The Electromagnetic Force between Two Parallel Electric Currents of 'Infinite' Length Attained Using Respectively Ampère's Law and Coulomb's Law Including a Relativistic Analysis.

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Abstract

In this paper an analysis is being made of a physical electric circuit that might correspond to the famous case of two parallel electric conductors of ‘infinite length’. The case, invented by Ampère, is not physically possible, of course. All electric currents must namely be guided back to their origin. Therefore one ought to present a physical circuit that contains as a part two conductors infinitesimally close to each other. It appeared that Ampére’s bridge would be a suitable choice as a starting model, provided a pair of conductors is inserted inside Practically, it resembles an “eight”, with two closed circuits coming close to each other along one branch. This is explained more rigorously in the text. A benefit is that a set of detailed computations on Ampère’s bridge by this author may be used as a mathematical basis. A comparison between the results attained by using Ampére’s law and Coulombs law respectively is being made. Further, the Lorentz Transformation of the Special Relativity Theory is being applied on the Coulomb result. The result is that also the latter method succeeds in predicting the force between two conductors. This result must further be chosen, since it has been shown elsewhere that the very definition of Ampère’s law is devoid of logically consistent argument, whereas Coulomb's law constitutes a 'simplest possible assumption. The usage of Coulomb’s law is completed with a relativistic analysis, relevant to the properties of the actual circuit, i.e. geometry, velocities.

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2. Introduction. Ampére's law vs. Coulomb's law

It is widely recognized that two parallel electric conductors are attracting each other, if the direction is the same. Here it will be shown by using a circuit with the basic structure of Ampère's bridge, how a theoretical result able to predict this can be derived. The model with two parallel conductors of infinite length of course does not correspond to a physical circuit. All currents must return to their origin. The idea of the two parallel conductors with infinite length is apparently an approximation, but a derivation of the formal basis for this would be beneficial for a strict treatment. However, if claiming this, it is by scientific argument also needed a physical circuit upon which the approximations are performed. As a matter of fact this author has already dealt with Ampère's bridge at several occasions and therefore a rigorous mathematical analysis already exists. It is rather simple to re-use these results with respect to this case. However, this author has also been giving good arguments for Coulomb's law as a better one than Ampère's law [1], [2], [3]. This will be repeated in this case. It has also been written papers by this author, which succeed in denouncing Ampère's law.

2.1. An analysis of the impact the branches have on the total force

For the convenience a set of Ampère's bridge is shown below in order to make the identification easier when solving the problem with two parallel currents.

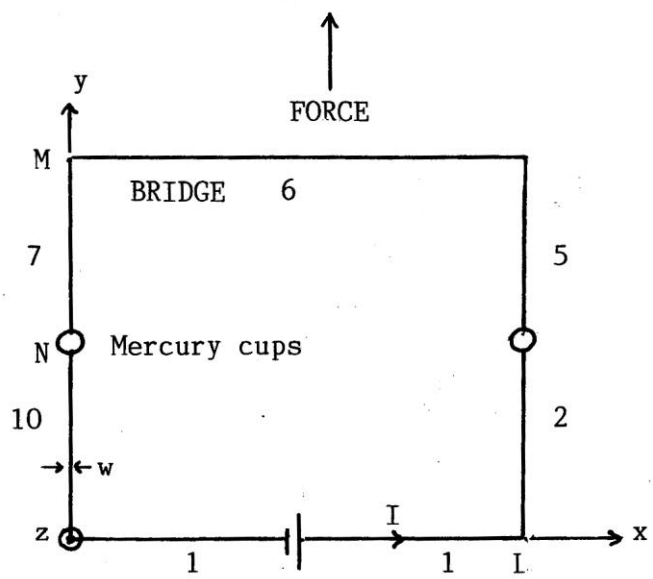

Figure 1: Ampère's Bridge [1], [2], [3] ,[6]

