Algorithm for Representation of Prime Numbers - Determinants of a Special Kind*

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This paper derives recursive relationships that can be considered as one of the variants of a big screen, with the essential difference that here, instead of prime numbers, mutual simplicity is used. *** From 2009.**

Introduction

Let us consider a square matrix of dimension n, with elements

$a_{i,i}=1, \text{for } i=1,2,...n$;

$a_{i,i+1}=0, \text{for } i=1,2,...n-1$; $a_{i,i+2}=1, \text{for } i=1,2,...,n-2$;

$a_{i,i+k}=0, \text{for } i=1,2,...,n-k \text{ and } k=3,4,...,n-i$;

$\left|a_{i,i-j}\right|=1 \text{for } i=2,3,...,n \text{ and } j=1,2...,n-2$.

For this matrix, let some special factors $E_i^{(n)}$ for $i=1,2..,n-1$ be calculated from the following formula from [1]:

$$E_{i+1}^{(n)}=\frac{-1}{E_i^{(n)}\left(1+\sum_{j=1}^{n-2} a_{i,i-j}\times\prod_{l=1}^{j} E_{i-l}^{(n)}\right)} \quad i=1,2...,n-2\,. \tag{*}$$

The following statement is simple to prove by decomposition of the determinant of the specified matrix:

$$\Delta_{k-1}E_k^{(n)}E_{k-1}^{(n)}=-\Delta_{k-2}+p_{k-1}E_k^{(n)} \text{ for } k=2,3,....,n-1\,. \tag{+}$$

Here Δ_k is the determinant of dimension k. It will become clear below that the p_k for $k=2,3,...$ is always an odd number.

Order 3

Let us consider the following square matrix of dimension 3:

$$\begin{bmatrix}1&0&1\\-1&1&0\\-1&-1&1\end{bmatrix}$$

The determinant of this matrix is equal to 3, the third prime number. This fact is checked directly. From the positive result at $n=3$ it follows that

$$E_2^{(3)}E_1^{(3)}=-1\quad. \tag{1}$$

Here it is accepted that $\Delta_0=1,\ p_1=0.$. We can write for the specified matrix of third dimension the expression

$$UX_2=E_2^{(3)}E_1^{(3)};E_2^{(3)}\quad.$$

The dimension of this expression is less than the dimension of the matrix. According to (1), we write $UX_2=-1;E_2^{(3)}$. From [1], it follows that $E_2^{(3)}=1/\Delta_2$, where Δ_2 is the determinant received from the initial deletion of the last row and last column of the matrix. It is equal to 1. Then $UX_2=(-1;1)/\Delta_2$. We will consider the column

$$DUX_n=UX_n\Delta_n\quad. \tag{2}$$

In particular, at $n=2$ it is found that

$$DUX_2=-1;1\quad, \tag{3}$$

and

$$DUX_2(2)=1\quad. \tag{4}$$

Further below, it becomes clear that $DUX_n(n)=p_n$. According to [2], the size of the determinant of third order is calculated as

$$\Delta_3=A_3DUX_2+\Delta_2\quad, \tag{5}$$

where A_3 is the third line in the matrix of third order without one element. Really $\Delta_3=-1(-1)+(-1)\cdot(-1)+1=3$. And, as shown in [2], this rule remains for any determinant of dimension n.

$$\Delta_n=A_nDUX_{n-1}+\Delta_{n-1}\quad, \tag{6}$$

where A_n is the n^{th} line in the matrix of order n without the element $a_{n,n}$.

Order 4

Let us consider the matrix of fourth order having precisely same structure:

$$\begin{bmatrix}1&0&1&0\\-1&1&0&1\\-1&-1&1&0\\a_{4,1}&a_{4,2}&a_{4,3}&1\end{bmatrix}$$

According to [1], for this matrix $N=4$; $M=2$; $L=3$. So it is designated in [1]. Here and below $N=n$.

$$E_{i+1}^{(4)}=\frac{-a_{i,i+2}}{\left(E_i^{(4)}(a_{i,i}+\sum_{j=1}^{3}a_{i,i-j}\times\prod_{l=1}^{j}E_{i-l}^{(4)})+a_{i,i+1}\right)}\quad i=1,2,3 \tag{7}$$

Cases $i=1$ and $i=2$ are already known to us. According to the expression labeled (+) above,

$$E_2^{(4)}E_1^{(4)}=-1\quad,\quad E_3^{(4)}E_2^{(4)}=-1-E_3^{(4)}\quad. \tag{8}$$

At $i=3$ it is found that

$$E_4^{(4)}=-a_{3,5}\Big/\left(E_3^{(4)}(a_{3,3}+\sum_{j=1}^{3}a_{3,3-j}\times\prod_{l=1}^{j}E_{3-l}^{(4)})+a_{3,4}\right) \tag{9}$$

According to [1], it is believed equal to zero, as such an element in the matrix of fourth order is not present. As the numerator in (9) is equal to zero, and $E_4^{(4)}$ in the general case is not equal to zero, the denominator in (9) is equal to zero. And from here

$$E_2^{(4)}=1\Big/\left(1+E_1^{(4)}\right) \tag{10}$$

One more parity connects $E_2^{(4)}$ and $E_1^{(4)}$. Then $E_1^{(4)}=-1/2; E_2^{(4)}=2; E_3^{(4)}=-1/3$. Let us pay attention to the fact that

$$E_3^{(4)}=-1/\Delta_3 \tag{11}$$

we will write down line UX_3 as

$$E_3^{(4)}E_2^{(4)}E_1^{(4)};E_3^{(4)}E_2^{(4)};E_3^{(4)}=-E_3^{(4)};-1-E_3^{(4)};E_3^{(4)} \tag{12}$$

or
$$UX_3=\frac{1}{\Delta_3}(1;-2;-1)\ \text{and}\ DUX_3=1;-2;-1 \tag{13}$$

$$DUX_3(3)=p_3=-1\quad. \tag{14}$$

And from (6), at $n=4$ we have

$$\Delta_4=a_{4,1}-2\cdot a_{4,2}-a_{4,3}+3 \tag{15}$$

We while do not know to that are equal $a_{4,1};a_{4,2};a_{4,3}$. It is known only, that each of them is equal or (+1), or (-1). Δ_4 – the fourth prime number is not known also.

