THE UNIFICATION OF MACROSCOPIC PHYSICS *

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"The whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of Nature and then from these forces to demonstrate the other phenomena."
NEWTON: preface to the *Principia*

THE ancient Greek atomists maintained that forces between bodies could only be communicated by pressure or impact, a view that was supported by Aristotle and St. Thomas Aquinas: it appears in the scholastic axiom that 'matter cannot act where it is not.' Duns Scotus and his followers did not agree; William of Ockham using his Razor to cut out any intermediate actions which were unobservable and saying that there was no reason to object to action-at-a-distance.

The movement of a magnet, causing movement of iron filings, appears to be action-at-a-distance, and there is no evidence to think the appearance misleading. We ourselves, however, are usually unable to produce effects at a distance except by disturbing the air (medium transmissions) or by throwing things (emission of particles, 'emanations'), and this fact is possibly the cause of the reluctance with which most natural philosophers have looked upon action-at-a-distance, including Descartes and Newton.† Descartes proceeded to fill all space with a *plenum* in which light was a statical pressure. Hooke maintained that it was a vibratory motion, a view which was supported and developed by Huygens. Cotes, however, in his preface to the second edition of the *Principia* defended action-at-a-distance on Ockhamist lines, namely that it was the only theory that did not introduce unverifiable and unnecessary suppositions; and

* Much of this article is, in part, based on a paper which appeared in the *Proceedings of the Physical Society* in 1955.

† Maxwell actually says ". . . we are unable to conceive of propagation in time, except either as the flight of a material substance through space, or as the propagation of a condition of motion or stress in a medium already existing in space." *A Treatise on Electricity and Magnetism*, 3rd Ed., p. 492, Oxford (1892).

Boscovitch attempted to explain everything physical in terms of action-at-a-distance between point particles. But the ether never really died, and Faraday's work, as developed by Maxwell, greatly added to its vitality so that it gradually displaced the action-at-a-distance theories of Continental thinkers who included Riemann, Carl Neumann, Weber, and Clausius.

Maxwell, however, was quite clear that, without an ether, his equations were mere formulæ, the symbols in which had no physical meaning: speaking of the expressions for energy, tension, and pressure in the electrostatic field, he said:

"If the action of the system E_2 on E_1 does in reality take place by direct action-at-a-distance, without the intervention of any medium, we must consider ... [these] quantities as mere abbreviated forms for certain symbolical expressions, and as having no physical significance." *

This was in 1892, several years after the famous experiment of Michelson and Morley had been performed. This was designed to detect motion through the ether and gave a null result. The simplest hypothesis is that there was either no motion or no ether. It is difficult to believe that, with the whole apparatus being turned round at different positions in the Earth's orbit, there was never any relative motion with respect to the ether. The suggestion that the ether was dragged along with the Earth could not be accepted after Sir Oliver Lodge had shown that the velocity of light is not affected by the motion of neighbouring matter, and that if this drags the ether with it, the velocity given to the ether does not exceed one two-hundredth part of its velocity.† We must therefore consider the other alternative—that there is no ether.

If there is no ether, then we must have action-at-a-distance or else ballistic transmission. The experiments of Majorana‡ eliminate the possibility of ballistic transmission as well as the ether, for they lead to the inference that the time taken for interaction is independent of the relative motion of source and receiver. In an ether the interaction time is independent of the relative motion of source and receiver only if the receiver is at rest in the ether, and in any ballistic theory relative motion of source and receiver always affects the time interval. Nothing could show more clearly than this independence that interaction and matter are distinct, and that attempts to explain the former in terms of the latter by inventing subtle particles

* *Loc. cit.*, p. 158.

† *Phil. Trans.*, clxxxiv, p. 727 (1893).

‡ *Phil. Mag.*, **35**, p. 163 (1918) and **37**, p. 145 (1919). These experiments might well be repeated with greater accuracy.

of various kinds, taking 'contact force' as not requiring further elucidation, are bound to fail. It is hardly necessary to add that it has never been shown that 'contact' is not action-at-a-distance with the distance very small.

