Recasting Copenhagen Doctrine in a Classical Vein

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This paper attempts to improve the logical and conceptual relations between the Bohr Condition, the Quantum Hall Effect, Copenhagen Views & de Rham Theory.

1. Introduction

In two years, it will be a century ago that the Bohr quantization condition became a first step into the quantum nature of atomic structure and thirty years ago von Klitzing et al revealed a surprise method of measuring quanta of action and electric charge with much enhanced consistency. The surprise was a newly discovered quantum version of the Hall Effect of 1879, which in conjunction with Josephson's AC effect produced these new data. While the tested mathematical relation was sensationally simple the ensuing theory became notoriously intricate, especially after a discovery at Bell laboratories in 1982 of what became known as *fractiona*l quantum Hall effect. The latter was believed fundamentally different from the earlier *integer* effect.

The following shows how, unlike when using Schrödinger's process, both quantum Hall effects very simply relate to Bohr quantization plus Cooper type conditions accommodating boson accumulation as in BCS theory. The latter facts are supported by an until now unaccounted for induced superconductivity of Hall samples in plateau states.

Generations in physics have been brought up believing in Schrödinger exactness versus an approximate nature of the pre-1925 Bohr-recipe, yet the just cited experience pleads for a proposition of giving the recipe an exactness reality realm of its own, it only differs from that of Schrödinger. Since the latter is identified as statistical, the Bohr realm would have to be pre-statistical. Copenhagen's point of view though does not make allowances for that possibility, its statistics is presumed to be nonclassical, which is defined as not adjustable; normal statistics are physically reducible to pre-statistic.

Since the Bohr QHE approach leads to very incisive changes in Copenhagen Doctrine it raises questions whether Bohr's simple quantization recipe seems part of a wider scheme of pre-statistic quantization. The Bohr-Sommerfeld integral was an example thereof and led to an early quantitative account of hydrogen fine structure. Frits London extended the principle from point particles to fields, this development then culminated in 1959 as the Aharonov-Bohm (AB) integral of the four-vector potential. It became an effective tool accounting for AB electron interference effects. Notwithstanding this positive achievement, there exists a near-silent undercurrent of AB doubt in some orthodox Copenhagen circles.

As evidence of this slumbering schism consider how a vast majority has been using Schrödinger's statistical methods on the epitome of non-statistic order known as the QHE. This path at understanding had its warnings of needing too many artifacts to get to any conclusion at all. The result has been an awkward integer-fractional dichotomy and no understanding as to why superconductivity might be coinciding with plateau states. Over an interval of thirty years this majority has not acknowledged the existence of a global approach invoking the AB integral [1] for a variety of very appropriate applications. This existence of two sectors of the physics community completely ignoring one another is an onset of anarchy that should have no place in science, because it shows an unwillingness of identifying problems, reflecting on their solutions or absence thereof.

The objective of this endeavor calls attention to this kind of isolationism in science wherever and whenever a way out is possible. The way out of this predicament points at recognizing next to a statistic Schrödinger reality the existence of a pre-statistic reality that has so far been denied by built-in Copenhagen inadequacies, e.g., excess *nonclassical* paradigms, which, in the course of time, have settled in hidden corners of our mind.

2. A Bohr-type Assessment of the Quantum Hall Effect

When Klaus von Klitzing et al discovered the surprising transitions between normal- and quantum Hall effects, it was inferred that the drift pattern of conduction was a matrix of cyclotron electron states, floating through a 2-dimensional interaction space of Mosfet samples. The proximity of adjacent layers providing a Cooper-type screening, accounting for a highly reduced mutual interaction between electrons in the same cyclotron orbit. An observed induced super-conductivity of the sample current calls for Cooper-pairing. Hence rather than calling on a Schrödinger Ψ statistics, the high degree of order of the case in question seems suited by a method similar to the one that gave Bohr his stationary states of hydrogen. For multiple electrons in the same orbit this process banks on a near-total electrostatic isolation between electrons in the same orbit.

