Linearization of the Einstein Equation and The 1993 Press Release of the Nobel Prize in Physics

C. Y. Lo
Applied and Pure Research Institute, 7 Taggart Drive, Unit E, Nashua, NH 03060
e-mail: c_y_lo@yahoo.com

In spite of Gullstrand's warning, theorists including Nobel Laureates and Field Medalists, failed to see that linearization of the Einstein equation to obtain an approximate dynamic solution is not valid in mathematics. This error is manifested in the 1993 press release of the Nobel Committee, with its misinterpretation of Einstein's equivalence principle. In 1995, however, Lo demonstrated that the Einstein equation cannot have a bounded dynamic solution for a two-body problem or gravitational wave solutions; and the Hulse-Taylor binary pulsars experiments actually support the modified Einstein equation. Nevertheless, due to earlier errors, these mathematical challenges to the linearization of the Einstein equation were not recognized. Consequently, theorists failed to see the necessity of unification between electromagnetism and gravitation. To address this problem, this paper will present two examples to illustrate: 1) The basic mathematical errors at the undergraduate level in the "bounded wave solution" of Misner, Thorne & Wheeler, and 2) The violation of the principle of causality in the unbounded "weak plane-wave" of Bondi, Pirani, & Robinson.

Key Words: Linearization; Einstein's equivalence principle; irreducibly unbounded; principle of causality; plane-wave; the modified Einstein equation of 1995. 04.20.-q, 04.20.Cv

"Science sets itself apart from other paths to truth by recognizing that even its greatest practitioners sometimes err. … We recognize that our most important scientific forerunners were not prophets whose writings must be studied as infallible guides—they were simply great men and women who prepared the ground for the better understandings we have now achieved." -- S. Weinberg, *Physics Today*, November 2005

1. Introduction

A problem in general relativity is the validity of linearization as an approximation of the Einstein equation. This approach uses the linearized Einstein equation [1] - that the bending of light is obtained, gravitational waves are derived, and Newtonian gravity is shown - as an approximation of the Einstein equation. Thus, the validity of linearization is important in general relativity.

An implicit assumption of linearization is the existence of a weak solution. In physics, this is guaranteed by the principle of causality, which implies that for weak sources the solution would also be weak [1-3]. Physical principles are applicable only to equations that are valid in physics, and since observation confirms Einstein's predictions, it would appear that there is no barrier to the validity of linearization. Therefore, many have argued that two-body solutions should exist and thus Einstein's derivation of the perihelion should be valid [4, 5].

Gullstrand [6], however, a member of the 1921 Committee for the Nobel Prize, suspected that Einstein's derivation was not valid since mathematically there was insufficient evidence for the existence of a solution for the two-body problem, which should exist if the equation were valid. Consequently, Einstein was awarded a Nobel Prize for his work on photo-electric effects [7], instead of his work on general relativity as many were expecting. However, although nobody can give a convincing counter argument on Einstein's behalf, some speculated the influence of Gullstrand as largely due to the fact that he is Swedish.

It was known in 1995 that there is no exact solution for the two-body problem and the gravitational radiation from realistic sources [8-10]. However, many still believed that approximate solutions could always be obtained through the approach of perturbation, demonstrating a lack of understanding of the new features of the non-linear Einstein equation.

The position of The Royal Swedish Academy of Sciences on general relativity changed in the press release awarding the 1993 Nobel Prize in Physics jointly to Hulse and Taylor, Jr., both of Princeton University, for the discovery of a new type of pulsar. The discovery was credited with not only opening up new possibilities for the study of gravitation, but also with the idea, "*Einstein's theory has passed the tests with flying colours*" [11]**.** In so doing, the Academy also considered, just as the deflection of light, the perihelion motion of Mercury as a valid small general-relativity contribution. However, such a claim has been proven incorrect. It was found [8-10] to be impossible for the Einstein equation to have a bounded solution of a two-body problem or a gravitational wave solution. [1)] Moreover, the Hulse & Taylor experiments actually support a modified Einstein equation that implies the existence of gravitational waves (see Section 2).

The impossibility of having a dynamic solution implies unequivocally that, for the dynamic cases, the Einstein equation is invalid in physics. Apparently, these conclusions were surprising to many theorists, including Nobel Laureates [4, 5] and Field Medalists (see Section 2), since they mistakenly believed that what had been derived from the linearized Einstein equation should be approximately valid for the non-linear equation. Mathematically, the linearized equation is valid for an approximate solution if weak solutions exist. Moreover, physics requires that a weak solution would always exist if the source term is weak enough. Thus, a bounded solution would not exist only if the field equation were not valid in physics.

Moreover, there was little reason to doubt the validity of the Einstein equation. Einstein's accuracy in his predictions from static solutions created a barrier to the objective re-analysis of his theories. Many incorrectly viewed linearization as valid because of its established acceptance. While linearization had been established for the static case, the fact is that the validity of linearization to obtain an approximate solution has never been proven. To establish this, it would be necessary to prove the existence of bounded dynamic solutions, i.e. Gullstrand [6] would be wrong.

What challenged this long-standing belief was NASA's discovery of the pioneer anomaly. Their data suggest the existence of theoretical problems in Einstein's theory [12-16], which lead theorists to re-examine Einstein's theory. In fact, for a non-linear field equation, a weak solution may not exist - independent of the strength of the source [8-10] although this is possible only if the field equation is not physically valid for such a situation.