Action-at-a-distance remains, and experiment convinces us that for light and oscillatory electrical forces this action is not instantaneous. We must therefore consider non-instantaneous, or retarded, action-at-a-distance and must realise that in a world consisting of interacting matter, the properties of the interaction must be deduced from experiment and accepted as 'given' in just the same way that the properties of the ultimate particles of matter are taken as given. This task is made much more difficult by certain features of language which have developed, of course, independently of science. We constantly use what is now called *thing-language* in referring to light. We talk about "a bent ray" or "light passing through a hole in a screen" or as if something were travelling, occupying successive intermediate positions in time. But rays are only useful conceptions; the oscillatory forces of light do not pass through holes any more than the steady forces of gravitation do; it is a matter of superposition and phase relationship: and nothing has ever been found travelling.

Let us start, then, with a macroscopic physical theory of Nature, namely that all 'stuff' is composed of unchangeable interacting material particles, and that all change is due to motion (and not to generation or annihilation). Force is the cause of motion, and physical objects (matter) are the causes of force. We shall restrict the theory to interactions on a sufficiently large scale to avoid the necessity for 'quantum' considerations.

We are attempting a description of the physical universe: we are *not* constructing a mathematical symbolism because, in physics, we are concerned with what Dr. Johnson kicked "so mightily that he rebounded from it." This was not a symbol, by definition: it was *matter*, which by its interaction with other matter is the supposed external cause of the sensations (knowledge of force arises from a special set of deep-seated nerves). Our description of the universe must be such that, if we imagine moving observers placed in it, the relationship between their measurements can be deduced from the theory and not treated as postulates as in relativity theory —a procedure that inevitably leads to subjectivism. In any case, physicists are rarely concerned with comparing moving observers' measurements: they are interested in one observer's measurements of the difference in interaction between bodies at rest and in motion.

In order to simplify the problem before us, we must make a physical hypothesis which has some justification in experiment, but whose value must be judged by results :

> All macroscopic action-at-a-distance has the same *retardation constant* c ($c \simeq 3 \times 10^{10}$ cm./sec.).

This allows us to assert, as follows :

The law of retarded action : All particles in the universe are continually interacting, but a particle at time t can only affect another particle with respect to which it is in motion at a distance r, at a later time $\left(t + \dfrac{r}{c}\right)$.

Time is measured in the ordinary way, using recurring physical effects, and with the addition of corrections from both theory and observation (tidal friction, irregular rotation of the Earth). On January 1st, 1960, this will be the new *Ephemeris Time.*

We proceed now using Newtonian method : that is to say that we start with a physical theory and then derive the metrical expression of it from experiment and observation. In this way Newton, starting with the physical idea of universal gravitational attraction, and inertia with respect to the body of God (strictly the "Sensorium," *i.e.* absolute space) * derived the inverse square formula from Tycho Brahe's observations (on which Kepler founded his laws), the laws of motion from Galileo's experiments, and the third law from experiments made specially for the Royal Society by Wren, Wallis, and Huygens, and of course, his own experiments.

When we are attempting to deal with a universe consisting of bodies acting on one another at a distance, it is obvious that, in the process of analysis, we come at last to the force between *two particles.* The formula for the force between two particles must inevitably be the cardinal formula of physics. We suppose—because it is probably impossible analytically to suppose anything else—that this force is independent of the rest of the universe, and that the forces we observe are due to the superposition of many forces. Newton, following Galileo, treated forces as acting independently, so that the gravitational force between two particles depended only on the particles themselves (represented by the coefficient *mass*) and their distance apart ; and, in cases where several such forces were acting on the particle at the same instant, the resultant force could be found by using the principle of superposition and the parallelogram of forces. We shall adopt the principle of superposition and the parallelogram of forces as being confirmed by experiment, and extend

* Newton did, however, consider the possibility that inertial forces were due to surrounding distant matter. *Cf. Nature,* **151**, 85 (1943).

the principle of independence to cover relative motion and retarded action as follows:

The principle of physical relativity: Every particle acts on every other particle with a force which depends only on the particles themselves (measured by mass and charge), their relative separation and motion, and the retardation constant c.*