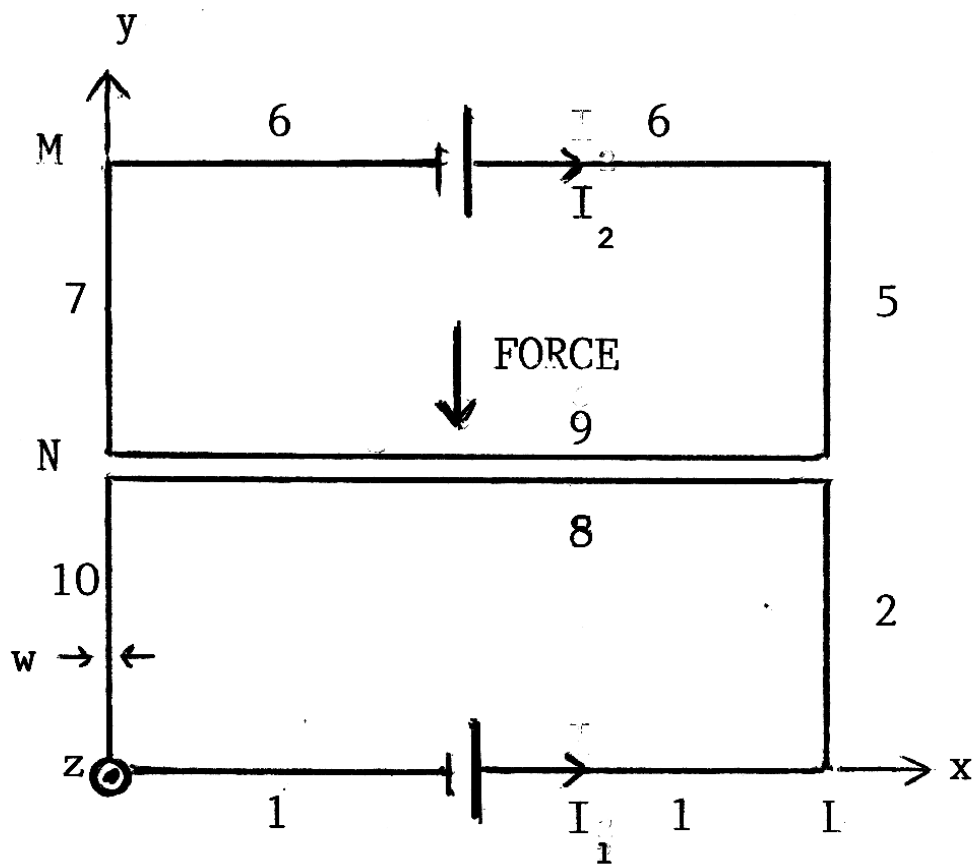

Figure 2: The configuration according to Ampère's bridge used in order to realize a "two-infinite-parallel-conductor" circuit

When the distance between two of the branches is very small compared to the distances between other branches, it is apparent that the result will be dominated by the closely situated branches. What the contributions are have earlier been rigorously derived in another paper [1] and the expressions will be used straightforwardly , without giving all the details once again.

2.2. Ampére's law predicting the attractive force between two electric conductors

Ampère's law predicts the force that a current will affect another one with. The formulas have earlier been given in [1], [2] and are as follows, on differential form:

$$\frac{d^6\vec{F}}{dx_1dx_2dy_1dy_2dz_1dz_2} = \vec{r}(-2\frac{\vec{J}_2 \bullet \vec{J}_1}{r^3} + 3\frac{(\vec{J}_2 \bullet \vec{r})(\vec{J}_1 \bullet \vec{r})}{r^5}) \qquad (1)$$

$$d^2\vec{F} = I_2 I_1 \vec{r}(-2\frac{(d\vec{s}_2 \bullet d\vec{s}_{1)}}{r^3} + 3\frac{(d\vec{s}_2 \bullet \vec{r})(d\vec{s}_1 \bullet \vec{r})}{r^5}) \qquad (2)$$

These formulas may be applied on an electric circuit, provided the geometry has been adequately defined. In the case that two currents come infinitesimally close to each other, the second formula has to be used (implying thus volume integrals to be used). In the other cases, line integrals will suffice. In the following sections it will be shown what force is expected to arise due to the different combinations of branches.