Theorists without rigorous training in both mathematics and physics (for instance such as 't Hooft [17-19]), would be less equipped to provide a compelling proof of this possibility.

In this paper, as a first step, some fundamental examples will be offered to illustrate this theoretical invalidity. In so doing, we hope to stimulate the reader to probe further into the related literature and recent theoretical developments on this issue (see Section 5). However, let us first point out some background on the theoretical errors related to linearization.

2. Theoretical Errors at Princeton University

Einstein claimed in his 1923 Nobel lecture that his considerations led to the theory of gravity, which yields the Newtonian theory as a first approximation, as well as yielding the motion of the perihelion of Mercury, the deflection of light by the sun, … In so doing, he did not respond to those questions raised by Gullstrand on the perihelion of Mercury. In particular, he failed to show the existence of a bounded dynamic solution, whose existence is crucial for the perturbation approach to calculate the perihelion of Mercury. However, The Nobel Prize Committee, well known for its cautious attitude, would not have changed its opinion based on Einstein's statements alone; Gullstrand would have to have been proven wrong mathematically.

A search of the literature between 1921 and 1993 reveals a "proof" of the existence of bounded dynamic solutions. Two Princeton professors in mathematics, Christodoulou and Klainerman, published a 500 page book in 1993 [20], claiming that bounded (in amplitude) dynamic solutions of the Einstein equation had been constructed. This book was highly regarded by Princeton University and considered a classic in mathematics. Moreover, Wheeler [2], an advocate of Einstein then, mentored Christodoulou and claimed him to be a genius. Thus, under the full weight of the reputation of Princeton University, the Nobel Committee understandably changed their mind.

However, upon closer examination [21], their construction of "dynamic" solutions has problems. Some are as follows:

1. They did not show that their constructed solutions are compatible with Einstein's radiation formula.
2. Their construction is a set of time-dependent solutions only, since they did not show that such a solution has dynamic sources. This is necessary according to the principle of causality.
3. They have shown only that a static solution belongs to their set. However, they did not prove that it includes time-dependent solutions, or provide even an example.

In other words, these two authors failed to prove the crucial issue that their set of bounded dynamic solutions is non-empty [21]. This problem alone is sufficient to establish that their construction of dynamic solutions is at least incomplete even if one ignores other problems that demonstrate the invalidity.

Christodoulou & Klainerman were politely criticized by Volker Perlick [22]. He described the first section as "hard to follow", instead of exposing the inherent errors contained within it [21]. The final paragraph of his book review reads:

> "Before this book appeared in 1993 its content was already circulating in the relativity community in form of a preprint that gained some notoriety for being extremely voluminous and extremely hard to read. Unfortunately, any hope that final version would be easier to digest is now disappointed. Nonetheless, it is to be emphasized that the result presented in this book is very important. Therefore, anyone interested in relativity and/or in nonlinear partial differential equations is recommended to read at least the introduction."

The journal *General Relativity and Gravitation* [23] republished Perlick's review, with the editorial note, "one may extract two messages: on the one hand, (by seeing e.g. how often this book has been cited), the result is in fact interesting even today, and on the other hand: There exists, up to now no generally understandable proof of it… For the convenience of the reader, this review is provided as an appendix." The messages were also ignored, because the errors cannot be fixed [21].

It is difficult to image that elementary errors in set theory were made in a work published by Princeton University. The work was not challenged probably because the existence of bounded dynamic solutions was earlier assumed by Einstein, Infeld & Hoffman [24]. Moreover, before 1993 mathematicians (including the Field Medalists such as Edward Witten (1990), and S. T. Yau (1982) whose works have been closely related to general relativity) also failed to discover that the Einstein equation does not have a dynamic solution. Nevertheless, while their errors were finally proven and published in 1995 [8-11], it was too late for the Nobel committee of 1993.

Note that the Wheeler School [25-28] also made errors directly, those were either factually incorrect and/or invalid in physics [28-30]. Notably, Misner, Thorne & Wheeler [26] incorrectly interpreted [3] and finally rejected [25] Einstein's equivalence principle, a foundation of general relativity. This was an extraordinary effort of over twenty years. They first followed the error of Fock [29], who tried to discredit Einstein by aligning his principle with a 1911 assumption known to be invalid. This work of Wheeler et al would appear to eliminate all theoretical obstacles to their own theory of black holes. [4] This theory often uses Einstein's covariance principle, which is actually in conflict with Einstein's equivalence principle [9, 10].

Moreover, the view of Wheeler et al that gravity is due to only space-time curvature, is directly in conflict with Einstein's equivalence principle, which states that the effect of a uniformly accelerated system is equivalent to the effect of uniform gravity generated by a metric [1, 2]. Thus according to Einstein, gravity may not necessarily be due to space-time curvature. The book **Gravitation** [26], by Wheeler et al illustrates their shortcomings, in mathematics, physics, and logic (see also Sections 3 and 4).

For instance, the relationship between gravitational and inertial mass, as expounded by Galileo and Newton, was used as if it were Einstein's equivalence principle. They [26] also got an incorrect expression of the local time of the earth in the solar system as shown in their eq. (40.14). [5] The analysis of Weber & Wheeler [27] on Einstein's "cylindrical waves" is incorrect [18, 19]. Apparently, the Wheeler School and its followers [25-28, 31] did not realize that Einstein's equivalence principle is necessary for general relativity to be self-consistent [33].