Now it is well-known that Newton assumed instantaneous interaction in the case of gravitation. He did not measure the attraction between bodies at rest, as was done later by Cavendish, but accounted satisfactorily for Kepler's laws by the hypothesis that the force between two particles at any instant was proportional to the inverse-square of the distance between their instantaneous positions. Thus he was actually dealing with moving bodies, but the planetary velocities are small compared with c, and the hypothesis seemed satisfactory, so that Newton did not attempt to include any velocity or acceleration terms in the inverse-square formula. The movement of the perihelia was not dealt with until later, but when disagreement appeared it suggested that extra terms of some kind were required in the law.†

Now we know that in electromagnetic phenomena the force between two charged particles varies with relative motion. We have only to charge up the insulated rim of a rotating disc to find that a nearby magnet or solenoid is affected, whereas with the disc charged and at rest no such action occurs.‡ The extra terms, therefore, in the formula for the force between charged particles must include velocity terms.

In order to proceed we need to make another hypothesis based on simplicity.§ Let us suppose that the gravitational and electrodynamic forces between two particles vary in the same way with relative motion and distance. We are led to suspect this possibility by the well-known fact that, when they are at rest, the force varies in the same way with distance, being $\frac{qq'}{r^2}$ (with the charges q and q' in e.s.u. and the distance r in cm.) and $\frac{mm'}{r^2}$ with the masses m and

* For the advantages of referring to c as the retardation constant rather than the velocity of light, *see* "A New Treatment of the Theory of Dimensions," *Proc. Phys. Soc.*, **53**, 418 (1941).

† *Cf.* Whittaker, *A History of the Theories of Aether and Electricity*, Vol. I, p. 207, Vol. II, p. 148. Nelson, London (1951).

‡ *Cf.* Rowland and Hutchinson, *Phil. Mag.*, **27**, 445 (1889).

§ This does not, of course, mean that we necessarily think Nature to be simple, but that we know *we* are—*see* Brown, *Science: its Method and its Philosophy*, p. 135. Allen & Unwin, London (1950).

m' in dynamical units (*i.e.* such that, when m, m', and r are unity, the force of attraction is 1 dyne).

Making this hypothesis, therefore, we can now proceed to determine the extra terms in the force-formula, because the electrodynamical forces are so enormously greater than the gravitational forces, and we can measure them with considerable accuracy. We assume with Ampère that all magnetic effects are due to the motion of charged particles, and that in neutral conductors carrying steady currents we have negatively charged particles moving with a resultant 'drift' velocity v, and that, in oscillators, these particles have an acceleration f.

Now Ampère succeeded in deriving, from many experiments with different kinds of circuits, an expression for the force between two steady currents i_1 and i_2 flowing in elements of circuits ds_1 and ds_2. With the help of the experiment of Rowland and Hutchinson already mentioned, which showed that moving electric charges produce magnetic effects similar to those of electric currents, and quantitatively the same if $\frac{qv}{c}$ is substituted for ids, we can write Ampère's formula so that it expresses the velocity terms in the formula for the force between two moving charged particles.* It should be noted that, if our theories of metallic conduction can be relied upon, the formula applies to values of v very much less than c.

Acceleration of electric charge occurs when a current is started or stopped in an ordinary closed circuit, and the forces produced were investigated by Faraday in his famous researches on electromagnetic induction. Acceleration also occurs in open circuits in which electric oscillations are taking place, notably in wireless aerials. The forces which these oscillations produce in receiving circuits are what make radio broadcasting possible, and these are known with a fair degree of accuracy. From these forces we can derive the acceleration terms in our formula more easily than from the former induction experiments, because if we consider the force at a point at a large distance from an aerial, all parts of the aerial are at sensibly the same distance from the point, and so no complication

$$^{*}\ \mathrm{E}_x = \frac{qq'}{r^2}\left\{\left[\frac{\mathrm{K}+2}{2}\cdot\frac{v^2}{c^2} - \frac{3(\mathrm{K}+1)}{2}\cdot\frac{v_r^{\,2}}{c^2}\right]\cos(rx) + \frac{\mathrm{K}v_x v_r}{c^2}\right\}$$

where E_x is the force along any direction x, and v_x, v_r, are the relative velocity resolved along x and r respectively. The appearance of the constant K allows for the fact that any force which added up to zero on integrating round a closed circuit is compatible with Ampère's results. A more detailed mathematical treatment will be found in "A Theory of Action-at-a-distance," *Proc. Phys. Soc. B.*, LXVIII, p. 672 (1955).

arises from the fact that the interaction is not instantaneous: this merely affects the phase.