Order 5

Let us consider the matrix of fifth order having precisely same structure. $N=5; M=2; L=4$

$$E_{i+1}^{(5)}=\frac{-a_{i,i+2}}{\left(E_i^{(5)}(a_{i,i}+\sum_{j=1}^{4}a_{i,i-j}\times\prod_{l=1}^{j}E_{i-l}^{(5)})+a_{i,i+1}\right)}\quad i=1,.,4 \tag{16}$$

Cases $i=1,2,3$ are already known to us. According to (+),

$$E_2^{(5)}E_1^{(5)}=-1\ ,\ E_3^{(5)}E_2^{(5)}=-1-E_3^{(5)}\ ,\ 3E_4^{(5)}E_3^{(5)}=-1-E_4^{(5)} \tag{17}$$

At $i=4$ it is found that

$$E_3^{(5)}=-1\Big/\left(y_4+z_4E_2^{(5)}\right) \tag{18}$$

And then

$$E_4^{(5)}=\frac{p_4}{\Delta_4}=\frac{p_4}{\Delta_3+x_4} \tag{19}$$

Here it is designated

$$x_4=a_{4,1}-2a_{4,2}-a_{4,3}$$
$$y_4=-a_{4,1}+a_{4,3}$$
$$z_4=a_{4,2}$$
$$p_4=-a_{4,1}-a_{4,2}+a_{4,3}.$$

As all a on the module are equal to 1, then x_4 and y_4 are even numbers, and z_4 and p_4 are odd. And, according to (15), and under the obvious condition that $\Delta_4>\Delta_3$, x_4 is still a positive number. And, as it has appeared, these laws further remain.

Further, we find that

$$\begin{aligned}UX_4&=E_4^{(5)}E_3^{(5)}E_2^{(5)}E_1^{(5)};E_4^{(5)}E_3^{(5)}E_2^{(5)};E_4^{(5)}E_3^{(5)};E_4^{(5)}\\&=-E_4^{(5)}E_3^{(5)};E_4^{(5)}E_3^{(5)}E_2^{(5)};E_4^{(5)}E_3^{(5)};E_4^{(5)}\\&=-E_4^{(5)}E_3^{(5)};-E_4^{(5)}-E_4^{(5)}E_3^{(5)};E_4^{(5)}E_3^{(5)};E_4^{(5)}\\&=\tfrac{1}{3}(-3E_4^{(5)}E_3^{(5)};-3E_4^{(5)}-3E_4^{(5)}E_3^{(5)};3E_4^{(5)}E_3^{(5)};3E_4^{(5)})\\&=\tfrac{1}{3}\left(1+E_4^{(5)}\right);\tfrac{1}{3}\left(1-2E_4^{(5)}\right);\tfrac{1}{3}\left(-1-E_4^{(5)}\right);E_4^{(5)}\quad.\end{aligned}$$

Substituting (19) here, we have

$$UX_4=\frac{1}{\Delta_4}(1-U_4;1-V_4^{(1)};-1+U_4;p_4)\quad, \tag{20}$$

Where $U_4=a_{4,2}$ and $V_4^{(1)}=-a_{4,1}+a_{4,3}$, and in this case they coincide with z and y. The following statements are thus valid:

$$x_4+y_4=-2U_4; x_4+p_4=-3U_4.$$

Thus we have derived four linear homogeneous equations with six unknown parameters, about which it is known that they integers, and also are either even, odd, or even and positive.

Let. $x_4=2$. It is known that it is -1, or +1. But the case is excluded, so in this case the first and third components of vector (20) result in a zero, which becomes clearly undesirable further below.

Let us accept that $U_4=-1$. Then

$$y_4 = 0; p_4 = 1; z_4 = -1; V_4^{(1)} = 0.$$

And all these 6 unknown numbers satisfy with all 4th linear homogeneous to the equations and all specified restrictions. In this case $a_{4,2} = -1; a_{4,3} = a_{4,1}$.

As in the determinant of third order, below the main diagonal yields -1. We will write down definitively

$$a_{4,1} = -1;\ a_{4,2} = -1;\ a_{4,3} = -1. \tag{21}$$

Δ_4 is the fourth prime number:

$$\Delta_4 = 5 \tag{22}$$

$$DUX_4 = 2;1;-2;1 \tag{23}$$

$$DUX_4(4) = p_4 = 1 \tag{24}$$

And from (6), at $n = 5$ we have

$$\Delta_5 = 2 \cdot a_{5,1} + a_{5,2} - 2 \cdot a_{5,3} + a_{5,4} + 5 \tag{25}$$

We while do not know $a_{5,1}; a_{5,2}; a_{5,3}; a_{5,4}$. It is known only, that each of them is equal to (+1), or (-1). That Δ_5 is the fifth prime number is also not known.

Order 6

Let us consider the matrix of sixth order having precisely same structure. $N = 6$; $M = 2$; $L = 5$.

$$E_{i+1}^{(6)} = \frac{-a_{i,i+2}}{\left(E_i^{(6)}(a_{i,i} + \sum_{j=1}^{5} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(6)}) + a_{i,i+1}\right)} \quad i = 1,.,5 \tag{26}$$

Cases $i = 1,2,3,4$ are already known to us. According to (+),

$$\begin{aligned} E_2^{(6)} E_1^{(6)} &= -1 \\ 3E_4^{(6)} E_3^{(6)} &= -1 - E_4^{(6)} \\ 5E_5^{(6)} E_4^{(6)} &= -3 + E_5^{(6)} \end{aligned} \tag{27}$$

At $i = 5$ it is found that

$$E_4^{(6)} = -1 \Big/ \left(y_5 + z_5 E_3^{(6)}\right) \tag{28}$$

And then

$$E_5^{(6)} = p_5 / \Delta_5 = p_5 / (\Delta_4 + x_5) \tag{29}$$

Here it is designated

$$\begin{aligned} x_5 &= 2a_{5,1} + a_{5,2} - 2a_{5,3} + a_{5,4} \\ y_5 &= -a_{5,2} + a_{5,4} \\ z_5 &= -a_{5,1} - a_{5,2} + a_{5,3} \\ p_5 &= a_{5,1} - 2a_{5,2} - a_{5,3} + 3a_{5,4}. \end{aligned}$$

As all a on the module are equal 1, it follows that x_5 and y_5 are even numbers, and z_5 and p_5 are odd numbers. The condition that $\Delta_5 > \Delta_4$, makes x_5 a positive number.

Further, we find

$$x_5 - 2p_5 = 5U_5; x_5 + 2z_5 = -U_5; U_5 = a_{5,2} - a_{5,4} = -y_5$$

are even numbers.