Nevertheless, the influence of the Wheeler School kept growing [28]. This was first due to barriers, earlier mentioned, to a critical analysis of general relativity. As such, their errors were not openly exposed to the public. In 1994, the Wheeler School [25] finally openly rejected Einstein's equivalence principle just as Fock [29] did [2] after their invalid conclusions were reflected in the 1993 press release of the Nobel Committee [11].

The Einstein equation is invalid for the dynamic case since it cannot have physical solutions for this case [8]. Therefore, it is unclear whether the linearized equation is valid since it is obtained from the Einstein equation. However, the Hulse-Taylor experiments on binary pulsars suggest that the gravitational wave does exist [8-10], and thus it should be possible to show the validity of the linearized equation, independent of the Einstein equation. Based on Einstein's equivalence principle, it has been shown [3] that, with massive sources, the linearized equation is independently valid. [6] Such an equation is called the Maxwell-Newton Approximation [8], and thus Einstein's notion of weak gravity is crucial to general relativity as a viable theory.

3. The Einstein Equation and the Conditional Validity of $E = mc^2$

In general relativity, the Hilbert-Einstein equation [31] of 1915 for gravity of space-time metric $g_{\mu\nu}$ [1, 2] is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K T(m)_{\mu\nu} \qquad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and K ($= 8\pi\kappa/c^2$, and κ is the Newtonian coupling constant) is the coupling constant.· Thus,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \quad \text{or} \quad R_{\mu\nu} = 0 \qquad (1')$$

at vacuum. However, Eq. (1') would imply that a gravitational wave does not carry away energy-momentum. This violates the principle of causality since a wave does carry away energy and momentum.[7] Thus the Einstein equation cannot have a bounded dynamic solution that includes waves as required by calculations of the Hulse-Taylor experiments [8]. Einstein's notion of gravitational energy-stress as a pseudo-tensor has been proven incorrect in physics. However, the Field Medalists, Witten & Yau, did not correctly interpret the related physics,[8] and thus overlooked the impossibility of a bounded dynamic solution.

One might argue that a function is always bounded in a restricted domain. However, not having a bounded dynamic solution is only a symptom of the violation of the principle of causality. For example, Bondi, Pirani & Robinson [34] considered an unbounded wave as from a distant source. However, they failed to recognize that the principle of causality is violated because such a metric cannot be reduced to the flat metric that a physical metric would have when gravity is absent (see Section 4). Another example is the cylindrical symmetric wave of Einstein and Rosen [27]. It is unbounded and does not have a valid source [18, 19]. The question that thus follows is whether the Einstein equation can be modified for the dynamic case.

In 1995, based on the binary pulsars experiments, it was found that the modified Einstein equation [8-10] should be,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K \left[T(m)_{\mu\nu} - t(g)_{\mu\nu} \right] \qquad (2)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. From Eq. (2), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K t(g)_{\mu\nu} . \qquad (2')$$

Note that $t(g)_{\mu\nu}$ is equivalent to Einstein's gravitational pseudo-tensor or $G^{(2)}_{\mu\nu}$ (the second order terms in $G_{\mu\nu}$) in terms of his radiation formula [8, 9, 35]. Thus, the modified Eq. (2) would support the existence of the gravitational wave although the Einstein equation failed. When the gravitational wave is present, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero as physics requires [8-10]. Then, the linearization of Eq. (2) is the Maxwell-Newton Approximation [6],

$$\frac{1}{2} \partial^{\alpha} \partial_{\alpha} \bar{\gamma}^{(1)}_{\mu\nu} = -K T(m)_{\mu\nu} \qquad (3)$$

where $\quad g_{\mu\nu} = \gamma_{\mu\nu} + \eta_{\mu\nu}, \quad \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu}$

and

$$\bar{\gamma}^{(i)}_{\mu\nu} = \gamma^{(i)}{}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \left(\gamma^{(i)}{}_{cd} \eta^{cd} \right) \qquad (4)$$

Here $\eta_{\mu\nu}$ is the flat metric, $\bar{\gamma}^{(1)}_{\mu\nu}$ is of the first-order, and $\gamma^{(2)}{}_{\mu\nu}$ the second order. Moreover, the binary pulsars experiments support the validity [8-10] of Eq. (2) since linear Eq. (3) is derived directly from Einstein's equivalence principle [3]. However, Eq. (3) can be invalid in physics if the source is not massive matter [36]. Experimental confirmations of the static Einstein equation beyond Eq. (3), would be to verify the Reissner-Nordstrom metric [26] for a charged particle.

Now let us give a brief description on how Eq. (2) is obtained. It started from Einstein's radiation formula where gravitational radiation energy is non-zero for a dynamic case. Now, consider a system of two equal masses. Since the gravity is weak, the metric and the motion of the masses would be essentially periodic functions of time [8-10]. Then, taking a time average for a chosen period of the Einstein Equation (1) in vacuum, one may have the average of the sum of the first order terms to be zero. However, according to Einstein's radiation formula, the average of the sum of second order terms cannot be zero. Thus, it is necessary to modify Eq, (1). To accommodate the gravitational waves, clearly the modified equation should be Eq. (2), since $t(g)_{\mu\nu}$ is equivalent to Einstein's gravitational pseudo-tensor or $G^{(2)}_{\mu\nu}$.