From radio experiments, therefore, we can derive the terms

$$E_x = \frac{qq'}{rc^2}\left[f_r \cos (rx) - f_x\right]$$

We can then proceed with the help of a mathematical theorem (Taylor's theorem) to modify this expression to allow for 'retardation' when r and $\cos (rx)$ are *not* the same for all parts of the inducing circuit, as in the case near to the aerial, and we then find, finally,

$$E_x = -\frac{qq'}{2rc^2}\left[f_r \cos (rx) + f_x\right]^*$$

We have now found the extra velocity and acceleration terms to be added to the electrostatic force-formula.† If this is correct, it ought to enable us to "demonstrate the other phenomena": for example, we ought to be able to deduce the phenomena of electromagnetic induction.

If we consider the inductive effect of an element of a circuit on another element of the same circuit, or of another circuit, we have to find the force tending to separate positively and negatively charged particles in the element in the direction of the element and sum this up for the whole inducing circuit; from this force, when it refers to unit charge, can be calculated the induced electromotive force. When this is done it is found that the velocity terms, and the acceleration terms, independently, yield e.m.f. $= -\frac{dN}{dt}$, which is Neumann's formulation of Faraday's results. This represents uniformly moving circuits with *steady* currents (velocity terms) and circuits at rest with *changing* currents (acceleration terms) respectively. This deduction, therefore, is satisfactory.

As has been noted, the velocity terms in the force-formula were derived from the results of experiments in which v was very small compared with c. Nevertheless it is interesting to see whether the formula will apply to moving free electrons. In the case where the electron moves at right angles to the electric force, we can show

* The electrostatic term is added to the preceding formula to allow for the free charges at the ends of the dipole, before modifying it.

† Thus the complete formula is

$$E_x = \frac{qq'}{r^2}\left\{\left[1 + \frac{(K+2)}{2}\cdot\frac{v^2}{c^2} - \frac{3(K+1)}{2}\cdot\frac{v_r^2}{c^2} - \frac{rf_r}{2c^2}\right]\cos (rx) + \frac{Kv_xv_r}{c^2} - \frac{rf_x}{2c^2}\right\}$$

that the force on it is no longer the classical $4\pi\sigma e$ (where the symbols have the usual meanings) but becomes $4\pi\sigma e\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$.

Now if we divide this by the inertial mass of the electron m_i, the acceleration could be written, if $\frac{v^2}{c^2}$ is small,

$$\frac{4\pi\sigma e}{m_i\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

so that if this result of the variation of electric force with velocity were treated as due to a change in the inertial mass, as is done in relativity theory, this would agree with the well-known relativity formula. Unfortunately the accuracy of this type of experiment is not sufficient to decide whether the force-formula is correct, or whether it requires the addition of higher powers of v^2.* The case in which the electron moves in the direction of the electric force, as in accelerators, is difficult to deal with owing to lack of information of the rapidly varying non-uniform force employed, but the formula indicates a limiting velocity. The force of a neutral current (or magnet) on a moving changed particle is correctly given.

The Origin of Inertia

In accordance with the hypothesis that gravitational and electrical forces vary in the same way with distance and motion we convert the force formula to represent the gravitational force of attraction between two particles by merely substituting $-mm'$ for qq'.

Now this equation tells us the gravitational force between two moving particles but nothing about the resultant change of motion. However, in the actual universe all particles are surrounded by many others, and observation shows that this surrounding matter is more or less uniformly distributed, and to about the same distance in all directions. Let us, therefore, investigate by means of the formula what effect a uniform spherical distribution of matter would have on a particle at, or near, the centre.

