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A Coherent Reframing of the Master Equations of Electromagnetic Theory

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Abstract The electromagnetic expressions for capacitance, inductance, and resistance are reformulated in terms of the index of refraction, n, of the dielectric material surrounding the two conductors of the so-called electric circuit together with the vacuum speed of light, c, and vacuum wave impedance, Zo. The "quantities-per-unit-length" concept is shown to be closer to the physics than is the "lumped-element" concept. The critical features of three different theories- of alleged electric circuits, of transmission lines, and of optics, are shown to be aspects of a single, coherent opto-electromagnetic theory of *TEM Wave Electrodynamics*. A necessary part of putting this show in order requires dismissing the actors known as electric permittivity, magnetic permeability, electric current, magnetic flux, voltage, and electric charge. These are the misdirected characters starring in a drama about static fields and electric currents that never were.

Introduction

Is it not a noble farce, where kings, republics, and emperors have for so many ages played their parts, and to which the whole vast universe serves for a theatre?— Michel de Montaigne

The electromagnetic expressions for capacitance, inductance, resistance, and impedance have been reformulated in terms of the optical index of refraction of the material surrounding the two conductors of a transmission line. The relation between the refraction index and relative permittivity is at once well-known and yet obscured. It is used every day by optical engineers while apparently remaining all but unheard of in basic electromagnetic theory: from common electrical theory all the way through classical electrodynamics [e.g. 2-5], quantum mechanics [e.g.6], quantum electrodynamics, and beyond, much as has occurred with the obscuring of the vacuum wave impedance, Z_o in those same books. I re-discovered this relationship myself in 2008 by accident. Due in part to the censorship resulting from practices like secret peer-review I don't know if the results below are original or not.

This document is an introduction to the ramifications and implications of the index-dielectric relationship, which one optics man calls "Maxwell's Relation" [7], taken together with the many and profound contributions of Ivor Catt [e.g. 8-12]. The results below cover all types of two-conductor transmission lines, which are identical to all types of two-conductor electric circuits. Each of these two nomenclatures refers to the same electromagnetic system composed, in the simplest case, of source, two conductors, and load (Fig 1.). The particular length of the conductors adds no new physics, being only of the nature of an aspect ratio. There is no legitimate need to clutter up the picture by speaking of circuits and transmission lines as if they were two entirely different things. The layout of a mechanism cannot affect its underlying physics.

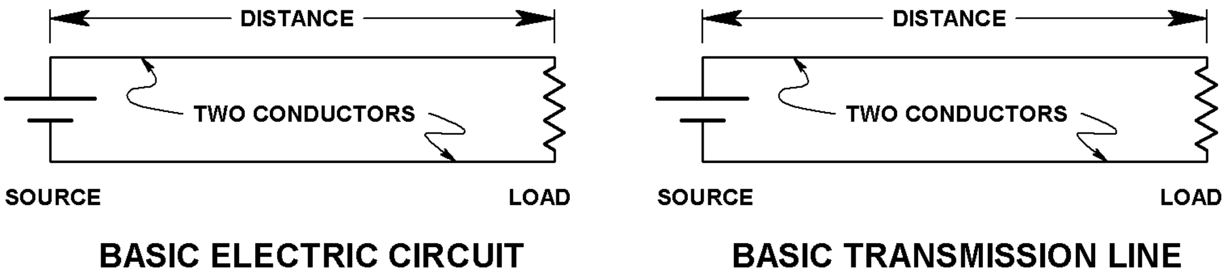

Figure 1.

There is no new physics added to the presented results, no previously hidden terms in the Maxwell-Heaviside equations, nor any hypothesized new phenomena. It is, rather, a *conservative* subtraction of clutter: in particular the relative permittivity (also called dielectric constant among other names), magnetic permeability, voltage, electric charge, magnetic flux, and electric current. The constants of interest are chosen based on what is actually measured: magnetic force, electric force, time, and length. Charge is shown again to be a replaceable alleged-physical constant in a separate, forthcoming paper [13]: removed and replaced by the vacuum wave impedance, Z_o.