2.2.1. Integral from branch 10 to branch 5

The force that branch 10 will give rise to on branch 5 has been attained by using Ampère's law according to the guidelines given above and are as follows[1]:

$$\vec{F}_{10\to 5} = \vec{F}_{2\to 7} =$$

$$I^2(\frac{M-N}{\sqrt{(M-N)^2+L_2}} - \frac{M}{\sqrt{M^2+L^2}} + \frac{N}{\sqrt{N^2+L^2}} - \ln\frac{M-N+\sqrt{(M-N)^2+L^2}}{L}) + \ln\frac{M+\sqrt{M^2+L^2}}{N+\sqrt{N^2+L^2}})$$

(3)

2.2.2. Integral from branch 1 to branch 7 (excluding the effect due to the current source in branch 1-please see chapter 2.3)

In this case the result is [1]:

$$\vec{F}_{1\to 7} = I^2(\ln\frac{M}{N} + \frac{M}{\sqrt{M^2+L_2}} - \frac{N}{\sqrt{N^2+L^2}} - \ln\frac{M+\sqrt{M^2+L^2}}{N+\sqrt{N^2+L^2}}) \qquad (4)$$

2.2.3. Integral from branch 1 to branch 5 (excluding the effect due to the current source in branch 1 –please see chapter 2.3)

In this case the result is [1]:

$$\vec{F}_{1\to 5} = I^2(\ln\frac{M}{N} + \frac{M}{\sqrt{M^2+L_2}} - \frac{N}{\sqrt{N^2+L^2}} - \ln\frac{M+\sqrt{M^2+L^2}}{N+\sqrt{N^2+L^2}}) \qquad (5)$$

2.2.4.. Integral from branch 1 to 6(excluding the effect due to the current source in branch 1-please see chapter 2.3)

In this case the result is [1]:

$$d^2\vec{F}_{1\to 6} = I^2(2\sqrt{(1+(\frac{L}{M})^2} - \frac{2M}{\sqrt{L^2+M^2}}) \qquad (6)$$

2.2.5. Integral from branch 8 to 9

This case is similar to the preceding one (i.e. section 2.2.4, 'Integral from branch 1 to 6') in that two parallel conductors are regarded. The result will therefore be alike Eq. (6) above, but with other values of the variables. Since the distance between the two actual branches is only r_{12} , instead of M one should use r_{12} . Hence,

$$d^2\vec{F}_{8\to 9} = I^2(2\sqrt{(1+(\frac{L}{r_{12}})^2} - \frac{2M}{\sqrt{L^2+r_{12}{}^2}}) \qquad (7)$$

2.2.6. Integral from branch 2 to 9(and 10 to 9)

In order to attain the results for the contribution from branches 2 and 10 respectively on branch 9, it must be realized that these branches come close to each other at one point. This case is mathematically similar to one being described in [11]

The result of this integral might therefore be used, if replacing $M-N$ with r_{12}

The result due to branches 2 and 7 is repeated here for convenience:

$$\vec{F}_{2\to 7,b} =$$

$$I^2(\frac{M-N}{\sqrt{(M-N)^2+L_2}} - \frac{M}{\sqrt{M^2+L^2}} + \frac{N}{\sqrt{N^2+L^2}} - \ln\frac{M-N+\sqrt{(M-N)^2+L^2}}{L}) + \ln\frac{M+\sqrt{M^2+L^2}}{N+\sqrt{N^2+L^2}})$$

(8)

$$\vec{F}_{2\to 9,b} = I^2(\frac{r_{12}}{\sqrt{r_{12}^2+L^2}} - \frac{M}{\sqrt{M^2+L^2}} + \frac{N}{\sqrt{N^2+L^2}} - \ln\frac{r_{12}+\sqrt{r_{12}^2+L^2}}{L}) + \ln\frac{M+\sqrt{M^2+L^2}}{N+\sqrt{N^2+L^2}})$$

(9)

2.3. The impact of the current source in contributing to the force.

Jonson also discovered that the very current (voltage) source also plays a role in contributing to the total force between two currents [3]. There were found to come contributions to all the branched of the bridge (i.e. branches 6,7, and 5 respectively.