Because of the Maxwell-Newton Approximation in Eq. (4), the radiation of the binary pulsar can be calculated without detailed knowledge of $t(g)_{\mu\nu}$. From Eq. (6), the approximate value of $t(g)_{\mu\nu}$ at vacuum can be calculated through $G_{\mu\nu}/K$, since the first-order approximation of $g_{\mu\nu}$ can be calculated through (4). In view of that $Kt(g)_{\mu\nu}$ is of the fifth order in a post-Newtonian approximation, that the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation [4] and that the perihelion of Mercury was successfully calculated with the second-order approximation from Eq. (2), the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation of Eq. (2) by using Eq. (4).

Thus, the approaches of Damour & Taylor [37, 38] would be valid except that they did not realize that linear Eq. (3) is actually an approximation of the modified Eq. (2) [8-10] that supports the existence of gravitational waves. Thus, *the Hulse & Taylor experiments actually confirm the modified Eq. (2) of 1995*. The Royal Swedish Academy of Sciences was advised by those who failed to understand the Einstein equation.

Note that P. Morrison of MIT had gone to Princeton three times to discuss the calculations of binary pulsars with Taylor [39]. Taylor finally told Morrison that Damour was responsible for the calculations. Feynman [5] described those theorists as incompetent. Nevertheless, the experiment of Hulse & Taylor would still warrant the Nobel Prize since it confirms a new direction by supporting the modified equation of 1995, in addition to the discovery of the binary pulsars.

Note that Eq. (2) shows that the coupling constants may not be the same sign. Thus, validity of $E = mc^2$ should be conditional. However, this error has its roots in Einstein's assumption of photons having only electromagnetic energy [2]. Einstein thought that he had proven that electromagnetic energy is equivalent to mass [2], because Einstein, as well as his peers, failed [31] to see that electromagnetic energy is not equivalent to mass [40], although the energy of photons is [41].

Thus, the paper on conditional validity of $E = mc^2$ [40] was reviewed for more than 4.5 years before theorists failed to defend the unconditional validity. Moreover, a new repulsive force was established; and a repulsive force mq^2/r^3 between a charge q and a mass m, separated with a distance r was derived from the Reissner-Nordstrom metric [26] for a charged particle Q with mass M and charge q. The metric is as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \qquad (5)$$

where r is the radial distance (in terms of the Euclidean-like structure [42]) from the particle center. Here, the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign.

This should have confirmed the discovery of a repulsive gravity. However, journals such as *Science* and *Nature* held onto the misinterpretation [43]. Subsequently, Herrera, Santos & Skea [44] attempted to revive the misinterpretation. They argued that M in Eq. (5) includes the external electric energy, but overlooked that this would create a double counting of the electric energy in two different ways [26]. Also, the gravitational forces would be different from the force created by the "effective mass" $M - q^2/2r$. If M included the external electric energy, then the inertial mass m_0 of the electron would be much smaller than M. Moreover, according to the Einstein equation, since the electromagnetic energy-stress tensor is traceless, curvature R is independent of the electromagnetic energy-stress tensor. Hence, the electromagnetic energy cannot be equivalent to the mass.

Nevertheless, the editor of the *Physical Review D*, Eric J. Weinberg, demanded a proof beyond electromagnetism for $E = mc^2$ being conditional. Fortunately, the effect of such charge-mass repulsive force was inadvertently detected by Tsipenyuk & Andreev [45]. They discovered that the weight of a metal ball is reduced after it is irradiated with high energy electrons. However, they could not explain this [2], because they did not account for the static charge-mass repulsive force [40] discovered in 1997. This experiment confirms unequivocally the necessity of unification between electromagnetism and gravitation, and further verifies that mass is not equivalent to electric energy.

4. Examples to Illustrate Invalidity of Linearization for the Dynamic Case

This simplest situation is the plane-waves, which are local idealization of waves. For the case of electromagnetic wave, the vacuum Maxwell equation is valid because an electromagnetic wave does not carry charges. However, for the gravitational waves, the vacuum Einstein equation is not valid because gravitational waves do carry energy-momentum, which is also a source of gravity. Thus, the investigation of gravitational plane-waves is crucial.

A "wave" form considered by Misner, Thorne, & Wheeler [26] would be a good example. The metric is as follows:

$$ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right) \qquad (6)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \qquad (7)$$

Misner et al [26] claimed that Eq. (7) has a bounded solution, compatible with a linearization of metric (1). It will be shown later that Misner et al are incorrect, and Eq. (7) does not have a physical solution that satisfies Einstein's requirement on weak gravity. In fact, $L(u)$ is unbounded even for a very small $\beta(u)$.

On the other hand, from the linearization of the Einstein equation (the Maxwell-Newton approximation) in vacuum, Einstein [46] obtained a solution as follows:

$$ds^2 = c^2dt^2 - dx^2 - (1 + 2\varphi)dy^2 - (1 - 2\varphi)dz^2 \qquad (8)$$

where ϕ is a bounded function of u ($= ct - x$). Note that metric (8) is the linearization of metric (6) if $\phi = \beta(u)$. Thus, the problem of waves illustrates that the linearization may not be valid for the dynamic case when gravitational waves are involved since Eq. (7) does not have a weak wave solution. [9] This can be proven with mathematics at the undergraduate level.