Then we will write down

$$\begin{aligned} UX_5 &= E_5^{(6)}E_4^{(6)}E_3^{(6)}E_2^{(6)}E_1^{(6)}; E_5^{(6)}E_4^{(6)}E_3^{(6)}E_2^{(6)}; E_5^{(6)}E_4^{(6)}E_3^{(6)}; E_5^{(6)}E_4^{(6)}E_5^{(6)} = -E_5^{(6)}E_4^{(6)}E_3^{(6)}; E_5^{(6)}E_4^{(6)}E_3^{(6)}E_2^{(6)}; \\ & E_5^{(6)}E_4^{(6)}E_3^{(6)}; E_5^{(6)}E_4^{(6)}; E_5^{(6)} = -E_5^{(6)}E_4^{(6)}E_3^{(6)}; -E_5^{(6)}E_4^{(6)} + E_5^{(6)}E_4^{(6)}E_3^{(3)}; E_5^{(6)}E_4^{(6)}E_3^{(6)}; E_5^{(6)}E_4^{(6)}; E_5^{(6)} = \\ &= \frac{1}{3}(-E_5^{(6)}3E_4^{(6)}E_3^{(6)}; -3E_5^{(6)}E_4^{(6)} + E_5^{(6)}3E_4^{(6)}E_3^{(6)}; E_5^{(6)}3E_4^{(6)}E_3^{(6)}; 3E_5^{(6)}E_4^{(6)}; 3E_5^{(6)}) = \frac{1}{3}(E_5^{(6)} + E_5^{(6)}E_4^{(6)}; \\ & -2E_5^{(6)}E_4^{(6)} + E_4^{(6)}; -E_5^{(6)} - E_5^{(6)}E_4^{(6)}; 3E_5^{(6)}E_4^{(6)}; 3E_5^{(6)}) = \frac{1}{15}(5E_5^{(6)} + 5E_5^{(6)}E_4^{(6)}; -2 \cdot 5E_5^{(6)}E_4^{(6)} + 5E_5^{(6)}; \\ & -5E_5^{(6)} - 5E_5^{(6)}E_4^{(6)}; 3 \cdot 5E_5^{(6)}E_4^{(6)}; 15E_5^{(6)}) = \frac{1}{15}(6E_5^{(6)} - 3; 6 + 3E_5^{(6)}; 3 - 6E_5^{(6)}; -9 + 3E_5^{(6)}; 15E_5^{(6)}) = \\ &= \frac{-1 + 2E_5^{(6)}}{5}; \frac{2 + E_5^{(6)}}{5}; \frac{1 - 2E_5^{(6)}}{5}; \frac{-3 + E_5^{(6)}}{5}; E_5^{(6)}. \end{aligned}$$

Let us note that it is possible to manage, and without this conclusion. Free members in the received expression 1; 2;-1;-3 coincide with factors at a in dependence for p_5 only with a sign a minus, and factors at E_5 2;-1;-2;-1 coincide with factors at a in expression for x_5. And this tendency in the further remains.

Substituting here (29), we have

$$UX_5 = \frac{1}{\Delta_5}(-1 - U_5; 2 - V_5^{(1)}; 1 + U_5; -3 + z_5; p_5), \tag{30}$$

where $V_5^{(1)} = -a_{5,1} + a_{5,3} - a_{5,4}$ is an odd number.

Thus we have received system of five linear homogeneous the equations with seven unknown numbers about whom it is known that they integers, and also either even, or odd, or even and positive.

Let. $x_5 = 2$. It is U_5 rather known, that it is 0 or +2, or-2.. But $U_5 = \pm 2$ there cannot be as in this case p_5 is an even number, that it is impossible. So $U_5 = 0$. Then $y_5 = 0$; $p_5 = 1; z_5 = -1; V_5^{(1)} = -1$.

And all seven unknown numbers satisfy these five linear homogeneous equations, and all specified restrictions. In this case we will write down definitively

$$a_{5,1} = 1; a_{5,2} = -1; a_{5,3} = -1; a_{5,4} = -1. \tag{31}$$

$$\Delta_5 = 7 \text{ , the fifth prime number} \tag{32}$$

$$DUX_5 = -1; 3; 1; -4; 1 \tag{33}$$

$$DUX_5(5) = p_5 = 1 \tag{34}$$

And from (6) at $n = 6$ we will have

$$\Delta_6 = -a_{6,1} + 3 \cdot a_{6,2} + a_{6,3} - 4 \cdot a_{6,4} + a_{6,5} + 7 \tag{35}$$

Let us notice, that z_5 coincides with p_4 only instead a_4 of it is necessary to write a_5. And p_5 similarly in the same sense with x_4. It is necessary to add to it $\Delta_3 \cdot a_{5,4}$. We while do not know to that are equal $a_{6,1}; a_{6,2}; a_{6,3}; a_{6,4}; a_{6,5}$. It is known only, that each of them is equal or (+1), or (-1). Δ_6 – the sixth prime number is not known also.

Order 7

Let us consider the matrix of seventh order having precisely same structure.

$N = 7$; $M = 2$; $L = 6$

$$E_{i+1}^{(7)} = \frac{-a_{i,i+2}}{\left(E_i^{(7)} \left(a_{i,i} + \sum_{j=1}^{6} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(7)}\right) + a_{i,i+1} \right)} \quad i = 1,.,6 \tag{36}$$

Cases $i = 1, 2,., 5$ are already known to us. According to (+),

$E_2^{(7)} E_1^{(7)} = -1$

$$E_3^{(7)} E_2^{(7)} = -1 - E_3^{(7)}.$$

$$3E_4^{(7)} E_3^{(7)} = -1 - E_4^{(7)}$$

$$5E_5^{(7)} E_4^{(7)} = -3 + E_5^{(7)}$$

$$7E_6^{(7)} E_5^{(7)} = -5 + E_6^{(7)}. \tag{37}$$

At $i = 6$ it is found that

$$E_5^{(7)} = \frac{-3}{y_6 + z_6 E_4^{(7)}} \tag{38}$$

And then

$$E_6^{(7)} = \frac{p_6}{x_6 + 7} \tag{39}$$

Here it is designated

$$x_6 = -a_{6,1} + 3a_{6,2} + a_{6,3} - 4a_{6,4} + a_{6,5}$$
$$y_6 = a_{6,1} + a_{6,2} - a_{6,3} + 3a_{6,5}$$
$$z_6 = a_{6,1} - 2a_{6,2} - a_{6,3} + 3a_{6,4}$$
$$p_6 = 2a_{6,1} + a_{6,2} - 2a_{6,3} + a_{6,4} + 5a_{6,5}.$$

As all a on the module are equal 1, then x_6 and y_6 are even numbers, and z_6 and p_6 are odd numbers. The condition that $\Delta_6 > \Delta_5$ makes x_6 still a positive number.