2.3.1 The sum of the correction terms due to branches 5, 6 and 7

The result of the operations were[1]:

$$I^2 \frac{L+M}{6}\left(\frac{-LM}{((\frac{L}{2})^2+M^2)3/2}+(\frac{2L}{M}-\frac{2M}{L})\frac{1}{\sqrt{(\frac{L}{2})^2+M^2}}-\frac{NL}{((\frac{L}{2})^2+N^2)^{3/2}}+\frac{2N}{L}\frac{1}{\sqrt{((\frac{L}{2})^2+M^2}}\right)$$

(10)

Evaluation gives $F_{corr} \cong -0.51 I^2$ (11) [4]

2.3.2 The sum of the correction terms due to branches 9

The new branch that will be affected in this set is branch 9. The result will be similar to branch 6 [1], if only changing the sign and replacing M with N.

$$\frac{2}{3} I^2 N(L+N)\left(\frac{+L}{((\frac{L}{2})^2+N^2)^{\frac{3}{2}}}-\frac{1}{N^2}\frac{2L}{\sqrt{L^2+N^2}}\right) \qquad (12)$$

2.4. Analysis of the relative impact on the total force of the respective contributions

A short glance at the contributions above gives that all expressions except Eq. (7) are of order ~1

(L, M and N are of the same order, between about one half and one meter each).

Eq. (7), however, behaves as

$$\sim \frac{L}{r_{12}} \text{ when } r_{12} << L, M, N \qquad (13)$$

which is commonplace when regarding two conductors as ‘close’ to each other, being of infinite length.

The conclusion is that in this circuit, when two of the branches are aligned along each other on a distance regarded as small compared to other distances, the force is dominated by the combination of those two branches. The force thus attained resembles that predicted by Ampère’s law, but also the Lorentz force law.

2.5. The case of the force between only two parallel conductors.

As appears from the discussion in the previous section, a circuit, where two parallel conductors are situated infinitesimally close to each others may in a mathematical sense be regarded, as if these two conductors were the only ones in the circuit. This is consistent with the way one is usually treating two parallel conductors. Usually, one uses the Lorentz force law in order to derive the force, and the fact that Ampère's force law for this special case arrives at the same result may have inferred physicists to believe that the two laws are also formally identical. However, in the paper by this author being referred to [1] that treats Ampére's bridge, it is convincingly shown that this is not the case.

2.5.1. A nonrelativistic analysis

This case is a classical one being taught within undergraduate courses. The most common way of explaining the attractive force between two current carrying conductors is using the so-called F=BiL rule, based upon a usage of the Lorentz force. Now it can be easily be tested, whether Ampère's law is an alternative, using the calculations in this very paper. There is a suitable formula for the part of Ampère's bridge, containing two parallel conductors, Eq. (7) above. The most convenient circuit demonstrating the force was described by Neumann [8].

The mentioned equation may be applied using instead of M the radial distance between two parallel conductors, r_{12}. When the distance between the conductors is very small, infinitesimal analysis allows for neglecting terms that becomes very small in comparison to the 'main term'. The term that remains within Eq. (7) is the $(\frac{L}{M})^2$ term within the left square root This gives $d^2\vec{F}_{1\to 6} \to I^2(2\frac{L}{r_{12}})$ (14)

Realizing that there is needed a minus sign if putting the directions of the two currents the same, the force between the currents becomes negative,

$$d^2\vec{F}_{1\to 6} \to -I^2(2\frac{L}{r_{12}}) \qquad (15)$$

which indicates a negative, hence attractive force between the currents, in good accordance with experience. This seems to imply that the 'cross product Lorentz's law' is unnecessary.

