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Signal Transmission in Digital

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All the literature treats a transmission line as a series of infinitesimally small inductors and capacitors; this article presents the more realistic approach where at no time do we have to consider infinitesimally small segments of the line.

When the output of a logic gate switches from false to true, it raises the voltage difference between the output pin of the gate and the reference through a voltage v, which is called the signal amplitude of the gate. It is important to think of the signal as being a differential mode signal between the front end of the signal line and the reference, which serves as the return path. This reference might be called 'earth'. This change in voltage difference between the signal line and the reference line does not appear instantaneously at the point further down the signal line because there is capacitance between the signal line and the reference line. This means that, before the new logic level (voltage difference) can be established between the signal line and the reference line at all points, an amount of charge +q must have been delivered by the gate to the signal line, and an equal and opposite charge must have been removed from the reference line by the gate, where

$$q = Cv$$

C being the capacitance between the signal line and the reference line. We might suppose that if the logic gate could instantaneously deliver that amount of charge (+q) to the signal line and remove it from the reference line, the signal would appear without delay at the far end.

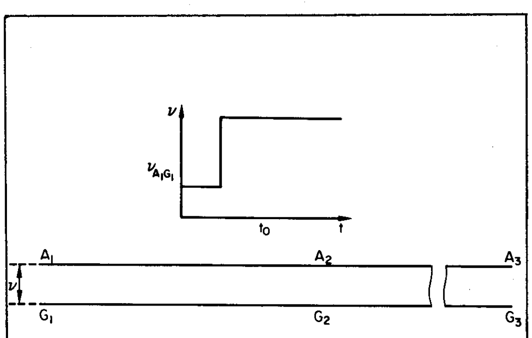

Figure 1 Output from logic gate

Unfortunately this is not so, because of inductance.

We know that movement of electrical charge is accoı panied by the appearance of magnetic flux. Change rate of flow of electric charge down the line, (which necessary if we are suddenly to transfer a charge q to tl line) involves change of flux, dΦ/dt, which, by Faraday law, creates an induced back e.m.f. which tends to oppo the change of current.

So, if we try to introduce a charge +q instantaneous into the line, we get an infinite dΦ/dt and so an infini back e.m.f.

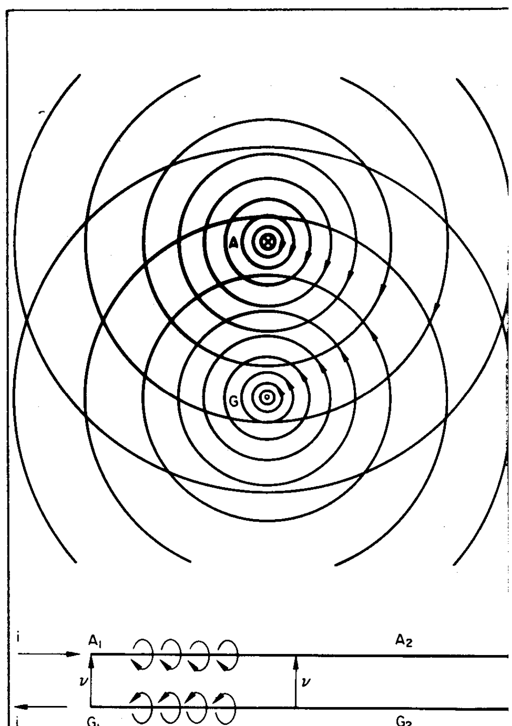

Figure 2 Logic signal on transmission line.

Systems

This means that, in order to drive instantaneously the voltage drop between signal line and reference line through a voltage change v, we would need an infinite driving voltage for a short time to overcome the back e.m.f.

An alternative way of explaining why the signal cannot travel instantaneously is to say that, as the electric charge travels down the line, energy appears around the line in the form of magnetic and electric flux. By the principle of conservation of energy, this energy must have been supplied by the only available source—the logic gate. Now the gate can (ideally) supply voltage and current instantaneously.

However, $v \times i$ is power, but not energy—power requires a third dimension, time, to give the full dimensions of energy. So, in the absence of infinite voltage or infinite current, time is necessary for the signal to develop throughout the length of the signal line. Note that in this discussion the geometry of signal wire and reference was not specified.