Suppose we have two particles A and B, at rest with respect to one another, and that we ignore the small gravitational interaction between them. Stimulated by the suggestion of Bishop Berkeley

* *Cf.* Zahn and Spees, *Phys. Rev.*, **53**, 511 (1938), and also Faragó and Jánossy, *Publications of Central Research Institute for Physics*, Budapest, 1956.

which was examined by Newton, let us consider the forces exerted by the rest of the universe upon them. Now we do not know the relative motions of the various particles of matter in the surrounding universe and consequently we do not know the forces exerted, but if A and B are free and not too far apart, the resulting motion will be the same for both. Let A be given a uniform velocity v with respect to B. The *extra* force produced by interaction with the universe can be calculated from the formula by using the principle of physical relativity and taking A to be in a uniform spherical distribution of matter which is moving with velocity $-v$. This force is found to be zero at the centre and negligible for the greater part of the interior. Thus we can say that uniform velocity of a particle with respect to another particle does not produce any *extra* interaction with the universe as a whole.*

Similarly, if we give A an acceleration with respect to B, we can calculate the extra force produced, and this turns out to be, at the centre,

$$\frac{4}{3}\frac{\pi\rho R^2 m}{c^2}.f$$

acting in a direction opposite to the acceleration f. The average density of matter in the universe is ρ (in dynamical units), and R is the radius as at present estimated from observation.† If the particle A is not at the centre but at a distance p from it, the resisting force becomes

$$\frac{20R^2 - 4p^2}{15}.\frac{\pi\rho m}{c^2}.f \quad \text{for radial motion and}$$

$$\frac{20R^2 - 8p^2}{15}.\frac{\pi\rho m}{c^2}.f \quad \text{for tangential motion.}$$

Since $R \simeq 10^{27}$ cm. and even in a thousand years the solar system only moves about 10^{16} cm., no local difference would be observable. Thus we can say that uniform acceleration of a particle with respect to any other particle produces extra interaction with the surrounding matter of the universe resulting in a force opposite in direction to the acceleration and proportional to it.

It is clear that the last deduction from this theory of retarded

* This paragraph is very carefully worded to overcome the difficulty, when there is no ether, of saying what uniform velocity is with respect to—with Newton, of course, it was the body of God (absolute space).

† With dynamical units of mass, the gravitational constant G is dimensionless, and mass has the dimensions L^3T^{-2} (*see* "A New Treatment of the Theory of Dimensions," *loc. cit.*).

action-at-a-distance now enables problems in dynamics to be dealt with. We started with the physical theory of Nature, namely that all physical phenomena can be accounted for by moving changeless particles which interact with one another by what we call force. We found the formula for the macroscopic force of attraction or repulsion by experiment, and use the force of attraction of the Earth on pieces of matter for the introduction of a numerical measure-system (weighing with a balance, *i.e.* static force). Knowledge of the force between two particles, as has been pointed out, tells us nothing of the resultant motion; but we have seen that if we consider the particles surrounded by a more or less uniform spherical distribution of particles, as is the case in practice, we find that the theory indicates the phenomenon of inertia: that is to say, a particle of gravitational mass m will have an acceleration f in the direction of the force such that $\frac{4}{3}\frac{\pi\rho R^2}{c^2}.m$ multiplied by f is equal to the force of attraction or repulsion which is the cause of the motion.

Thus if we choose to write $\frac{4}{3}\frac{\pi\rho R^2}{c^2}.m = m_i$ and call it the 'inertial mass', we derive the well-known form of Newton's Second Law defining inertial force

$$F = m_i f$$

We have already obtained a result similar to the First Law, *viz.* that a particle moving with uniform velocity with respect to any other free particle experiences no extra resistance from the surrounding matter of the universe and thus will continue so to move.

The Third Law is involved in the view, on the theory of retarded action-at-a-distance, that force is mutual, and exists only between *two* particles. A force 'at a point in space' is a pure conception with no experimental evidence. Thus the force between particles A and B has no independent existence in such a way that it could be different on A from on B. The accelerations resulting from this mutual force, when A and B are free, and surrounded by matter as in our universe, is found in the usual way by dividing this force by the respective inertial masses.

The expression for the attractive force of gravitation, when it contains velocity and acceleration terms, allows us, therefore, to derive laws of motion which, in the central regions of a uniform spherical distribution of matter (where our galaxy appears to be), are closely in agreement with Newton's laws which were, of course, treated as axioms. The universe is taken to be, on the whole, electrically neutral, so that no extra similarly calculable 'electrical inertia' acts on a charged particle.