The Vacuum Transverse Electromagnetic Wave Constants

There are two mutually-exclusive sets of the Transverse Electromagnetic (TEM) wave constants, composed of two of the constants in each set cast in terms of the remaining other two. They are either the set composed of the electric permittivity and the magnetic permeability $\{\varepsilon_0, \mu_0\}$-

$$\varepsilon_0 = \frac{1}{Z_0 \mathrm{c}} \tag{1}$$

and

$$\mu_0 = \frac{Z_0}{\mathrm{c}} \tag{2}$$

or they are the set composed of the wave impedance and the speed of light $\{c, Z_o\}$-

$$\mathrm{c} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{3}$$

and

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{4}$$

One part of removing the clutter in contemporary electromagnetic theory is to express equations using either one or the other of these two sets of wave constants. Mixing them together introduces spurious relationships, such as described and corrected in my 2007 paper [14] for the famous fine-structure constant, alpha. That correction shows, among other things, that alpha has nothing to do with the speed of light or permittivity, nor does **π** figure in anywhere. To illustrate, that concise result was of immediate use to a Brookhaven physicist working on the impedance of the electron [15]. In the development below $\{c, Z_o\}$ is selected as the exclusive set of vacuum electromagnetic constants. Permittivity and permeability are thereby removed from further use as they are already expressed by the constants on the rhs of Equs. (1) and (2).

The Relationship Between the Index of Refraction and the Dielectric Constant

It is well established, if not definitive, that the absolute index of refraction, n, is the ratio of the speed of light in vacuum, c, to the speed of light in an isotropic, bulk dielectric material, c_M

$$n = \frac{c}{c_M}, \quad \text{or} \qquad c_M = \frac{c}{n} \tag{5}$$

The "velocity factor" called out in coaxial cable specifications is the reciprocal of the index of refraction of the dielectric material. The dielectric constant κ, also called the relative permittivity ε_r ,is defined as

$$\kappa \equiv \varepsilon_r = \varepsilon / \varepsilon_o \tag{6}$$

From Equ (5), and with the caveats of non-magnetic media and elastic, non-resonant scattering,

$$n = \frac{c}{c_M} = \frac{\sqrt{\mu_o \varepsilon}}{\sqrt{\mu_o \varepsilon_o}} = \sqrt{\frac{\varepsilon}{\varepsilon_o}} = \sqrt{\kappa} \tag{7}$$

This can be extended to the isotopic and magnetic cases through the application of Equs. (1) and (2). I had to 'discover' this relation myself, by accident. This particular expression, though it is now found on the internet [e.g. 16], is nowhere to be found in over 100 relevant electrodynamics textbooks and reference works that I have reviewed, nor was this crucial relationship ever mentioned in my 'formal' physics and engineering coursework. One reason may be found in the exposition to follow.

The Geometric Relationship Between C_L, L_L, R_L, and Z

Before stating the master equations in their final, compact form another overlooked unity should be considered. Consider two long, flat-plate conductors of width b separated by a distance a with a dielectric material of resistivity ρ between the plates. Neglecting fringing-field effects, the dielectric resistance per unit length is

$$R_L = \rho\frac{a}{b} = \rho f \tag{8}$$

As an aspect ratio relates two different dimensions of a single object, so the generalized geometric factor f is a dimensionless ratio relating the size of the two conductors to the distance between them [8]. Both of these lengths lie in the plane transverse to the direction of motion of the transverse electromagnetic (TEM) wave. For capacitance per unit length, C_L, the expression is

$$C_L = \varepsilon\frac{b}{a} = \frac{\varepsilon}{f} \tag{9}$$

For inductance per unit length, L_L, the expression is

$$L_L = \mu\frac{a}{b} = \mu f \tag{10}$$

and for the characteristic impedance (see below) in the transverse plane of impedance,

$$Z = \sqrt{\frac{L_L}{C_L}} = \sqrt{\mu f\left(\frac{f}{\varepsilon}\right)} = f\sqrt{\frac{\mu}{\varepsilon}} \tag{11}$$