Ampére's law also has the benefit of being able to explain the repulsive force in the mercury basin between the electric poles and a copper boat described by Ampère [9], which the Lorentz' force law is unable to.

2.6. Coulomb's law predicting the attractive force between two electric conductors

2.6.1. A paradox

It may also be mentioned that Coulomb's law cannot account for the attractive force.]The force namely becomes repulsive between two parallel conductor, as appears when looking at the equation, here repeated for convenience. As described in [5], Coulomb's law is represented by the 'b-term' (times one-third).

$$d^2\vec{F}_{1\to 6,b} = I^2(-2\sqrt{(1+(\frac{L}{M})^2} + 4 - \frac{2M}{\sqrt{L^2+M^2}}) \qquad (16)$$

Making the same infinitesimal analysis as with respect to Eq. (7) above, namely gives a minus sign in front of the $(\frac{L}{M})^2$ term within the left square root and after having changed sign, in analogy with above, the final sign with respect to two parallel current flowing in the same direction becomes positive, hence a repulsive force**.** This would seem to prove the unfeasibility to use Coulomb's law in the actual experimental situation, and, therefore disproving the whole theory. A the disproval of a theory in one sole case namely implies that the theory is false. However, it must be kept in mind that thus far the Special relativity theory(SRT) has not been applied. The mere fact that the electron velocities are extremely low compared to the speed of light makes one easily a victim of the temptation not to investigate the effects of the SRT at all.

As will be seen in the following section, the usage of the SRT will infer another result to arise, one that satisfies the sign requirement (i.e. an attractive force between the two parallel currents).

2.6.2. The solution to the problem through the usage of Special relativity theory (SRT).

What has been left to be done is a relativistic analysis. If a non-relativistic approach is insufficient in order to explain a physical phenomenon involving velocities, it is reasonable to se to what extent an application of the Special Relativity Theory (SRT) will change the result. The need is in this case felt acute, since it does not exist any explanation to Ampère's Law. Efforts to relate Ampère's law to the Lorentz force have been made, but have been disproved [2] In the preceding section it was shown that that Coulomb's law did not succeed using non-relativistic methods. So far this constitutes a huge problem : Simultaneously it is impossible both to derive Ampère's law and to use Coulomb's law. Ampère's law is rather arbitrarily defined by Ampère as an ad hoc definition and Coulomb's law predicts the wrong sign of the force. Admittedly, Keele [12], [13] has made an effort to use apply the Special relativity theory(SRT) on Coulomb's law, but his analysis has appeared to be incomplete with respect to retarded action and to the usage of the SRT [14]. The thus far best solution will appear if taking into account all known facts concerning geometry, retarded action and the SRT. It appeared to be rather complicated to do in reality, but if doing every part in a very strict mathematical way, success followed.

2.6.3. The four contributions to the force between the two conductors

Since each conductor has both immobile positive ions and moving electrons, there will appear four separate force terms due to the four ways the charges of the first conductor may interact with charges of the second conductor. In order to make the properties of the forces easily conceivable, the analysis is restricted to the simplistic case with two parallel conductors. This means that the angle with which a distance vector crosses both the conductors, φ and ψ, are equal, even though the simplification is not immediately conducted. In this case the so-called Standard Configuration of the SRT may be applied, with one system K, which is stationary with respect to the positive ions of the two conductors, hence the whole circuitry, and K' following the movement of the electrons of the first conductor, along the x_1 axis, and with velocity v_1. The velocity of the electrons of the second conductor is assumed to be v_2. In this analysis only the simplistic case with equal velocities, $v_1 = v_2$, will be regarded, when comes t.

These definitions make it possible to compare the effects of the electrons of respective conductor on each other.

The focus in the analysis is on the force perpendicular to the currents, which is also the force that acts repelling or attracting between them.