We can make a quantitative test: we know $\frac{m}{m_i} = \sqrt{G}$, and so $\sqrt{G}$ should be $\frac{3c^2}{4\pi\rho R^2}$. Taking the observational values $G = 6{\cdot}7 \times 10^{-8}$ and $R = 2 \times 10^{27}$ cm., the mean density of matter in the universe turns out to be 10^{-27} gm./cm.^{3}, a result which agrees with present estimates.

Movement of the Perihelia of Planets

So far we have not needed to determine the constant K in the force-formula. If we examine the orbit of a planet circling the Sun under the extended gravitational force-formula, we find this to be an ellipse the perihelion of which has a rotation per revolution of $\frac{(3 - K)\pi GM}{c^2 a(1 - e^2)}$, where M is the mass of the Sun in grams, e is the eccentricity and a is the semi-major axis of the orbit. If we choose $K = -3$ we get the formula first obtained by Gerber in 1898 on the assumption of retarded gravitational potential with velocity c. It was also derived by Einstein in 1916, and is in good agreement with observation in the case of Mercury which is the only one large enough to be known fairly accurately.

At present there does not seem to be another physical phenomenon involving K for which accurate observational results are available to check the value chosen. This value is, however, required for the relations which follow.

Momentum and Energy

As has been mentioned earlier, relativity theory rests on postulates about relationships between moving observers' measurements, whereas a genuine physical theory has to describe a universe such that, if there should be moving observers and they happen to be making measurements, the relationship between their measurement can be calculated, although in practice this case is rarely of interest. Strictly speaking, therefore, the present physical theory and relativity theory are not comparable. We can, however, compare the *formulæ* derived on the two theories.

Now it is well-known that certain relativity formulæ have been widely confirmed in atomic physics although not with great accuracy.* It is interesting, therefore, to see whether these formulæ can be given an interpretation on the present theory.

Suppose that we consider elastic collisions : we require the force-formula with the force along the line joining the two particles. This becomes

$$E = \frac{qq'}{r^2}\left(1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{rf}{c^2}\right)$$

The ratio of the acceleration term to the velocity term is $2\frac{rf}{v^2}$. If we are considering the close approach of molecules, r is of the order 10^{-8} cm., so that f can be neglected unless it is of the order $10^8 v^2$. If f can be neglected

$$E = \frac{qq'}{r^2}\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$

and substituting from the inertial formula for a particle of mass m_i

$$m_i f = \frac{qq'}{r^2}\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$

Now if, for the sake of comparison with relativity theory, we want to say that it is m_i that changes with velocity (and not the electrical interaction) † we could write

$$\frac{m_i}{\sqrt{1 - \frac{v^2}{c^2}}}.f = \frac{qq'}{r^2}$$

which would lead to the momentum being $\frac{m_i v}{\sqrt{1 - \frac{v^2}{c^2}}}$ which is equivalent, within present limits of accuracy, to the well-known relativity formula.‡

Turning next to the energy-mass relationship, we must remember that mass in physics is a coefficient which enables us to calculate gravitational interaction and it is clear from the force-formula (with $-mm'$ substituted for qq') that we *could* include the terms in brackets

* *Cf.* Faragó and Jánossy, *loc. cit.* (1956).

† All ' mass-variation ' experiments have involved electrical interaction.

‡ Strictly speaking, relativity theory leads to $\frac{m_0 v}{(1 - v^2/c^2)^{3/2}}$ but relativists ignore this on conservation grounds and write $\frac{m_0 v}{(1 - v^2/c^2)^{\frac{1}{2}}}$ (*see* Tolman, *Relativity Thermodynamics and Cosmology*, p. 55, Oxford (1934)).

with m', and say that this is how the mass m' varies with relative motion, *i.e.* when m' is moving with respect to m it must be taken to be $m'\{\ .\ .\ .\ \}$. But the principle of physical relativity would obviously entitle us to attach the bracketed terms with equal justification to the other mass m. The force between the particles, however, is mutual, so that if this changes with relative motion and the bracketed terms are attached to *its* measure, there is no need to decide which of the two particles has to be regarded as moving. Thus subjective considerations about observers and reference frames are avoided. Similar remarks apply, of course, to electrical force. We are not content on this physical theory to say that a coefficient varies with motion : we ask what physical change causes us to have to change the coefficient and our answer is that the force changes.

