations for single particles. In other words, the photon/particle is conceived of as a superposition of continuous wave properties and discrete particle properties in the same physical space. This is in contradiction to complementarity, regardless of whether the equations are applied to photons or, as in the diffraction of electrons, to particles. The simultaneous manifestation of wave and particle properties is prohibited.

Although its theory is fundamentally flawed, the wave-packet model of uncertainty can be recovered in practice. Wave-particle complementarity is circumvented by viewing particle parameters as potentialities that are only realized upon detection. A further complication becomes evident, however, because the detection process implies that a singularity in time occurs. Energy and momentum that are delocalized in theory and extend to infinity appear instantaneously when a particle is detected in what is known as the "collapse of the wave-function." If one insists on maintaining a causal description, then this leads to a variety of possibilities that are highly paradoxical.[1] In order to avoid the appearance of physical inconsistency, the mathematics is said to be devoid of physical content.

3. THE CONCEPT OF PHOTON

3.1 The Single Photon

Both methods of extrapolating the uncertainty relations to the level of single-particle interactions illustrate the central problem in trying to verify their authenticity and thus formulate a consistent photon model. Although the verity of the uncertainty relations as reciprocal delimitations of measurement accuracy is unquestioned, the means by which they were obtained is in doubt since a derivation by actual experiment or by thought experiment that is in complete conformance with the laws of nature *does not exist*. Because thought experiments upholding the uncertainty principle require wave and particle properties to be manifested simultaneously, not only is complementarity violated, but the photon must be singular or behave singularly. A singular field structure carries with it the attendant conceptual difficulties common to all singular models.

3.2 The Singular Photon

Even though wave properties are not manifested in single-particle interactions, they must nevertheless be included conceptually in a model of the photon. However, difficulties present themselves if wave properties are introduced singularly. Sinusoidal electromagnetic waves include electric fields of both positive and negative polarity. The coexistence of fields of opposite polarity at a point in space causes a cancellation of field and denies the possibility of external influence altogether. Similarly, their coexistence at a point in time requires that they emanate from that point with the precise timing necessary for rhythmic wave motion. We are faced with a paradox. If the photon acts over a distance equal to the wavelength and a time equal to the period, then how does it do this singularly? A model of the photon that includes these requirements as a single contiguous whole will inevitably lead to considerable physical complexity and conceptual ambiguity.

If a particle is singular, it has a symmetric field and we are able to speak of the possibility of it having an exact position. However, the sinusoidal electromagnetic fields of a photon possess neither a point nor a plane of symmetry, and so we must treat them as asymmetric. Thus to assign an exact or singular position to the photon is neither physically nor mathematically tenable.

3.3 Experimental Model

The quantum-mechanical photon that is actually used to predict the outcome of experiments consists of a wave-packet of length equal to the coherence length, linear momentum equal to $\hbar k$, and spin one. Because the photon may be located anywhere within the wave-packet, it is said to be delocalized. Delocalization is achieved by defining photons in terms of the spatial modes of a quantum-harmonic oscillator. As a result, the meaning of the term “photon” is highly subjective.(7) “The concept [of photon] survives as an operational definition in terms of photon detection and it provides a useful qualitative description of the nature of the state.” Because the model actually used in quantum optics by experimentalists has very little in common with models used in other areas of physics, the concept of photon has fallen into disfavor.(8) Clearly it is impossible to define the photon in a simple and unequivocal manner. The fact that a universal model does not exist is a source of internal inconsistency for quantum mechanics.

3.4 Spatially Extended Model

High-energy electron-photon collisions occur in less than 10^{-11} s, demonstrating that single-particle interactions are very nearly instantaneous.(9) By contrast, a detection event in quantum optics, though discrete, occurs over a time period more than two orders of magnitude larger, or 10^{-9} s. “Photon” detection could be the electrical discharge of photodetection, an exposed silver halide crystal in a film emulsion, or any other discrete event caused by electromagnetic radiation. Therefore the “single photon” of quantum optics differs from the single photon of high-energy interactions. The transfer of momentum by an “optical” photon is field related because detections are averaged over detector surfaces containing large numbers of molecules and time periods much longer than the wave period $1/\nu$. However, high-energy photons interact for extremely short instants of time, so momentum must be transmitted by means of particle properties rather than field properties.

In quantum optics, because a photon’s field is only detectable as a multiple of 2π, phase is not a measurable quantity and the photon is treated as a localization of field or wave-packet. There is also direct evidence that a photon’s fields are discrete. Momentum exchange during the reflection of light occurs in less than 10^{-14} s, a time period much shorter than the duration of optical detections.(10) This means that optical photons do not always exchange momentum by means of time-averaged transverse fields but may do so nearly instantaneously as well. Voluminous evidence indicates that two types of interaction are possible, by field or by particle properties. If we combine experimental evidence with the requirements of complementarity, an acceptable model of uncertainty may be defined very simply by introducing the concept of spatial extension. For example, if photons are viewed as indivisible entities of length equal to the wavelength λ, then there will be an uncertainty of position for single-particle interactions $\Delta x \geq \lambda$. An uncertainty relation due to spatial extension follows naturally:

$$\lambda \cdot \Delta p \geq h. \tag{3}$$

Thus if photons are conceived of as spatially extended entities, *no uncertainty relation is necessary for single-particle interactions*.