The fundamental assumption that is made here is that Coulomb's Law is the only cause behind the force between electric charges, which implies that the very idea of magnetic fields is abandoned. It is necessary to emphasize that, since it is often claimed that Coulomb's law is consistent with the existence of specific magnetic force terms within the expression of the total electromagnetic force.

What brings about a change of the shape of the expression of the force compared with the original Coulomb's law is

1. The effects of the SRT (i.e. Lorentz transformation and derived expressions)

2. Delay effects concerning the retarded observation of fields being generated by charges

2.6.4. The four contributions to the force between the two conductors

When taking into account the Special relativity theory, it is extremely important to very strictly define to which coordinate system the variables are referring to. Primed or not primed variables will indicate whichever system is used.

2.6.5. The force between the positive charges of respective conductor

Since no velocities are involved, when deriving the force that the positive ions give rise to, the force can easily be attained by writing down plain Coulomb's law for the case, when the charges are regarded as thin linear elements.

$$d^2 F_{y,+to+} \cong \frac{\rho_1 \rho_2 \Delta x_1 \Delta x_2}{r^3} y \qquad (17)$$

2.6.6. The force between the electrons of respective conductor

In order to make the calculation of the force as easy as possible, it is favourable to regard the electrons of either system as being at rest. For convenience the K' system is chosen.

Since the force as it is being felt within K shall be computed, it must be realized the distance Δx_1 is felt shorter by K', namely

$$\Delta x_1' = \frac{\Delta x_1}{\gamma(v_1)} \qquad (18)$$

here $\Delta x_1' = \dfrac{\Delta x_1}{\gamma(v)}$ (19)

The same holds also for the charge element of the second conductor with moving electrons.

They will regard

$$\Delta x_2' = \frac{\Delta x_2}{\gamma(v_2)} \qquad (20)$$

Further, the distance vector between the charge elements will not be $\vec{r} = (x, y, z)$.

Instead it will be $\vec{r}' = (\dfrac{x}{\gamma(v_1)}, y, z)$ (21)

It must also be taken into account that if having a force in K', that too will be transformed

The Coulomb force between the electrons of respective conductor will

Now it is the force that shall be transformed from K' to K and Resnick [16] shows how this must be done according to the SRT:

$F_x = F_x'$ (389) (22) $F_y = \dfrac{F_y'}{\gamma(v)}$ (390) (23) $F_z = \dfrac{F_z'}{\gamma(v)}$ (391) (24)

Finally, the effects of retardation mentioned above implies that if there is a difference between the two velocities, a delay factor $(1 - \dfrac{v_2 - v_1}{c}\cos\theta)$ must be multiplied [15]. However, if –which will be done here in the simplified case – they are assumed to be equal – this term can be neglected. However, there inevitably will appear ***two delay effects*** due to the fact that in K, the signal from a moving charge element is delayed dependent on direction, simultaneously the field arrives at the moving charge element of the second conductor with a delay that differs with respect to position at that charge element [15].

The resulting force can now be written:

$$d^2F_{y,-to-} \cong \frac{\rho_1\rho_2 \frac{\Delta x_1}{\gamma(v_1)} \frac{\Delta x_2}{\gamma(v_2)} (1 - \frac{v_2 - v_1}{c}\cos\theta)}{((\frac{x}{\gamma(v_1)})^2 + y^2 + z^2)^{\frac{3}{2}}} y \bullet \frac{1}{\gamma(v_1)} (1 - \frac{v_1}{c}\cos\theta)(1 - \frac{v_2}{c}\cos\psi)$$

(25)

After some manipulations with the expression, assuming $\frac{v_1}{c} << 1$, that allows for a series expansion, gives:

$$d^2F_{y,-to-} \cong \frac{\rho_1\rho_2\Delta x_1\Delta x_2}{r^3} y \bullet (1 - \frac{3}{2}(\frac{v_1}{c})^2 \cos^2\theta) - (\frac{v_1}{c})^2 - \frac{1}{2}(\frac{v_2}{c})^2 + \frac{v_1}{c}(\cos\theta)\frac{v_2}{c}\cos\psi)$$