Let r be the cyclotron radius, m the electron mass, B the magnetic induction, perpendicular to the sample's 2-dimensional interaction space, e the electronic charge, ω the circular frequency, and $h = 2\pi\hbar$ Planck's constant. The equation of motion gives for each electron now taken to be in isolated screened position on the circular orbit

$$mr\omega^2 = er\omega B \tag{1}$$

solving Eq. (1) for the frequency

$$\omega = \frac{e}{m}B \tag{2}$$

The Bohr condition requires

$$mr^2\omega = n\hbar = \frac{nh}{2\pi}; \qquad n = 1,2,3... \tag{3}$$

The flux Φ subtended by the electron orbit r equals

$$\Phi = \pi r^2 B \tag{4}$$

Using Eq. (2) to replace B by $m\omega/e$, we get $\Phi = \pi\left[mr^2\omega\right]/e$. The term in brackets can now be replaced by Eq. (3), hence the flux intercepted by the orbital charge equals

$$\Phi = \frac{nh}{2e}; \tag{5}$$

Note that flux quantum $h/2e$ is the one experimentally verified by Doll et al and Fairbank at al [2]; it is half the value of the unit cited by Aharonov-Bohm [1].

Eqs. (1) to (5) give the ingredients needed for the quantum Hall effect culminating in a quantized Hall impedance $Z(n,s)=$ Hall voltage over Hall current. The Hall voltage is the flux passing the contact sensor of the voltage circuit and the Hall current is the charge passing per unit time; $Z(n,s)=$ cyclotron flux over cyclotron orbital charge. For an ordered quantum situation the number of identical cyclotrons simultaneously passing the Hall sensor and the transition speed cancel. Hence a single one of the cyclotrons sets the stage for the flux over charge ratio. Assuming s electrons per orbit we have from Eq. (5)

$$Z(n,s) = \frac{n}{s}\frac{h}{2e^2}; \qquad n, s \text{ integer} \tag{6}$$

For n and s integers Eq. (6) reproduces a result indicated in a 1982 article [3] in which the Hall impedance was identified as the ratio of the integrals of Aharonov-Bohm and Gauss-Ampère; both holding the position of period integrals in a de Rham context. Eq. (6) simultaneously describes the *integer* effect with s injecting charge, and the *fractional* effect with n injecting flux. Quantum Hall Effect data are for all practical purposes based on Eq. (6). It had a role in officially approved precision improvements of h and e values. The current facts hint at the existence of exact primary pre-statistical quantization conditions.

3. Reinstating Pre-statistical Quantization

Taking a look in a perspective of time at this vast macro- to micro-physical field of description, some aspects emerge that are neither macro- nor typically microphysical in nature. Since the metric field gives us the sense of micro-macro distinction these description aspects may be taken to be metric-independent; they can be said to be innately ***scale*** invariant. In the scalar domain *flux, charge and action* have a *metric-independent* feature, because they are known as countable quantum items the invariance of which is metric independent. The integral Eqs. (7,8,9) testify to that.

Aharonov-Bohm $$\oint_{c_1} A = nf; \qquad n = 1,2,3... \tag{7}$$

Similarly linked by c_1 and $f = \tilde{h}/2\tilde{e}$.

Gauss-Ampère $$\oiint_{c_2} \tilde{G} = s\tilde{q}; \qquad s = 1,2,3... \tag{8}$$

Net charge "enclosed" by c_2 and $\tilde{q} = \pm\tilde{e}$.

Kiehn product $$\oint\!\!\oint\!\!\oint_{c_3} A \wedge \tilde{G} = ns\tilde{\alpha} \tag{9}$$

Action quanta inside c_2 and $\tilde{\alpha} = \pm\tilde{h}/2$.

The one-form A is defined by the four-vector potential and $\tilde{G}$ the 2-form of Minkowski's joint expression for displacement D and magnetic field H. The tilde specifies *impair* features of the form 2. The use of Cartan form-language here implies Eqs. (7,8,9) obey *general relativistic* invariance yet as a consequence of a by Cartan [4] cited metric-independence (1924) they also meet an added feature of *scale* independence characteristic of a topological situation.