To prove Eq. (7) having no wave solution, it is sufficient to consider the case of weak gravity since a reduction of source strength would lead to weak gravity [3]. According to Einstein, for weak gravity of metric (6), one would have

$$L^2 e^{2\beta} \cong 1 \qquad \text{and} \qquad L^2 e^{-2\beta} \cong 1 \tag{9a}$$

It follows that

$$L^4 \cong 1, \qquad e^{\pm 2\beta} \cong 1 \qquad \text{and} \qquad L(u) >> |\beta(u)| \tag{9b}$$

Since $L(u)$ is bounded, $L'(u)$ cannot be a monotonic function of u. Then, there is an interval of u such that the average,

$$\langle L'' \rangle = 0 \tag{10}$$

On the other hand, the average of the second term of Eq. (7) is always larger than zero unless $\beta'(u) = 0$ (i.e., no waves).

From Eqs. (7) and (9), one would obtain $L\ (\cong 1) > 0$, and one has $0 > L''(u)$ if $\beta'(u) \neq 0$. Thus, $-L'(u)$ is a monotonic increasing function in any finite interval of u since $\beta'(u) = 0$ means $L'' = 0$, i.e., no wave. In turn, this implies that $L(u)$ is an unbounded monotonic function of u. Therefore, this would contradict the requirement that $L^4 \cong 1$. In other words, (7) does not have a bounded wave solution. Moreover, the second order term L'' would give a very large term to L, after integration. Therefore, linearizing (7) to $L'' = 0$ is mathematically invalid.

Now, let us investigate the errors of Misner et al [26; p. 958]. Their assumption is that the signal $\beta(u)$ has duration of $2T$. For simplicity, it is assumed that definitely $|\beta'(u)| = \delta$ in the period $2T$. Before the arrival of the signal at $u = x$, one has

$$L(u) = 1, \qquad \text{and} \qquad \beta(u) = 0 \tag{11}$$

If the assumption of weak gravity is compatible with Eq. (7), one would have $L(u) \cong 1$. It thus follows from Eq. (7), one has

$$L'(u) = 0 - \int_x^u \beta'^2 dy \approx -\int_x^u \delta^2 dy = \delta^2 (u - x) \quad \text{for} \quad x + 2T > u > x,$$

or
$$\approx -\delta^2 2T \quad \text{for} \quad u > x + 2T \tag{12}$$

Hence

$$\left. \begin{aligned} L(u) &= 1 + \int_x^u L' dy \\ &\approx 1 - \int_x^u \delta^2 (y - x) dy = 1 - \frac{\delta^2 (u - x)^2}{2} \end{aligned} \right\} \quad \text{for} \quad x + 2T > u > x$$

or
$$\left. \begin{aligned} &\approx 1 - \int_x^{x+2T} \delta^2 (y - x) dy - \delta^2 2T \int_{x+2T}^u dy \\ &= 1 - \delta^2 2T (u - T - x) \end{aligned} \right\} \quad \text{for} \quad u > x + 2T \tag{13}$$

Thus, independent of the smallness of $2\delta^2 T$ (or details of $|\beta'(u)|^2$), L could be approximately zero and violates the condition for weak gravity. Thus, Eq. (7) has no weak wave solution. In contrast to their belief [26; p. 958], this situation is physically different from that $g_{tt} \approx 0$ (and $g_{rr} \approx \infty$) of the Schwarzschild solution in which the gravity could be strong. Moreover, $|L(u)|$ is not bounded since it would become very large as u increases. Thus, restriction of $2\delta^2 T$ being small [26] does not help.

The root of their errors was that they incorrectly [8-10] assumed that a linearization of a non-linear equation would always produce a valid approximation. Thus, they obtained an incorrect conclusion by adopting invalid assumptions. Linearization of (7) yields $L'' = 0$, and in turn this leads to $\beta'(u) = 0$. In turn, this leads to a solution $L = C_1 u + 1$ where C_1 is a constant. Therefore, if $C_1 \neq 0$, it contradicts the requirement $L \approx 1$ unless $|u|$ is very small. Thus, the linearization is valid only in a very small region.

Thus, one cannot get a weak wave solution through linearization of Eq. (7), which has no bounded solution. This shows also that the assumption of metric form (6) [26], which has a weak form (8), is not valid for the Einstein equation.

Another well-known counter example is the metric obtained by Bondi, Pirani, & Robinson [34] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2 \begin{bmatrix} \cosh 2\beta \left(d\eta^2 + d\varsigma^2\right) \\ + \sinh 2\beta \cos 2\theta \left(d\eta^2 - d\varsigma^2\right) \\ -2 \sinh 2\beta \sin 2\theta \, d\eta \, d\varsigma \end{bmatrix} \tag{14a}$$

where ϕ, β and θ are functions of u ($= \tau - \xi$). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right) \tag{14b}$$

They claimed this is a wave from a distant source. (14b) implies ϕ *cannot be a periodic function.* The metric is irreducibly unbounded because of the factor u^2. Both Eqs. (7) and (14b) are special cases of $G_{\mu\nu} = 0$. However, clearly linearization of (14b) does not make sense since variable u is not bounded. Thus, they claim Einstein's notion of weak gravity invalid.