Further, we find

$$x_6 + y_6 = 4U_6; x_6 + z_6 = U_6; 2x_6 + p_6 = 7U_6; U_6 = a_{6,2} - a_{6,4} + a_{6,5}$$

are odd numbers. Then we write down

$$UX_6 = \frac{-2 - E_6^{(7)}}{7}; \frac{-1 + 3E_6^{(7)}}{7}; \frac{2 + E_6^{(7)}}{7}; \frac{-1 - 4E_6^{(7)}}{7}; \frac{-5 + E_6^{(7)}}{7}; E_6^{(7)}.$$

Substituting (39) here, we have

$$UX_6 = \frac{1}{\Delta_6}(-2 - U_6; -1 - V_6^{(1)}; 2 + U_6; -1 - y_6; -5 + z_6; p_6) \tag{40}$$

where $V_6^{(1)} = -a_{6,1} + a_{6,3} - a_{6,4} - 2a_{6,5}$ is an odd number. As well,

$$-x_6 + 3p_6 = -7V_6^{(1)}; \text{ and } x_6 - V_6^{(1)} = 3U_6.$$

Thus we have received system of six linear homogeneous equations with eight parameters about which it is known that they integers, and also either even, or odd, or even and positive.

We postulate, that each of components of vector UX represents a rational number, that is fraction, in numerator and a denominator which are mutually simple integers.

Let. $x_5 = 2$. It is U_6 somewhat known; *i.e.* that it is $\pm 1; \pm 3$. But $U_6 = \pm 3$ cannot exist, because $|y_6| \le 6$. Let $U_6 = 1$. But then

$$\frac{-2 - U_6}{x_6 + \Delta_5} = \frac{-3}{9} = \frac{-1}{3} \quad ,$$

and that is unacceptable. Let $U_6 = -1$. And let

$$y_6 = -6; z_6 = -3; p_6 = -11; V_6^{(1)} = 5.$$

But then

$$\frac{-1 - V_6^{(1)}}{x_6 + \Delta_5} = \frac{-6}{2 + 7} = \frac{-2}{3} \quad ,$$

and that is unacceptable. Let $x_6 = 4$. Then U_6 can accept the same two values ± 1. Let $U_6 = 1$. Then

$$y_6 = 0; z_6 = -3; p_6 = -1; V_6^{(1)} = 1.$$

This variant works in every respect. So it is definitive

$$a_{6,1}=1; a_{6,2}=1; a_{6,3}=-1; a_{6,4}=-1; a_{6,5}=-1. \quad (41)$$

$$\Delta_6=11 \text{ is the sixth prime number} \quad (42)$$

$$DUX_6=-3;-2;3;-1;-8;-1 \quad (43)$$

$$DUX_6(6)=p_6=-1 \quad (44)$$

And from (6) at n =7 we will have

$$\Delta_7=-3a_{7,1}-2a_{7,2}+3a_{7,3}-a_{7,4}-8a_{7,5}-a_{7,6}+11 \quad (45)$$

Let's notice, that y_6 coincides with z_5 only instead a_5 of it is necessary to write $-a_6$. It is necessary to add to it $\Delta_3 \cdot a_{6,5}$.

Order 8

Let us consider the matrix of eighth order having precisely same structure. $N=8$; $M=2$; $L=7$;

$$E_{i+1}^{(8)}=\frac{-a_{i,i+2}}{E_i^{(8)}(a_{i,i}+\sum_{j=1}^{7}a_{i,i-j}\times\prod_{l=1}^{j}E_{i-l}^{(8)})+a_{i,i+1}} \quad i=1,.,7 \quad (46)$$

Cases $i=1,2,...,6$ are already known to us. According to (+),

$$E_2^{(8)}E_1^{(8)}=-1$$

$$E_3^{(8)}E_2^{(8)}=-1-E_3^{(8)}$$

$$3E_4^{(8)}E_3^{(8)}=-1-E_4^{(8)}$$

$$5E_5^{(8)}E_4^{(8)}=-3+E_5^{(8)}$$

$$7E_6^{(8)}E_5^{(8)}=-5+E_6^{(8)}$$

$$11E_7^{(8)}E_6^{(8)}=-7-E_7^{(8)}. \quad (47)$$

At $i=7$ it is found that

$$E_6^{(8)}=\frac{-5}{y_7+z_7E_5^{(8)}} \quad (48)$$

And then

$$E_7^{(8)}=\frac{p_7}{x_7+11} \quad (49)$$

Here it is designated

$$x_7=-3a_{7,1}-2a_{7,2}+3a_{7,3}-a_{7,4}-8a_{7,5}-a_{7,6}$$
$$y_7=-a_{7,1}+2a_{7,2}+a_{7,3}-3a_{7,4}+5a_{7,6}$$
$$z_7=2a_{7,1}+a_{7,2}-2a_{7,3}+a_{7,4}+5a_{7,5}.$$
$$p_7=-a_{7,1}+3a_{7,2}+a_{7,3}-4a_{7,4}+a_{7,5}+7a_{7,6}$$

As all a on the module are equal 1, then x_7 and y_7 - even numbers, and z_7 and p_7 are odd numbers. Condition, that $\Delta_7>\Delta_6$, x_7 to that still positive number. Furthermore we find

$$x_7-3y_7=-8U_7; 2x_7+3z_7=-U_7; x_7-3p_7=-11U_7;$$
$$U_7=a_{7,2}-a_{7,4}+a_{7,5}+2a_{7,6}-$$

an odd number. Then we write down

$$UX_7=\frac{1-3E_7^{(8)}}{11};\frac{-3-2E_7^{(8)}}{11};\frac{-1+3E_7^{(8)}}{11};$$
$$\frac{4-E_7^{(8)}}{11};\frac{-1-8E_7^{(8)}}{11};\frac{-7-E_7^{(8)}}{11};E_7^{(8)}.$$

Substituting here (49), we have

$$UX_7=$$
$$\frac{1}{\Delta_7}(1-U_7;-3-V_7^{(1)};-1+U_7;4+V_7^{(2)};-1-y_7;-7+z_7;p_7) \quad ,$$

where (50)

$$V_7^{(1)}=$$
$$-a_{7,1}+0a_{7,2}+a_{7,3}-a_{7,4}-2a_{7,5}+a_{7,6}\vdots ev\vdots 3x_7+2p_7$$
$$=11V_7^{(1)}; x_7-3V_7^{(1)}=-2U_7.$$
$$V_7^{(2)}=$$
$$-a_{7,1}-a_{7,2}+a_{7,3}+0a_{7,4}-3a_{7,5}-a_{7,6}\vdots od\vdots 4x_7-p_7$$
$$=11V_7^{(2)}; x_7-3V_7^{(2)}=U_7.$$

Thus we have received system of seven linear homogeneous the equations with nine unknown numbers about which it is known that they integers, and also either even, or odd, or even and positive.