Logic signal transfer down a uniform transmission line. Let us assume that a logic swing v is introduced at A_1 G_1 between the signal line A and the reference line G (Figure 1) at time t_0. We can expect the logic swing to propagate down to the right, so that later in time (t_1) the signal has reached a point A_2 G_2. Let us suppose that the capacitance per unit length between A_1 A_3 and G_1 G_3 is C. This means that, if a steady potential difference v existed between the lines, the charge on each line would be (q) =Cv per unit length.

Let the self-inductance per unit length of the pair of lines A_1 A_3 and G_1 G_3 be L. This term L means that, if a steady current +i were flowing down A_1 A_3 and a steady current –i were flowing back along G_1 G_3 the magnetic flux passing through a surface A_1 A_3 G_3 G_1 bounded by the two wires would be $\Phi = Li$ per unit length of the pair of lines.

Since the cross-sectional geometries and the surrounding dielectric of the pair of lines A G does not vary along their length, it is reasonable to assume that the signal travels at a constant velocity **c**. We know that the impedance which the pair of lines AG presents to an impressed signal V_{A1G1} is not infinite, so there must be current as well as voltage contained in the signal. If a current +i is flowing down line A and a current –i is flowing back along line G, we have a magnetic flux field as shown in Figure 2; and this results in a net flux passing between the pair of lines AG.

Then in time Δt, the signal will have travelled a distanc equal to $\mathbf{c}\Delta t$, and the flux passing through the surfac $A_1 A_3 G_3 G_1$ will have increased by

$$\Delta\Phi = Li\,\mathbf{c}\Delta t \qquad (1)$$

Now Faraday's law of induction says that, if th flux through a loop increases steadily at the rate $\Delta\Phi/\Delta t$, a e.m.f. is induced equal to $-\Delta\Phi/\Delta t$ By Lenz's law (*Physics* by Hausmann, Slack Van Nostrand, p. 381), this oppose the original signal, and it can be called a back e.m.f.

Apart from the original signal v introduced acros $A_1 G_1$, this back e.m.f. is the only voltage around th loop $A_1 A_3 G_3 G_1$. By Kirchoff's second law, the sum o the voltages around the loop equals zero.

$$\text{Impressed voltage v} + \text{back e.m.f.} = 0$$

$$v + (-\Delta\Phi/\Delta t) = 0$$

$$\text{therefore } v = \Delta\Phi/\Delta t = Li\,\mathbf{c} \text{ from equation (1)}$$

This gives us the first important relationship betwee voltage, current and velicity:

$$v/i = L\,\mathbf{c} \qquad (2)$$

The second relationship will be derived from th principle of conservation of charge. We know that a current i is entering the line A at A_1. So, in time Δt, the total charge in line $A_1 A_3$ will have increased by

$$q = i\Delta t \qquad (3)$$

During time Δt, the signal will have advanced a distance $\mathbf{c}\Delta t$, and this new section of line will have been charged up through a voltage v.

The charge absorbed is given by:

capacitance $\times$ voltage
= capacitance per unit length $\times$ distance $\times$ voltage
= $C.\,\mathbf{c}\Delta t.\,v$

By the principle of conservation of charge this must be equal to the charge introduced into the line, $i\Delta t$ (equation 3). Therefore $i\Delta t = C\,\mathbf{c}\,\Delta t\,v$

This gives us the second important relationship between voltage, current and velocity:

$$v/i = 1/\mathbf{c}C \qquad (4)$$

Now if we divide equation (2) by equation (4), we eliminate v and i, and find that

$$\mathbf{c} = \pm\frac{1}{\sqrt{LC}} \qquad (5)$$

This means that *all signals, whatever their amplitude, travel at the same velocity,*

$$\mathbf{c} = \frac{1}{\sqrt{LC}}$$

If we multiply equation (2) by equation (4) we eliminate $\mathbf{c}$, to get

$$v/i = \pm\frac{\sqrt{L}}{\sqrt{C}} \qquad \pm\sqrt{\frac{L}{C}} \qquad (6)$$

This means that the ratio between v and i is a constant for the line. This constant has been called the characteristic impedance Z of the line.

References

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(2) I. Catt, 'Crosstalk (noise) in digital systems', *I.E.E.E. Trans. Electronic Computers*, EC-16, pp. 743-763, December, 1967.