Moreover, when gravity is absent, it is necessary to have $2\phi = \sinh 2\beta = \sin 2\theta = 0$. These reduce (14a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right) \tag{14c}$$

However, this metric is not equivalent to the flat metric. Thus, metric (14c) violates the principle of causality; and (14a) also does since there is no physical parameter to be adjusted such that metric (14a) becomes equivalent to the flat metric.

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editorials of the *Royal Society Proceedings A* and the *Physical Review D*; thus there is no general consensus. Nevertheless, in 1999 Lo [3] defended Einstein's notion of weak gravity. The Royal Society correctly pointed out [47, 48] that Einstein's notion of weak gravity is inconsistent with his covariance principle. However, Einstein's covariance principle has been proven invalid since counter examples have been found [49-51].[10] Moreover, Einstein's notion of weak gravity is supported by the principle of causality.

Another major problem among theorists in general relativity is that they ignore the principle of causality. For example, another "plane wave", which is intrinsically non-physical, is the metric accepted by Penrose [52] as follows:

$$ds^2 = du\,dv + Hdu^2 - dx_i dx_i\,, \quad \text{where} \quad H = h_{ij}(u)x_i x_j \quad (15)$$

where $u = ct - z, v = ct + z$. However, there are arbitrary non-physical parameters (the choice of origin) that are unrelated to any physical causes. Being a mathematician, Penrose [52] overlooked the principle of causality.

It has been illustrated that the linearized equation for a dynamic case is incompatible with the Einstein equation, which has no bounded dynamic solutions. Thus, Eq. (7) and Eq. (14b) serve as good simple examples that can be shown through explicit calculation that linearization of the Einstein equation is not valid.[9) Also, metric (15) suggests that the cause of having no physical solution could be due to inadequate source terms [8, 10, 53].

5. Conclusion

In 1995, a paper [8] was published pointing out the invalid claims in the 1993 press release of The Royal Swedish Academy of Sciences [11]. Clearly, the 1993 press release was not supported by some Nobel Laureates. However, the Wheeler School was in a dominant position in the field of gravitation. Understandably, the mathematician Yau chose an indirect way to inform Christodoulou & Klainerman that their book [20] had problems. [11) If the Wheeler School prevailed, then their errors would be accepted by the Nobel Prize Committee without engaging its customary collective caution and wisdom.

Unfortunately, Chandrasekha passed away shortly after the publication of the dissenting paper [8], and it was essentially ignored until 't Hooft challenged it [17-19].[12) For example, Wald [35] still did not see that the linearization of Einstein equation was not generally valid in mathematics [6, 8-10]. In fact, Wald [35; p. 183] even incorrectly maintained extending the process to the case that the initial metric of the perturbation is not flat. Moreover, as Richter [54] pointed out, some protect themselves by treating dominant theories of physics as a religion, and thus untouchable. Due to the dominance of certain views, some journals simply refuse, without adequate scientific reason, to consider papers that criticize an influential theorist. Thus errors accumulate and are disseminated widely, with little scrutiny. Thus, the full meaning of the Hulse & Taylor experiments was not given adequate consideration.

Due to an insufficient understanding of mathematics [55], 't Hooft challenged the non-existence of gravitational wave solutions with his own examples that failed to satisfy the linearized equation [18, 19]. He did not distinguish between mathematics and physics [17, 56], and thus he did not understand the principle of causality sufficiently to understand that, for a dynamic problem, the approach of linearization to obtain an approximation is not valid. [13) He also failed to see that Eq. (7) does not have a bounded dynamic solution. In short, he pursued the approach of perturbation for dynamic cases that have only unbounded solutions. Nevertheless, he is not alone. For instance, Bertschinger [57] and Wald [35] still accepted linearization for the dynamic case; and Hehl [58] still believed in the approach of perturbation as unconditionally valid.

Moreover, the editorials of *Physica Scripta* [59] [14), also mistakenly suggested that there is a "Standard Theory" of general relativity [15) that has been generally accepted [32, 33]. A board member of the Royal Society claimed that linearization and weak field approximation on finite time intervals can still be valid [60]. However, this claim is a result of confusing physical requirements with mathematical results of an equation. [16) As expected, this board member was unable to give a supporting example. Nevertheless, the editorials of the "Royal Society Proceedings A" consider the board member to be an expert [61]. [17) This further confirms the claim of Feynman [5] that many theorists in gravitation are just incompetent. [18)

For example, Eq. (7) does not have a weak field solution. Metric (14) is unbounded and thus cannot be approximated with the approach of linearization. The failure of linearization as an approach for an approximation, in terms of physics, is due to a violation to the principle of causality. [19) Moreover, many experts actually do not understand even special relativity adequately [32, 33] since they fail to see that special relativity has been mistakenly used to justify measurements for general relativity [1, 2]. Further evidence for not having a bounded wave solution is that the calculated result of gravitational radiation depends on the approach of approximation chosen [62].

The non-existence of a wave solution for the Einstein equation can be traced back to J. E. Hogarth (1953 Ph. D. Thesis, Dept. of Math., Univ. of London) who wrote that, for the existence of a gravitational wave solution, the energy-stress tensor is necessarily non-zero at vacuum [9]. However, it was NASA's discovery of the pioneer anomaly that would change the conventional wisdom in general relativity, and allow theorists to be more willing to re-examine the invalidity of linearization, which is crucial to understanding the Hulse-Taylor experiments although many theorists would still mistakenly believe that long acceptance of a mathematical approach is sufficient to establish its validity.