Let $x_7=2$. U_7 can accept the following values $\pm1;\pm3;\pm5$. But approaches only one value $U_7=-1$, as in all other cases y_7 not the whole number or $|y_7|>12$. Then

$$y_7=-2; z_7=-1; p_7=-3; V_7^{(1)}=0; V_7^{(2)}=1.$$

This the variant works in every respect. So it is definitive

$$a_{7,1}=-1; a_{7,2}=-1; a_{7,3}=1; a_{7,4}=-1; a_{7,5}=1; a_{7,6}=-1. \quad (51)$$

$$\Delta_7=13 \text{ is the seventh prime number} \quad (52)$$

$$DUX_7=2;-3;-2;5;1;-8;-3. \quad (53)$$

$$DUX_7(7)=p_7=-3. \quad (54)$$

And from (6) at $n=8$ we will have

$$\Delta_8=2a_{8,1}-3a_{8,2}-2a_{8,3}+5a_{8,4}+a_{8,5}-8a_{8,6}-3a_{8,7}+13. \quad (55)$$

Order 9

Let's consider the matrix of ninth order having precisely same structure.

$N = 9$; $M = 2$; $L = 8$

$$E_{i+1}^{(9)} = \frac{-a_{i,i+2}}{E_i^{(9)}(a_{i,i} + \sum_{j=1}^{8} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(9)}) + a_{i,i+1}} \quad i = 1,.,8 \tag{56}$$

Cases $i = 1,2,...,7$ us are already known. According to (+),

$$E_2^{(9)} E_1^{(9)} = -1$$

$$E_3^{(9)} E_2^{(9)} = -1 - E_3^{(9)}$$

$$3E_4^{(9)} E_3^{(9)} = -1 - E_4^{(9)}$$

$$5E_5^{(9)} E_4^{(9)} = -3 + E_5^{(9)}$$

$$7E_6^{(9)} E_5^{(9)} = -5 + E_6^{(9)}$$

$$11E_7^{(9)} E_6^{(9)} = -7 - E_7^{(9)}$$

$$13E_8^{(9)} E_7^{(9)} = -11 - 3E_8^{(9)}. \tag{57}$$

At $i = 8$ we have

$$E_7^{(9)} = -7 \Big/ \left(y_8 + z_8 E_6^{(9)}\right) \tag{58}$$

And then

$$E_8^{(9)} = p_8 \big/ \left(x_8 + 13\right) \tag{59}$$

Here it is designated

$$x_8 = 2a_{8,1} - 3a_{8,2} - 2a_{8,3} + 5a_{8,4} + a_{8,5} - 8a_{8,6} - 3a_{8,7}.$$
$$y_8 = -2a_{8,1} - a_{8,2} + 2a_{8,3} - a_{8,4} - 5a_{8,5} + 7a_{8,7}.$$
$$z_8 = -a_{8,1} + 3a_{8,2} + a_{8,3} - 4a_{8,4} + a_{8,5} + 7a_{8,6}.$$
$$p_8 = -3a_{8,1} - 2a_{8,2} + 3a_{8,3} - a_{8,4} - 8a_{8,5} - a_{8,6} + 11a_{8,7}.$$

As all a on the module are equal 1,then x_8 and y_8 - even numbers, and z_8 and p_8 are odd numbers. Condition, that $\Delta_8 > \Delta_7$, x_8 to that still positive number.

Further we will receive

$$x_8 + y_8 = -4U_8; x_8 + 2z_8 = 3U_8; 3x_8 + 2p_8 = -13U_8; U_8 = a_{8,2} - a_{8,4} + a_{8,5} + 2a_{8,6} - a_{8,7}.$$

is an even number. Then we write down

$$UX_8 = \frac{3 + 2E_8^{(9)}}{13}; \frac{2 - 3E_8^{(9)}}{13}; \frac{-3 - 2E_8^{(9)}}{13}; \frac{1 + 5E_8^{(9)}}{13}; \frac{8 + E_8^{(9)}}{13}; \frac{1 - 8E_8^{(9)}}{13}; \frac{-11 - 3E_8^{(9)}}{13}; E_8^{(9)}.$$

Substituting here (59), we have

$$UX_8 = \frac{1}{\Delta_8}(3 - U_8; 2 - V_8^{(1)}; -3 + U_8; 1 + V_8^{(2)}; 8 - V_8^{(3)}; 1 - y_8; -11 + z_8; p_8) \tag{60}$$

$V_8^{(1)} = -a_{8,1} + 0a_{8,2} + a_{8,3} - a_{8,4} - 2a_{8,5} + a_{8,6} + 3a_{8,7}.$

$\vdots od \vdots 2x_8 - 3p_8 = -13V_8^{(1)}; x_8 + 2V_8^{(1)} = -3U_8.$

$V_8^{(2)} = -a_{8,1} - a_{8,2} + a_{8,3} + 0a_{8,4} - 3a_{8,5} - a_{8,6} + 4a_{8,7}.$

$\vdots od \vdots x_8 + 5p_8 = 13V_8^{(2)}; x_8 + 2V_8^{(2)} = -5U_8.$

$V_8^{(3)} = -a_{8,1} + 2a_{8,2} + a_{8,3} - 3a_{8,4} + 0a_{8,5} + 5a_{8,6} + a_{8,7}.$

$\vdots od \vdots 8x_8 + p_8 = -13V_8^{(3)}; x_8 + 2V_8^{(3)} = U_8.$

Thus we have received system of 8-th linear homogeneous the equations with 10 unknown numbers about which it is known that they are integers, and also either even, or odd, or even and positive.

Let $x_8 = 2$. U_8 can accept seven values $0;\pm2;\pm4;\pm6$. But as $|y_8| \le 18$, that leaves five values $0;\pm2;\pm4$. At $U_8 = \pm 2$. z_8 will be even number that cannot be. Then

$$U_8^{(i)} = \pm 4i; y_8^{(i)} = -2 \mp 16i; z_8^{(i)} = -1 \pm 6i. \quad i = 0,1.$$

But in this case $\frac{-11 + z_8^{(i)}}{x_8 + \Delta_7} = \frac{-11 \pm 6i - 1}{2 + 13} = \frac{-4 \pm 2i}{5}$, which is unacceptable.