(26)

2.6.7. The force from the electrons of the first conductor to the positive ions of the second conductor.

The expression for the force can rather easily be constructed using Eq. (26), if replacing $\rho_2\Delta x_2'$ with $-\rho_2\Delta x_2$, the minus sign due to the opposite sign of the positive ions with respect to the electrons. Further, there will be no delay term, since the ions of the second conductor are at rest with respect to K. There will also in this case be a division by the gamma factor at the end, since the electrons of the first conductor are at rest in K'. Hence,

$$d^2F_{y,-to+} \cong \frac{-\rho_1\rho_2 \frac{\Delta x_2}{\gamma(v_2)} \Delta x_1}{((\frac{x}{\gamma(v_1)})^2 + y^2 + z^2)^{\frac{3}{2}}} y \bullet \frac{1}{\gamma(v_1)} \qquad (27)$$

After some manipulations with the expression, assuming $\frac{v_1}{c} << 1$, that allows for a series expansion, gives:

$$d^2F_{y,-to+} \cong \frac{-\rho_1\rho_2\Delta x_1\Delta x_2}{r^3} y \bullet (1 - \frac{3}{2}(\frac{v_1}{c})^2 \cos^2\theta) - \frac{1}{2}(\frac{v_1}{c})^2) \qquad (28)$$

2.6.8. The force from positive ions of the first conductor to the electrons of the second conductor.

In this case, there will be no Lorentz transformation from K' to K, since the positive ions of the first conductor that are giving rise to a force at the second conductor are already situated in K. The case is in that case similar to the first case above. The difference is now that the electrons of the second conductor are moving away from the observer, which makes the need for a delay term apparent. Please observe also the need for a change of sign due to the opposite sign of the positive ions with respect to the electrons. Hence,

$$d^2F_{y,+to-} \cong \frac{-\rho_1\rho_2\Delta x_1\Delta x_2(1-\frac{v_2}{c}\cos\psi)}{r^3}y \qquad (29)$$

2.6.9.. The sum of the four contributions to the force between the two conductors

Summing all four contributions, assuming also the simplest case with equal velocities, gives after some boring steps

$$d^2F_{y,total} \cong \frac{\rho_1\rho_2\Delta x_1\Delta x_2}{r^3}y(-(\frac{v}{c})^2+(\frac{v}{c})^2\cos\theta\cos\psi) \qquad (30)$$

which may also be written

$$d^2F_{y,total} \cong \frac{1}{c^2r^3}y(-\frac{I_1I_2}{c^2}+\frac{I_1I_2}{c^2}\cos\theta\cos\psi) \qquad (31)$$

3. Conclusions

In fact this expression above Eq.(30) has the same asymptotic properties as Eq. (2), even though the coupling constants are different.

The expressions will differ, however, if allowing for different velocities and directions, but in the time of Ampère, it seems reasonable that there were no opportunities to neither vary nor measure the electron velocities. Not even the electron had been discovered.

Therefore one can say that the above result (30) is in appropriate accordance with Ampère's law, which would be rather satisfactory, if taking into account that Ampère himself was hesitating between different coupling constants [17]..

The appropriate conclusion to be drawn is that Ampère's law appears as a consequence of applying the SRT to Coulomb´s law in the case of two current carrying conductors. And since this law is sufficient, the Lorentz force is no more needed. Ampère´s law can namely account for the attractive force between two parallel conductors.

4. Units

The units are SI-units.