General relativity is based on two principles: the principle of general relativity and Einstein's equivalence principle. The principle of general relativity was later extended by Einstein to his "principle of covariance" because of theoretical difficulties [1, 2]. The root of this problem is Einstein's adaptation of the notion of distance in Riemannian geometry as if valid in physics [48, 49]. Such an adaptation was criticized by Whitehead [63] as invalid in physics. Recently it has been found that Einstein's justification for such an adaptation is actually based on invalid applications of special relativity [32, 33]. It is ironic that many theorists rejected the equivalence principle that experiments support, but accepted the incorrect covariance principle [49, 50, 64].

Zhou Pei-Yuan [65], based on Einstein's equivalence principle, first pointed out that his covariance principle was invalid.[20) However, there is nothing other than Einstein's own work that gives a correct interpretation of his equivalence principle [30, 66]. Before 2006 many failed to understand the necessity of Einstein's principle of equivalence for self-consistency of general relativity [32, 33] and the necessity of unification among electromagnetism and gravitation [67]. The errors of 't Hooft [17-19] and Hehl [58] would illustrate that new features are often difficult to grasp.

Even the creator of a theory might fail to understand his creation in full. It turns out that the valid theory of measurement is found [64] to be just what Einstein has practiced in his calculations of light bending [1, 2]. While the Einstein equation gives accurate predictions for the static cases, it is an invalid equation for a dynamic case [6, 8]. Fortunately, the Hulse & Taylor experiments suggest that the Einstein equation can be rectified. However, because of entrenched views, many failed to see that the Einstein equation was invalid for the dynamic case [53].

Systematic errors can be difficult to detect and rectify because of the entrenchment of particular theoretical models. However, the nonexistence of dynamic solutions was recognized by Nobel Laureate Chandrasekha and Lo [8, 9] in 1995. 21) Concurrently, Einstein's controversial notion of gravitational energy-stress as a pseudo-tensor [68-70] was proven incorrect. Another serious error was the implicit rejection of Einstein's equivalence principle [11] as Fock [29], Wheeler [25], and others do explicitly. 22) It would seem essential then, to integrate the principles of physics into mathematical analyses [55]. The errors in general relativity persist, in part, because to rectify them, one requires training in both physics and pure mathematics. 23)

Nevertheless, Einstein was truly a genius and the meaning of general relativity is still emerging. Based on general relativity, the static charge-mass repulsive force has been discovered, and its verification is another confirmation of general relativity [45]. 24) However, some less informed relativists 25) would protect their own theoretical interests by resisting challenges [64]. This is exactly what happened with NASA's discovery of the pioneer anomaly [14-16]. It is also well known that some theorists attempted to shut down the Super Collider in Europe.

Unfortunately, institutes such as Princeton University [20], 26) the Royal Society [60] and others made errors. The editorials of the Royal Society were wrong on Einstein's equivalence principle [30, 39]. The Nobel Committee accepted their advice, in spite of the wisdom of Gullstrand [6]. Further, many theorists failed to see that $E = mc^2$ is conditionally valid [40, 61], and thus overlooked the necessity of unification [40, 67]. The lack of expertise in the field of gravitation has permitted the accumulation and spread of errors in general relativity [33]. For instance, Misner et al. [26] created a distortion of the Einstein-Minkowski condition [1, 2], the so-called "local Lorentz invariance". This would unfairly give further damage to the reputation of Einstein. 27)

The Nobel awards committees adopted a strategy of recognizing scientific discoveries that have withstood the test of time. However, nobody would have expected mathematical errors to dominate for more than 90 years [8, 33]. Consequently, it was overlooked that the necessity of unification can be proven [67], and that photons must include another energy, which is identified as gravitational energy [71].

The 1993 press release on the Nobel Prize in Physics is the first time that the Nobel Committee made erroneous public statements in physics. 28) It is possible that their error led theorists to disregard NASA's great discovery of pioneer anomaly as incorrect [14, 16].

In the interest of science, for which Mr. Nobel established his prizes, this paper is written to call attention to this problem and to honor the work of Gullstrand [6].

Acknowledgments

This paper is dedicated to Prof. J. E. Hogarth of Queen's University, Canada who is the first pointing out that for a gravitational wave solution must involve a non-zero energy-stress tensor at vacuum. The author acknowledges stimulating discussions with Claire Birch, S. –J. Chang, A. J. Coleman, F. W. Hehl, L. Liu, A. Napier, J. Schneps, and C. Wong. This work is supported in part by Innotec Design, Inc., U. S. A. and the Chan Foundation.

Endnotes

1) Although Einstein was able to deduce the perihelion of Mercury by treating the planet as a testing particle [1, 2], this effect in nature is actually only one of the perturbations [7]. Thus, the perihelion of Mercury cannot, in principle, be derived from the Einstein equation since it has no bounded dynamic solution [8]. However, this does not exclude that a modified Einstein equation can do such a job [8, 9]. Note that Krammer, Stephani, Herlt, & MacCallum [72] obtained that there is no exact solution for the two-body problem and the gravitational radiation from a realistic bounded source. However, this would still leave a room for the existence of approximate dynamic solutions that Hehl [58] insisted on.