Let $x_8 = 4$. U_8 can accept the same five values $0;\pm2;\pm4$. At $U_8 = 0;\pm4$. Then z_8 would be an even number, which it cannot be. Let us accept $U_8 = 2$. Then

$$y_8 = -12; z_8 = 1; p_8 = -19; V_8^{(1)} = -5; V_8^{(2)} = -7; V_8^{(3)} = -1.$$

This the variant works in every respect. So it is definitive

$$a_{8,1} = -1; a_{8,2} = -1; a_{8,3} = -1; a_{8,4} = 1; a_{8,5} = 1; a_{8,6} = 1; a_{8,7} = -1. \tag{61}$$

$$\Delta_8 = 17 \text{, the 8-th prime number} \tag{62}$$

$$DUX_8 = 1; 7; -1; -6; 9; 13; -10; -19. \tag{63}$$

$$DUX_8(8) = p_8 = -19. \tag{64}$$

And from (6) at $n=9$ we will have

$$\Delta_9 = a_{9,1} + 7a_{9,2} - a_{9,3} - 6a_{9,4} + 9a_{9,5} + 13a_{9,6} - 10a_{9,7} - 19a_{9,8} + 17 \tag{65}$$

Order 10

Let's consider the matrix of tenth order having precisely same structure. $N=10$; $M=2$; $L=9$

$$E_{i+1}^{(10)} = \frac{-a_{i,i+2}}{E_i^{(10)}\left(a_{i,i} + \sum_{j=1}^{9} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(10)}\right) + a_{i,i+1}} \quad i = 1,.,9 \tag{66}$$

Cases $i = 1,2,...,8$ are already known to us. According to (+),

$$E_2^{(10)} E_1^{(10)} = -1$$

$$E_3^{(10)} E_2^{(10)} = -1 - E_3^{(10)}$$

$$3E_4^{(10)} E_3^{(10)} = -1 - E_4^{(10)}$$

$$5E_5^{(10)} E_4^{(10)} = -3 + E_5^{(10)}$$

$$7E_6^{(10)} E_5^{(10)} = -5 + E_6^{(10)}$$

$$11E_7^{(10)} E_6^{(10)} = -7 - E_7^{(10)}$$

$$13E_8^{(10)} E_7^{(10)} = -11 - 3E_8^{(10)}.$$

$$17E_9^{(10)} E_8^{(10)} = -13 - 19E_9^{(10)}. \tag{67}$$

At $i = 9$ we have

$$E_8^{(10)} = -11 \Big/ \left(y_9 + z_9 E_7^{(10)}\right) \tag{68}$$

And then

$$E_9^{(10)} = p_9 \Big/ \left(x_9 + 17\right) \tag{69}$$

Here it is designated

$$x_9 = a_{9,1} + 7a_{9,2} - a_{9,3} - 6a_{9,4} + 9a_{9,5} + 13a_{9,6} - 10a_{9,7} - 19a_{9,8}.$$

$$y_9 = a_{9,1} - 3a_{9,2} - a_{9,3} + 4a_{9,4} - a_{9,5} - 7a_{9,6} + 11a_{9,8}.$$

$$z_9 = -3a_{9,1} - 2a_{9,2} + 3a_{9,3} - a_{9,4} - 8a_{9,5} - a_{9,6} + 11a_{9,7}.$$

$$p_9 = 2a_{9,1} - 3a_{9,2} - 2a_{9,3} + 5a_{9,4} + a_{9,5} - 8a_{9,6} - 3a_{9,7} + 13a_{9,8}.$$

As all a on the module are equal to 1, then x_9 and y_9 - even numbers, and z_9 and p_9 are odd numbers. The condition that $\Delta_9 > \Delta_8$ makes x_9 a positive number. Further, we find

$$x_9 - y_9 = 10U_9; 3x_9 + z_9 = 19U_9; 2x_9 - p_9 = 17U_9; U_9 = a_{9,2} - a_{9,4} + a_{9,5} + 2a_{9,6} - a_{9,7} - 3a_{9,8}.$$

is an odd number. Then we will write down

$$UX_9 = \frac{-2+E_9^{(10)}}{17}; \frac{3+7E_9^{(10)}}{17}; \frac{2-E_9^{(10)}}{17}; ; \frac{-5-6E_9^{(10)}}{17}; \frac{-1+9E_9^{(10)}}{17}; \frac{8+13E_9^{(10)}}{17}; \frac{3-10E_9^{(10)}}{17}; \frac{-13-19E_9^{(10)}}{17}; E_9^{(10)}.$$

Substituting here (69), we have

$$UX_9 = \frac{1}{\Delta_9}(-2-U_9; 3-V_9^{(1)}; 2+U_9; -5+V_9^{(2)}; -1-V_9^{(3)}; 8-V_9^{(4)}; 3-y_9; -13+z_9; ; p_9) \ , \tag{70}$$

where

$$V_9^{(1)} = -a_{9,1} + 0a_{9,2} + a_{9,3} - a_{9,4} - 2a_{9,5} + a_{9,6} + 3a_{9,7} - 2a_{9,8}.$$
$$\vdots od \vdots 3x_9 + 7p_9 = -17V_9^{(1)}; x_9 + V_9^{(1)} = 7U_9.$$

$$V_9^{(2)} = -a_{9,1} - a_{9,2} + a_{9,3} + 0a_{9,4} - 3a_{9,5} - a_{9,6} + 4a_{9,7} + a_{9,8}.$$
$$\vdots ev \vdots 5x_9 + 6p_9 = -17V_9^{(2)}; x_9 + V_9^{(2)} = 6U_9.$$

$$V_9^{(3)} = -a_{9,1} + 2a_{9,2} + a_{9,3} - 3a_{9,4} + 0a_{9,5} + 5a_{9,6} + a_{9,7} - 8a_{9,8}.$$
$$\vdots od \vdots - x_9 + 9p_9 = -17V_9^{(3)}; x_9 + V_9^{(3)} = 9U_9.$$

$$V_9^{(4)} = -2a_{9,1} - a_{9,2} + 2a_{9,3} - a_{9,4} - 5a_{9,5} + 0a_{9,6} + 7a_{9,7} - a_{9,8}.$$
$$\vdots od \vdots 8x_9 + 13p_9 = -17V_9^{(4)}; 2x_9 + V_9^{(4)} = 13U_9.$$

Thus we have received a system of 9 linear homogeneous equations with 11 unknown numbers about which it is known that they are integers, and also either even, or odd, or even and positive.