Supposing, however, that we do treat the bracket terms as part of m' we can easily see how the mass-energy formula arises. Consider two particles of mass m and m' at rest with respect to one another at a distance r. It is convenient to take the masses to be in grams. The attractive force between them is

$$\mathrm{F} = \frac{Gmm'}{r^2} \text{ along } r.$$

Now suppose m' has a velocity v in any direction. Its kinetic energy is now $\frac{1}{2}m'v^2$ by definition. The force along r now becomes,

$$\mathrm{F}' = \frac{Gmm'}{r^2}\left[1 - \frac{1}{2}\frac{v^2}{c^2}\right]$$

and we could write this

$$\mathrm{F}' = \frac{Gm}{r^2}\left[m'\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)\right]^*$$

i.e. we *could* say that m' has decreased by $\frac{1}{2}m'\frac{v^2}{c^2}$. Thus an increase in kinetic energy of $\frac{1}{2}m'v^2$ corresponds to a decrease in mass equal to this kinetic energy divided by c^2. So we obtain :

$$\text{kinetic energy} = \text{mass} \times c^2.$$

Change of potential energy is due to change of position, and not directly to motion so that no change of the interaction coefficient occurs.†

* The corresponding relativity formula is $\frac{Gm}{r^2}[m'(1 - 3v^2/c^2)]$.

† Relativists generalise the relation to include potential energy and treat it as a postulate (*see* Tolman, *loc. cit.*, p. 49).

The conservation of energy and momentum follows mathematically from the classical formula $F = mf$. As we have derived the same formula on the theory of retarded action-at-a-distance, the conservation laws also hold formally. The physics of the case, however, must not be overlooked, for there are no disembodied forces in Nature. F is always due to other matter, and so if there is relative motion with respect to this other matter, then F must be calculated from the force-formula which includes velocity and acceleration terms, so that the simple derivation of the conservation laws cannot apply. The classical conservation laws, owing to the extreme smallness of the velocity and acceleration terms in most ordinary cases, remain in very close approximation.

Conclusion

This paper attempts to show that the physical theory of Nature, together with retarded action-at-a-distance, is capable of giving a causal account of well-attested macroscopic phenomena discovered since Newton's time, in terms of matter, motion and force. Gravitation and electromagnetism are united by showing that the hypothesis that the force-formula is similar for both is satisfactory. Newtonian method, that is to say, starting with a physical theory and then proceeding to derive the metrical force-formula from experiment, has been used throughout, and from the theory and the formula are deduced other phenomena such as inertia and electromagnetic induction, thus bearing the " burden of philosophy " in the manner advocated by Newton.

The theory of relativity starts from postulates about relationships between moving observers' measurements, which avoids the genuine physical problem as to what property the physical universe must have in order that, if there are any observers and if they choose to make measurements, their results turn out to be what they are. The present theory gives a physical explanation of the mass-variation and mass-energy relations of special relativity. As regards the three well-known predictions of general relativity, only the value of the perihelion motion of Mercury is in satisfactory agreement with observation. The present theory accounts for perihelion motion but quantitative agreement requires the adjustment of a constant: general relativity requires two extra postulates (covariance and equivalence). The increased inertia near a large body can be calculated on the present theory, and this yields a value of the right order for the red-shift of spectral lines on the Sun's limb, but this is a problem not yet resolved. On an action-at-a-distance theory no ' bending of light ' in free space can, of course, occur. The current value for grazing incidence near the Sun, which is not the relativity value, would seem to be due to terrestial and coronal matter.*

One fundamental difference between the results of the two theories is this : on the present theory the surrounding matter of the universe does not cause any perceptible increase in inertia for bodies whose velocity approaches c and consequently relative velocities greater than c may occur. At present there is some indirect evidence that cosmic ray particles reach the Earth's surface from the outer atmosphere (where they are thought to be produced) in times which indicate that they have travelled faster than c (otherwise they would have ' decayed ' before arrival). Relativists hold this as evidence for ' time dilation '. Perhaps further research will enable this and other matters to be decided.†

* For criticism of this determination, *see* SCIENCE PROGRESS, **44**, 176, 619 (1956).

† The author would like to express his thanks to Dr. G. H. A. Cole for several helpful discussions. His general indebtedness to W. Ritz will be clear to all who know the history of electrodynamics.