2) Fock [29], Ohanian & Ruffini and Wheeler [25] have mistaken the invalid equivalence assumption of 1911 that a uniform Newtonian gravity is equivalent to a uniformly accelerated frame, as Einstein's equivalence principle of 1916 [1, 2]. (However, their errors continue, for instance, in the open course ware of MIT Phy. 8. 033 of the 2006 version.) Moreover, Fock [29] found it impossible to express a uniform Newtonian gravity with a spacetime metric. Making the same error, Ohanian & Ruffini [23] and Wheeler also abandoned Einstein's equivalence principle. (If Einstein's equivalence principle were wrong, his credit would be greatly reduced since the field equation was first obtained by Hilbert [31].) However, they are proven wrong *because the metric for a uniform gravity has been derived and published in 2007* [30]. Thus, validity of this principle is illustrated.

3) Misner et al. [26] do not provide the references to Einstein's equivalence principle (i. e. [1] and [2; p. 111]). Instead, they misleadingly refer to Einstein's invalid 1911 assumption [2; p. 99] and Pauli's version of misinterpretation [69]. Based on their misrepresentations, many made the wrong claims, and invalidly criticized Einstein's equivalence principle [30, 33, 73].

4) Theorists such as Will [43] & etc. [67] also insist on another error, the unconditional validity of $E = mc^2$ [15, 40, 56].

5) Wald [35] and Weinberg [4] do not made the same mistake, but Ohanian & Ruffini [25] do.

6) Thus, theories based on gravitomagnetism and gravitational radiation would remain to be valid. However, it was not known that dynamic solutions actually do not exist [57], and the related approach of perturbation is actually not valid.

7) Einstein defined a pseudo tensor, which is equivalent to $G^{(2)}\mu\nu$ (see after Eq. [2']) [35], as the gravitational energy-stress. Many theorists, including 't Hooft [17-19], mistakenly regarded such a misleading definition [8] as valid. Thus, 't Hooft considers eq. (2) as invalid.

8) An accomplishment of string theory is a derivation of the 1915 Einstein equation, which is invalid for the dynamic cases. This equation is used by Yau in his "Proof of the Positive Mass Theorem" [74-76].

9) L

iu & Zhou [77] also has investigated the gravitational plane-wave in vacuum. As shown by their calculations [10], it is clear that a bounded solution does not exist and linearization of Einstein equation is not generally valid.

10) For instance, the shortest distance from the center of the sun has to depend on the gauge chosen [50, 64]. Since such a distance is a physical quantity as shown by the calculations of Einstein [1, 2], his covariance principle is not valid in physics.
11) In their book [20], they acknowledge S. T. Yau for his early interest to their work.
12) If a theorist works against a prevailing view, to convince others, he may have to wait for an appropriate time after other related problems have been addressed [33, 48].
13) 't Hooft cannot distinguish physics from mathematics to see that his examples violate the principle of causality [18, 19], but they were used to support his negative comments to what he called "a "brilliant" general relativist, C. Y. Lo".
14) *Physica Scripta* is published by the IOP on behalf of the Royal Swedish Academy of Sciences for the Science Academies and the Physical Societies of the Nordic Countries.
15) There is no general consensus since the Physical Review and the Royal Society do not agree with each other [34, 48].
16) A common problem for some theorists is that they cannot tell the difference between mathematics and physics [9, 33].
17) In mathematics as well as in physics, a theory without a supporting example is likely wrong.
18) Feynman, a student of Wheeler, is reluctant to name names.
19) Some [34, 47] even rejected Einstein's requirement for weak gravity, which is based on the principle of causality [3].
20) It was surprising that Chinese theorists, including C. N. Yang, failed to understand Zhou [65]. Recently, it was found that they followed similar theoretical errors [78-81] as the Wheeler School and Yang additionally made errors based on the shortcomings of the Yang-Mills theory [82] before it is applicable to realistic physics.
21) The Nobel Laureates are selected by a Nobel Committee that consists of five members elected by The Royal Swedish Academy of Sciences. In its first stage, several thousand people are asked to nominate candidates. These names are scrutinized and discussed by experts until only the winners remain. The nomination and selection process for the Nobel Prize in Physics is usually long and rigorous. This is a key reason why these Nobel Prizes have grown in importance over the years to become the most important prizes in Physics. On the other hand, an error of the Nobel Committee in physics is almost certain an error of those considered as experts in the field.
22) Einstein's equivalence principle is generally valid although it is inadequate for the case beyond attractive gravity [83, 84].
23) For a modern theoretical physicist, now it seems that some training in pure mathematical analysis is highly desirable.
24) Two initially equal-weight balls, after irradiated with high energy electrons to one ball, it becomes lighter.
25) Surprisingly, many theorists in gravitation read narrowly, and do not study even the mathematics of non-linear equation.
26) Some of my respectable teachers are graduates of Princeton. Hogarth is a graduate of the University of London. Zhou is a graduate of California Institute of Technology. However, no research or research institution is flawless.
27) Experimental tests of the "local Lorentz invariance", an invalid notion created by Misner et al. [26], have unfavorable results [85].
28) Many statements in the 1993 press release cannot be derived from the Einstein equation [8-10]. In fact, they are either wrong or misinterpretations of physics. Moreover, they are obstacles for the investigation of the charge-mass interaction that would be the mainstream physics in the 21 century.

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