Let $x_9 = 2$ U_9 can accept seven values $\pm 1; \pm 3; \pm 5; \pm 7; \pm 9$. But as $|y_9| \le 28$ and $|z_9| \le 29$, that leaves two values, ± 1. Let us accept $U_9 = 2$. Then

$$y_9 = -8; z_9 = 13; p_9 = -13; V_9^{(1)} = 5; V_9^{(2)} = 4; V_9^{(3)} = 7; V_9^{(4)} = 9.$$

This the variant works in every respect. So it is definitive

$$a_{9,1}=1; a_{9,2}=-1; a_{9,3}=-1; a_{9,4}=1; a_{9,5}=-1; a_{9,6}=1; a_{9,7}=1; a_{9,8}=-1. \quad (71)$$

$$\Delta_9 = 19 \text{, the 9-th prime number} \quad (72)$$

$$DUX_9 = -3; -2; 3; -1; -8; -1; 11; 0; -13. \quad (73)$$

$$DUX_9(9) = p_9 = -13. \quad (74)$$

And from (6) at $n=10$ we will have

$$\Delta_{10} = -3a_{10,1} - 2a_{10,2} + 3a_{10,3} - a_{10,4} - 8a_{10,5} - a_{10,6} + 11a_{10,7} + 0a_{10,8} - 13a_{10,9} + 19 \quad (75)$$

Order 11

Let us consider the matrix of eleventh order having precisely same structure. $N=11$; $M=2$; $L=10$.

$$E_{i+1}^{(11)} = \frac{-a_{i,i+2}}{E_i^{(11)}(a_{i,i} + \sum_{j=1}^{10} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(11)}) + a_{i,i+1}} \quad i=1,.,10 \quad (76)$$

Cases $i=1,2,.,9$ are already known to us. According to (+),

$$E_2^{(11)} E_1^{(11)} = -1$$

$$E_3^{(11)} E_2^{(11)} = -1 - E_3^{(11)}$$

$$3E_4^{(11)} E_3^{(11)} = -1 - E_4^{(11)}$$

$$5E_5^{(11)} E_4^{(11)} = -3 + E_5^{(11)}$$

$$7E_6^{(11)} E_5^{(11)} = -5 + E_6^{(11)}$$

$$11E_7^{(11)} E_6^{(11)} = -7 - E_7^{(11)}$$

$$13E_8^{(11)} E_7^{(11)} = -11 - 3E_8^{(11)}.$$

$$17E_9^{(11)} E_8^{(11)} = -13 - 19E_9^{(11)}.$$

$$19E_{10}^{(11)} E_9^{(11)} = -17 - 13E_{10}^{(11)}. \quad (77)$$

At $i=10$ it is found that

$$E_9^{(11)} = -13 \Big/ \left(y_{10} + z_{10} E_8^{(11)}\right) \quad (78)$$

And then

$$E_{10}^{(11)} = p_{10} \big/ (x_{10} + 19) \quad (79)$$

Here it is designated

$$x_{10} = -3a_{10,1} - 2a_{10,2} + 3a_{10,3} - a_{10,4} - 8a_{10,5} - a_{10,6} + 11a_{10,7} + 0a_{10,8} - 13a_{10,9}.$$

$$y_{10} = 3a_{10,1} + 2a_{10,2} - 3a_{10,3} + a_{10,4} + 8a_{10,5} + a_{10,6} - 11a_{10,7} + 13a_{10,9} = -x_{10}$$

$$z_{10} = 2a_{10,1} - 3a_{10,2} - 2a_{10,3} + 5a_{10,4} + a_{10,5} - 8a_{10,6} - 3a_{10,7} + 13a_{10,8}.$$

$$p_{10} = a_{10,1} + 7a_{10,2} - a_{10,3} - 6a_{10,4} + 9a_{10,5} + 13a_{10,6} - 10a_{10,7} - 19a_{10,8} + 17a_{10,9}.$$

As all a on the module are equal to 1, then x_{10} and y_{10} are even numbers, and z_{10} and p_{10} are odd numbers. The condition, that $\Delta_{10} > \Delta_9$ makes x_{10} still a positive number. Further, we will receive

$$x_{10} + y_{10} = 0U_{10}; 2x_{10} + 3z_{10} = -13U_{10}; x_{10} + 3p_{10} = 19U_{10}; U_{10} = a_{10,2} - a_{10,4} + a_{10,5} + 2a_{10,6} - a_{10,7} - 3a_{10,8} + 2a_{10,9},$$

an odd number. Further,

$$UX_{10} = \frac{-1-3E_{10}^{(11)}}{19}; \frac{-7-2E_{10}^{(11)}}{19}; \frac{1+3E_{10}^{(11)}}{19}; \frac{6-E_{10}^{(11)}}{19}; \frac{-9-8E_{10}^{(11)}}{19}; \frac{-13-E_{10}^{(11)}}{19}; \frac{10+11E_{10}^{(11)}}{19}; \frac{19+0E_{10}^{(11)}}{19}; \frac{-17-13E_{10}^{(11)}}{19}; E_{10}^{(11)}.$$

Substituting here (79), we have

$$UX_{10} = \frac{1}{\Delta_{10}}(-1-U_{10}; -7-V_{10}^{(1)}; 1+U_{10}; 6+V_{10}^{(2)}; -9-V_{10}^{(3)}; -13-V_{10}^{(4)}; 10+V_{10}^{(5)}; 19-y_{10}; -17+z_{10};\ ; p_{10}) \quad (80)$$

where

$$V_{10}^{(1)} = -a_{10,1} + 0a_{10,2} + a_{10,3} - a_{10,4} - 2a_{10,5} + a_{10,6} + \vdots ev \vdots 7x_{10} + 2p_{10} = 19V_{10}^{(1)}; x_{10} - 3V_{10}^{(1)} = -2U_{10}. +3a_{10,7} - 2a_{10,8} - 3a_{10,9}.$$

$$V_{10}^{(2)} = -a_{10,1} - a_{10,2} + a_{10,3} + 0a_{10,4} - 3a_{10,5} - a_{10,6} + \vdots od \vdots 6x_{10} - p_{10} = 19V_{10}^{(2)}; x_{10} - 3V_{10}^{(2)} = U_{10}. +4a_{10,7} + a_{10,8} - 5a_{10,9}.$$

$$V_{10}^{(3)} = -a_{10,1} + 2a_{10,2} + a_{10,3} - 3a_{10,4} + 0a_{10,5} + 5a_{10,6} + \vdots ev \vdots 9x_{10} + 8p_{10} = 19V_{10}^{(3)}; x_{10} - 3V_{10}^{(3)} = -8U_{10}. +a_{10,7} - 8a_{10,8} + a_{10,9}.$$