These three integrals have been around well before they became related to quanta. In fact, after Faraday's electrolytic experiments of 1835, Eq. (8) emerged as a charge quanta counter and became herewith a first explicit messenger of quantization. It may not be officially recognized as such, yet theory accepts its validity, it is today taught as Gauss' law of electrostatics. These integrals replace the prematurely discarded pre-1925 quantum recipes. They may merely approach a Schrödinger statistical reality, but they do have their own pre-statistic reality in the conclusive renditions Eqs. (7,8,9).

In 1959 Aharonov and Bohm [1] introduced Eq. (7) as spacetime counter of flux units; using Schrödinger's single-valuedness of Ψ for its unit flux size *h/e*. Batelaan and Tonomura [5] have given a beautiful overview of interference effects from the AB angle as described by Eq. (7). The experiments of Doll et al and Fairbank et al [2] also measured a ***static*** flux unit *h/2e* in 1961, a counterpart of the AB *dynamic* unit *h/e*.

Yet despite this array of meaningful applications, the AB integral has been denied a fundamental status similar to that of the Gauss integral. The fact is that a ratio of the AB- and Gauss integrals surprisingly accounts for a unified description of the integer and fractional quantum Hall effects $Z_H = (n/s)\left(h/2e^2\right)$ governed by two quantum numbers n and s. Yet over the past 30 years the vast majority was either unaware of this surprise, or never asked questions whether this coincidence had any basis in true physics. Instead, Schrödinger's statistical process was used to account for a highly ordered non-statistical display of a near perfect quantum Hall order. An overly strict dogmatic adherence to Copenhagen Doctrine then led to placing a statistic cradle as a base for quantum Hall order. This manifestation of a silent distrust against the AB law may have been due to the bold Aharonov-Bohm move of extracting a non-statistical result from a statistical Ψ function. Despite an accepted Gauss' integral Eq. (8), attitudes remained strangely ambivalent about integral Eq. (7) as testified by past decades of QHE history.

For a joint justification of Eqs. (7,8,9) it is now appropriate to focus on developments in the differential topology of *closed differential forms* by de Rham [6]. Vanishing exterior derivatives (the ocean) complemented by realms of nonzero exterior derivative (islands in the ocean) mark the landscape. The cyclic integrals of the nonzero realms are topological invariants of the closed form topography and de Rham's existence theorem [6,8] makes powerful statements about the structure of those invariants calling for a comparison with Eqs. (7,8,9). Along with an unquestioned validity of the Gauss integral Eq. (8), an independently proven existence of the AB integral and its smallest period $f = h/2e$ as confirmed by [2] ought to rule out any misgivings about Eq. (7). These subtle mathematical developments truly exclude lingering doubts about Eq. (7). They are sketchily referred to here because they don't transcribe well into mathematics traditionally used in physics. Formally speaking Schrödinger's process now exists by the grace of that very Eq. (7) in providing independent justification for Ψ function single-valuedness by injecting new physical meaning into Schrödinger's original derivation. Bits and pieces so converge in de Rham's perspective linking quantization to a *pre-metric* topological manifestations.

Avoiding an explicit appeal to de Rham theory, Kiehn [7] recognizes the period integral similarity in the context of Brouwer's degree of a map and elevated all three period integrals (7,8,9) as part of a complete set probing quantum order and the changes thereof. Calling on de Rham's existence theorem [6,8] for *closed* forms the topology becomes more tangible. Kiehn's approach has options for topology *change*, which from a de Rham angle corresponds to topological changes in the closed form being topologically assessed. Charged pair creation-annihilation is a verification of topological change.