$x_1,\ x_2,\ y_1, y_2, z_1, z_2$ Cartesian variables of respective branch

I current

$I_1,\ I_2$ current, with indices, due to the respective number of circuit

$\vec{F}$ force (vector quantity)

$\vec{J}_1,\ \vec{J}_2$ current density (vector quantity)

$d\vec{s}_1,\ d\vec{s}_2$ infinitesimal element of the respective conductor

$r, \vec{r}$ distance between the two current elements (magnitude and vector)

r_{12} the distance between two parallel conductors when regarded as small compared to the other distances of the circuit

$\vec{F}_{10\to 5}$ the force due to the effect of one branch upon another (here 10 and 5 respectively)

$\vec{F}_{2\to 7,b}$ = the force due to the effect of one branch upon another (here 2 and 7 respectively), the effect of the 'second(i.e. 'b') term'

$d^2F_{y,-to+}$ the y component of the force due to the effect of one branch upon another (relativistic analysis; the force from electrons of the first conductor on the positive ions of the other)

$\rho_1,\ \rho_2$ charge density of the first and second conductor respectively

$\gamma(v_1), \gamma(v_2)$ Lorentz ('gamma') factor of respective current due to the velocity of the electrons

5. References

[1] Jonson, J.O.,' A Detailed 'Wesley Evaluation' of the Pappas-Moyssides Experiments on Ampere's Bridge Compared to Jonson's Evaluation Using Coulomb's Law and the Special relativity Theory'**.** Only published on www.worldsci.org

[2] Jonson, J.O.,'A Comparison between Ampère's Law, Coulomb's Law and the Lorentz Force' Proceedings of the NPA, Volume 7, pp. 232-239
2010, 17th Natural Philosophy Alliance Conference, Long Beach, CA, United States. A small correction of the formula has been made with respect to the original printing.

[3] Jonson, J.O., 'The Magnetic Force Between Two Currents Explained Using Only Coulomb's Law', Chinese Journal of Physics, 1997, Volume 35, No. 2, pp. 139-149

[4] Jonson, J.O.,' A Detailed 'Wesley Evaluation' of the Pappas-Moyssides Experiments on Ampere's Bridge Compared to Jonson's Evaluation Using Coulomb's Law and the Special relativity Theory'**.** Eq. (349). Only published on www.worldsci.org

[5] ibid, chapter 5.2

[6] J. P. Wesley, Spec. Sci. Tech. 10, *47 (1987).*

[7] The Grassmann-Ampere paper by this author

[8] Neumann, 'Vorlesungen über Elektriscje Stroeme', Poggendorffs Annalen der Physik, Leipzig, Teubner, 1884, § 33 p. 91, (Ref given by P. Graneau, 'Amperee Tension in Electrc Currents, IEEE Trams on Magn. VOL MAG 20 NO. 2, 1984)/2010NPAJOJ-filen

[9] A. M. Ampère, "Mémoire. Sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience, dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 et 26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825", 'Mémoires de l'Académie Royale des Sciences de l'Institut de France Année 1823, Tome VI, Paris, chez Firmin Didot, Père et fils, libraires, Rue Jacob, No 24, p 211. Copy belonging to the Stockholm University Library (www.sub.su.se)/

[10] J. P. Wesley, Spec. Sci. Tech. **10,** 47 (1987)

[11] 3.1.2. Integral from branch 2 to 7 in [1]

[12] J. Keele, "Theoretical Derivation of Ampere's Law", Galilean Electrodynamics, March/April 2002, Vol. 13, No. 2, p. 40.

[13] J. Keele, "SR Theory of Electrodynamics for Relatively Moving Charges", http://www.worldsci.org/php/index.php?tab0= Scientists& tab1=Scientists&tab2=Display&id=43

[14] J. O. Jonson, 'The Magnetic Force between Two Currents Further Analyzed Using Coulomb's Law and Special Relativity Theory', Chapter 3.3, Proceedings, *2009, 16th Natural Philosophy Alliance Conference, Storrs, CT, United States*

[15] Jonson, J. O., "The Magnetic Force between Two Currents Explained Using Only Coulomb's Law", Chinese Journal of Physics, VOL. 35, NO. 2, April 1997, p. 142,

[16] Robert Resnick, 'Introduction to Special Relativity', John Wiley & Sons, 1968, p. 148

[17] James R. Hofmann, André Marie Ampère: 'Enlightenment and Electrodynamics', Cambridge University Press, 1994.