$V_{10}^{(4)} = -2a_{10,1} - a_{10,2} + 2a_{10,3} - a_{10,4} - 5a_{10,5} + 0a_{10,6} + \vdots od \vdots 13x_{10} + p_{10} = 19V_{10}^{(4)}; 2x_{10} - 3V_{10}^{(4)} = -U_{10}. +7a_{10,7} - a_{10,8} - 8a_{10,9}.$

$V_{10}^{(5)} = -a_{10,1} + 3a_{10,2} + a_{10,3} - 4a_{10,4} + a_{10,5} + 7a_{10,6} + \vdots od \vdots 10x_{10} + 11p_{10} = 19V_{10}^{(5)}; x_{10} - 3V_{10}^{(5)} = -11U_{10}. +0a_{10,7} - 11a_{10,8} + 3a_{10,9}.$

Thus we have received system of 10-th linear homogeneous the equations with 12th unknown numbers about whom it is known, that they integers, and also either even, or odd, or even and positive.

Let $x_{10} = 2$. U_{10} can accept 14 values $\pm(2i+1), i = 0,1,2..,6$ But as $|z_{10}| \le 37$, that leaves ten values $\pm(2i+1), i = 0,1,2..,4$. Further there are only three values equal $U_{10}^{(i)} = -1 \pm 6i, i = 0,1$. In all other cases z_{10} not an integer. But in this case

$$\frac{-1-U_{10}^{(i)}}{x_{10}+\Delta_9} = \frac{-1+1\mp 6i}{2+19} = \frac{\mp 2i}{7}, i = 0,1,$$

which is unacceptable.

Let $x_{10} = 4$. U_{10} can accept 14 values: $\pm(2i+1), i = 0,1,2..,6$. But as $|z_{10}| \le 37$, that leaves ten values $\pm(2i+1), i = 0,1,2..,4$. Further there are only three values equal $U_{10}^{(i)} = 1 \mp 6i, i = 0,1$. In all other cases z_{10} not an integer.

Let us accept $U_{10} = 1$; then

$$y_{10} = -4; z_{10} = -7; p_{10} = 5; V_{10}^{(1)} = 2; V_{10}^{(2)} = 1; V_{10}^{(3)} = 4; V_{10}^{(4)} = 3; V_{10}^{(5)} = 5.$$

This the variant works in every respect. So it is definitive

$$a_{10,1} = -1; a_{10,2} = -1; a_{10,3} = 1; a_{10,4} = -1; a_{10,5} = 1; a_{10,6} = -1; a_{10,7} = -1; a_{10,8} = -1; a_{10,9} = -1. \tag{81}$$

$$\Delta_{10} = 23 \text{ is the } 10^{\text{th}} \text{ prime number} \tag{82}$$

$$DUX_{10} = -2; -9; 2; 7; -13; -16; 15; 23; -24; 5 \tag{83}$$

$$DUX_{10}(10) = p_{10} = 5 \quad . \tag{84}$$

And from (6) at $n = 11$ we have

$$\Delta_{11} = -2a_{11,1} - 9a_{11,2} + 2a_{11,3} + 7a_{11,4} - 13a_{11,5} - 16a_{11,6} + 15a_{11,7} + 23a_{11,8} - 24a_{11,9} + \ldots \tag{85}$$

Conclusion

And now for presentation we will write out a triangular array of the numerical the values of determinants constructed in the manner specified above:

$$\begin{array}{ccccccccccl}
1 & 0 & 1 & & & & & & & & \\
-1 & 1 & 0 & 1 & & & & & & & \\
-1 & -1 & 1 & 0 & 1 & & & & & & =3 \\
-1 & -1 & -1 & 1 & 0 & 1 & & & & & =5 \\
1 & -1 & -1 & -1 & 1 & 0 & 1 & & & & =7 \\
1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 & & & =11 \\
-1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 1 & & =13 \\
-1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 0 & & =17 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 0 & =19 \\
-1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & =23
\end{array}$$

$$\begin{array}{ccccccccccl}
1 & 0 & 1 & & & & & & & & =1 \\
-1 & 1 & 0 & 1 & & & & & & & =2 \\
-1 & -1 & 1 & 0 & 1 & & & & & & =3 \\
-1 & -1 & -1 & 1 & 0 & 1 & & & & & =5 \\
1 & -1 & -1 & -1 & 1 & 0 & 1 & & & & =7 \\
1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 & & & =11 \\
-1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 1 & & =13 \\
-1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 0 & 1 & =17 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 0 & =19 \\
-1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & =23
\end{array}$$

Most can accept this as the connection between the index of a prime number and its value: the order of a determinant, its index of prime number, and its numerical value. Earlier, something similar was known for Fibonacci numbers [3].

In [2] it is shown that for any set of integers, it is possible to present a corresponding set of determinants, where a determinant order corresponds to an integer number in this set.

Here, probably for the first time, such representation is received for the prime numbers. We will result still, in my opinion, a number of interesting consequences (77) it is possible to present expression in the form of continuous fraction.

$$E_{10}^{(11)} = \cfrac{-17}{-13 - \cfrac{247}{19 - \cfrac{187}{3 - \cfrac{91}{1 - \cfrac{55}{-1 - \cfrac{21}{-1 - \cfrac{5}{1 - \cfrac{3}{1 - \cfrac{1}{E_1^{(11)}}}}}}}}}.$$

And still, if in a considered matrix all elements located below the main diagonal are equal 1 such matrix will be individual. It is easy to prove it, having spread out it on elements of last column. But unlike its classical individual matrix own values will not be equal among themselves and equal 1, and will be to represent complex numbers.

The resulted recursive parities can be considered as one of the variants, with that essential difference that here, instead of prime numbers, mutual simplicity is used.

References

[1] Aleksandr Tsybin, “On Solving a System of Linear Equations” in this volume.

[2] Aleksandr Tsybin, “New algorism solution determinant ” in this volume.

[3] A.P. Mishina and I.V. Proskurjakov, Higher Algebra, Linear Algebra, Polynomials, the General Algebra The edition second, corrected (P.K. Rashevskiy, editor, 300 pages, Publishing house "SCIENCE", Moscow, 1965).