The above manner of expressing uncertainty has a meaning that is very different from that by Heisenberg given earlier. The uncertainty relation (2) places position and momentum on an equal footing, thereby implying that either position or momentum measurements may be performed to any desired accuracy by choosing a photon of appropriate energy. However, perfect spatial localization cannot be achieved no matter how much energy is available. At high energy we do not observe localization at all since pair production occurs. This indicates that a cutoff energy must be applied. At the other end of the energy spectrum Abbe's law gives a limiting resolution of $\lambda/2$ for microscopes, but the improved accuracy is due to diffraction effects and does not affect the detection of point particles.

Conversely, it *is* true that momentum can be measured to any desired accuracy. The momentum of an ideal mirror, for example, does not change appreciably ($\Delta p \approx 0$) for a determination of its position in one dimension. Therefore its position depends only on λ. In fact, actual experimental determinations of momentum that lie below the limit set by (2) have been performed and are referred to in the literature as "quantum nondemolition" measurements.(11)

4. QUANTUM KINEMATICS

Kinematics concerns the motion of a particle in the absence of force. Because the uncertainty relations (1), (2), and (3) all claim to be minimal descriptions of particle motion, their purpose is essentially to define the kinematics of a quantum system.

In (3) the duration of particle collisions is assumed to be instantaneous, so we attribute changes in momentum to a particle property, impenetrability, rather than to a force ($F = dp/dt$). In order to define the kinematics of the interaction, we set the particle's initial velocity equal to zero and use it to specify the origin of a coordinate system. The trajectory of the incident photon together with its polarization is then sufficient to describe a geometry for locating coordinate points, where wavelength determines the minimum spacing of the coordinates. A determination of the momenta after collision will suffice to complete a kinematical description of this, the simplest quantum system.

It is now evident why Heisenberg based his interpretation of uncertainty upon the action of a singular photon in absolute space. As he states, "Quantum mechanics [is] founded exclusively upon relationships between quantities which are in principle observable."(12) Quantum observables need not be described relative to a well-defined origin. A position measurement in a quantum system, for example, is expressed most naturally in terms of coordinate *differences*, not laboratory frame coordinates. This is because properly defined coordinates must be specified relative to something physical. Consequently, the choice of origin for describing microscopic observables is completely arbitrary and in fact there is no compelling need to specify an origin at all. Nevertheless, it is a required *first step* in applying the laws of mechanics to a particular system. The microscope experiment uses a *macroscopic* instrument yet it ignores the *macroscopically* derived principles of mechanics by not specifying an origin. Thus quantum mechanics is more properly described as a theory about matter's structure rather than its mechanics.

A comprehensive discussion of the relationship between reference frames and the microscopic observables they describe is given elsewhere.(13)

5. CONCLUSION

In the previous discussions concerning interactions that occur between electromagnetic radiation and matter it was revealed that the imprecision of measurement, or uncertainty, of single particles is not the same as the field-related indeterminacy of particle ensembles. This suggests that the principle of duality should be extended to include measurement theory as follows. On the one hand, there is a field-derived *indeterminacy* expressed by (1); on the other hand, a particle-derived *uncertainty* is given by (3). Thus two limitations on measurability would exist: one that is particle related and causal and one that is field related and statistical in origin. The same conclusion may also be arrived at through a consideration of the properties of *observers* (cf. Ref. 13).

Experiment cannot distinguish uncertainty from indeterminacy because time-averaged and instantaneous exchanges of momentum occur independently and are measured by different means. In the diffraction of an electron beam, for example, the minimum uncertainty in the future position of a point electron is given by (1) and is equal to $\hbar/(2\Delta p)$. However, the electron does not arrive at a point on the screen since it must be made observable by the expulsion of at least one electron from a fluorescing atom. The position of the expelled electron is determined by (3) since it has a minimum uncertainty of position in its atomic orbital given by $\Delta x \geq \lambda$, where λ corresponds to the escape energy. Therefore it is not strictly true to state that the wave-function provides "a quantum mechanically complete description of the behavior of a particle."(14) The wave-function is defined with respect to point particles, and the existence of spatial extension in the determination of a particle's parameters or the possibility of spatial extension in its structure cannot be ignored.

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Résumé

Les dérivations existantes de l'incertitude pour des interactions de particules isolées violent la complémentarité parce qu'elles exigent que un photon aille un comportment de onde et 'de particule simultanément; et ainsi singulièrement. Afin de remettre 'uniformité physique à la théorie quantique on propose un modèle du photon en ayant la prolongation spatiale égale à la longueur d'onde. En conséquence, l'incertitude et l'indéterminisme doivent être assignés des significations indépendantes. L'incertitude décrit la limite dans l'aspect de la mesure pour des échanges instantanés de moment et est due aux propriétés des particules de photon. Elle est causale d'origine et fournit un modèle pour la cinématique de la théorie du quantum. L'indéterminisme, d'autre part, est statistique en nature et est attribué à l'échange 'de moment par les champs moyennés dan le temps.. Le besoin de deux limitations sur l'aspect mesure est vu comme une prolongation du principe de dualité à la théorie de mesure.

Endnotes

[1] Schrödinger's cat and parallel universes are two commonly cited examples.

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