4. Consequences for the Quantum Hall Effect

The earlier mentioned ratio of integral (7) over integral (8) were yielding a de facto accuracy and precision good enough to lead to corrections in official determinations of h and e. Topologically the ratio of the two integrals (7) and (8) describes a cyclotron ring current linking a flux of the applied B field perpendicular to the interaction space of the Hall effect sample, yielding Z_H. Let the integer $n = 1,2,3...$ count flux units $h/2e$ linked in *magneto-static* situations and let the other integer s taken to be even 2,4,6... to cover options of multiple Cooper pairing within the cyclotron. The fact remains that prevalent odd enumerators of *n/s* [8] have been telltale signs of *reduced* ratios hinting at super-conduction. Yet those astute experimental observations have been sadly abandoned in the wake of that vast majority favoring Schrödinger-based QHE assessments.

Schrödinger-based assessments of the integer effect start out with a single charge cyclotron missing out on Cooper pairing to cover induced superconductivity. The fractional effect calls for a many body approach, leaving an integer-fractional dichotomy.

A matrix of identical cyclotrons slowly slides through the 2-dimensional interaction space. The Hall impedance Z_H, defined as Hall voltage over Hall current, equals linked flux over linking charge passing the Hall probe. Velocity and size of the cyclotron matrix cancel thus yielding $Z(n,s)$ Cooper pairing is how electron pairs can reside in the same cyclotron orbit; i.e., boson behavior of the Cooper pairs comes to the rescue. An BCS-type argument has electrons in the interaction space interact exclusively with nearest neighbors in adjacent layers of the interaction space, thus promoting Boson formation overcoming repulsive positive energy interactions between electrons in same orbits. A better understanding of when h/e or $h/2e$ prevails would welcome independent determinations of n and s measurements.

5. Consequences for the Copenhagen Doctrine

Having now argued how Gauss' theorem accumulates boson pairs Eq. (8), along with the experimentally proven flux count of Eq. (7) thus adds considerably to a pre-statistical applicability of both laws working in tandem. Eq. (9) is reducible to Eqs. (7) and (8) opening up potential of getting independent n and s info. In other words, contrary to Copenhagen Doctrine not all quantum info is statistical. Copenhagen's statistical exclusivity has been a major hurdle preventing full acceptance of the Aharonov-Bohm developments. The fact is, having to settle for a pre-statistic branch of quantum physics, alongside the familiar statistic branch. Compatibility requires mutual transitions exist but are excluded under nonclassical statistical rule. An outline of ensuing repercussions has been presented in a volume 181 of the Boston Studies in the Philosophy of Science [9]. It is in part Popper's [10] initiative replacing Copenhagen's single system by an ensemble of orientation- and phase-random systems, now governed by *classical*- not by a non-classical statistics. In retrospect Schrödinger-type quantum mechanics so becomes more classical. Instead of deriving the AB integral from the Schrödinger Ψ phase; the AB law now stipulates rather than postulates single-valued wave functions.

It was Schrödinger equation mystique inviting the *non-classical paradigm* and then it led to too much of the same. The experimental resurrection of the single system quantum aspect is now a special reminder how discreteness of matter is at the source of all quantum phenomena transcending from the tangible to the abstract. Planck's initial step of the quantum of action has so far remained physics' most enduring abstract step ever! The discrete structure of closed differential forms is a topological counterpart of quantization.

6. Conclusion

Let us once more extol the conceptual virtues of the two-tier change of Copenhagen Doctrine invoking a resurrection of single system pre-statistic quantization. *Real ensembles* of identical systems subject to a *classical* phase and orientation statistics replace Copenhagen's *single system* and its *nonclassical* statistics. Evidence thereof exists in calculations by Planck for phase- and Kompaneyets for orientation averaging [10]. The resurrection of pre-statistical quantization pioneered by Aharonov and Bohm is, as indicated by Kiehn [7], to be extended into a complete set of period integrals Eqs. (7,8,9). Exceeding invariance requirements of the general theory of relativity, they are metric-independent general invariant and accommodating scaling as characteristic of bringing out topological structure. This reinterpretation of bare facts puts an end to halve a century of Aharonov-Bohm schism, the cause of missing out on QHE unity.

Let that vast Copenhagen majority be invited to compare methodologies so that open forum may rule again guiding the way! This semi-hidden conflict pertaining to the use of the AB integral needs to be confronted, because it is too embarrassing to linger on.

7. Epilogue: Current State of Quantum Physics

After having become acquainted with minor changes in classical procedure in an a propos section, the main body of the text is an attempt at substantiating a solid set of changes aiming at reinstating classical reasoning procedures in what has been known in the form of Copenhagen Doctrine . In the end these results then have been summarized bringing out an undeniable gain in terms of logical coherence. A next question is whether the Copenhagen era in quantum theory from 1927-until the present, has been a unique anomaly in processing science data, or have there been other unexpected discrepancies between premise and ensuing theory?

It was Schrödinger's wave equation that really initiated Copenhagen's doctrine by cleverly combining frequency manifestations of de Broglie quantum interferometry and the characteristic (eigen)-values of a differential equation he had produced. The sheer magic of this procedure overwhelmed the world of physics making Schroedinger's equation and process the most prolific source of brand new physical results. The appearance of the Schrödinger equation occurred three years after the publication in 1924 of the famous Courent-Hilbert book on **Mathematical Methods in Physics**, which brilliantly and exhaustively covered the mathematics of the classical resonance problems of the pre-quantum era. Understandably, subsequent Courent-Hilbert editions also covered Schroedinger's celebrated extension of eigenvalue procedures to the quantum realm.

The just cited overlap of mathematical methodology between classical and quantum domains of physics was a blessing for the work during the years to come, yet by the same token it had the effect of viewing all quantization in an exclusive statistic context. This bias became clearly apparent in the subsequent interpretive Copenhagen picture that began to develop around Schrödinger's process by culminating as it did in a single system quantum state subject to an a priori-existing statistical disturbance that was said to be nonclassical in nature. All this strangely contradicted that Schrödinger's process had been most successful in sorting out spectral displays generated in random-phased and random-oriented collectives of identical quantum systems.

In the second half of the last century great strides made in low temperature physics began paying off in the form of new quantum manifestations that did not respond well to Schrödinger-type evaluations. The Josephson- and Aharonov-Bohm effects and finally the quantum Hall effects were a beginning of macro-ordered phenomena failing to make an easily understandable contact with Copenhagen's always present, a priori existing non-classical statistics. Aharonov and Bohm found the use of their phase integral better suited for dealing with what appeared to be pre-statistical situations. The same was found to be true for the quantum Hall effects, yet, as earlier delineated, here a vast majority stuck to viewing matters in their preferred Schrödinger perspective.

The just depicted situation has by now developed into a fully blown chasm and true schism of procedure in contemporary quantum physics yet even so not quite recognized as such. The physics community is loath to admit it is split in factions that are not communicating with one another. In fact a vast majority seems blissfully unaware of recognizing here a very dangerous breakdown in open exchange needed to resolve this predicament. A situation is developing here that goes against the very spirit of science pursuit, because its open exchange is in danger of drowning in an atmosphere of unhealthy competition.

At this point the most useful contribution that can resolve this chasm may well be the insights of de Rham theory in giving an independent support for the Aharonov-Bohm and Gauss integrals. This move brings out a two-tier nature of the quantum situation by casting light on the topological nature of quantization: *i.e.,* AB and Gauss integrals are quanta counters of flux and charge in situations of pre-statistic order, whereas Schrödinger's process remains in charge of ensembles of identical systems residing in states of phase- and orientation disorder.

Recently there have been attempts at injecting topology aspects into the quantum Hall situation in the perspective of Schrödinger's approach. Since it is difficult to extract conclusive topology information through an ensemble-based approach, it would seem the AB schism needs attention first. Finally the following thought may help to find our way. It was Bohr who guided us into the atomic realm. It was Bohr who may have chaired Copenhagen sessions perhaps going overboard in some of that *nonclassical* stuff. However, it was again Bohr who made a high-school approach to the quantum Hall effect possible. Just a reminder: rejects of Copenhagen's past can still hold promise for the future.

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