The rhs of the above four equations is general to all two-conductor geometries, *only the geometric factor f changes to account for different cross-section geometries.* For example, for a coaxial cable of an inner conductor diameter a and an inner diameter of outer conductor b,

$$f = \frac{1}{2\pi}\ln\left(\frac{b}{a}\right) \tag{12}$$

Putting that into the general expression for C_L yields the standard result for coax-

$$C_L = \frac{\varepsilon}{f} = \frac{\varepsilon}{2\pi}\ln\left(\frac{b}{a}\right) \tag{13}$$

The General Formulae for Electrical Properties Expressed in Terms of Wave Impedance and Dielectric Speed of Light

With the above background in place I can now state the general electromagnetic formulae in terms of the set of TEM wave constants $\{c, Z_o\}$, the vacuum wave impedance and the vacuum speed of light, together with the index of refraction and the three-dimensional geometry of the device in question, in a form the skilled reader should be able to understand.

For a bulk dielectric material of infinite extent in Euclidean space the dimensionless geometric factor f can be thought of as 1:1 = 1. The bulk dielectric wave impedance, sometimes called intrinsic impedance depending on author, of the non-magnetic dielectric of permittivity ε and permeability μ is

$$Z_M = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o \kappa}} = \sqrt{\frac{\mu_o}{\varepsilon_o n^2}} = \frac{Z_o}{n} \tag{14}$$

which is found by substitution from Equ. (7) and additionally by setting μ equal to μ_o (the non-magnetic caveat which can be dropped in a further extension). In the presence of the two conductors their geometry modifies the bulk dielectric wave impedance Z by an amount calculated using f:

$$Z = f\sqrt{\frac{\mu}{\varepsilon}} = fZ_M = \frac{fZ_o}{n} \tag{15}$$

Z is generally called the characteristic impedance of the transmission line. For the transmission line and electric circuit, the index of refraction enters twice, once to modify the speed of light and a second time to express the characteristic line impedance. Solving for the dimensionless geometric factor f

$$f = \frac{Zn}{Z_o} \tag{16}$$

Equ. (16) can be used to determine the effective geometric factor from the measured impedance. From Equ. (6) for dielectric constant, κ, Equ. (7) for index of refraction, n, Equ. (9) for capacitance per unit length, C_L, Equ. (1) for vacuum permittivity, and Equ. (15) for Z

$$C_L = \frac{\varepsilon}{f} = \frac{\varepsilon_o \kappa}{f} = \left(\frac{n^2}{Z_o \mathrm{c}}\right)\frac{1}{f} = \left(\frac{n^2}{Z_o \mathrm{c}}\right)\frac{Z_o}{Zn} \tag{17}$$

or

$$C_L = \frac{n}{cZ} \tag{18}$$

restating in terms of the speed of light measured in the dielectric,

$$C_L = \frac{1}{Zc_M} \tag{19}$$

This shows that the capacitance per unit length is strictly a function of the speed of light, the wave impedance, the geometry of the cross-section of the transmission line, and the index of refraction of its dielectric material. At no point was it necessary to invoke things like electric charge and voltage. At no point was it necessary to consider the material the conductors were made of. Equs. (18) or (19) suggest that 'electricity' does not travel inside of wires, but rather moves through the material that surrounds them. Similarly for the inductance per unit length L_L -

$$L_L = \mu_o f = \mu_o \frac{Zn}{Z_o} = \frac{Z_0}{\mathrm{c}} \frac{Zn}{Z_o} = \frac{Zn}{\mathrm{c}} \tag{20}$$

or, restating,

$$L_L = \frac{Z}{c_M} \tag{21}$$

Equs. (20) or (21) in plain English state that the inductance of two conductors is strictly determined by the impedance and the speed of light within the dielectric between them. As one check, dividing Equ. (21) by Equ. (19) and taking the root eliminates the speed of light to yield

$$Z = \sqrt{\frac{L_L}{C_L}} \tag{22}$$

a known result [8]. Multiplying Equ. (21) by Eq. (19) eliminates Z to yield a known result,

$$c_M = \frac{1}{\sqrt{L_L C_L}} \tag{23}$$

C_L and L_L, though still mathematical constructs, are closer to the physics than lumped-element capacitance C and inductance L because the two-dimensional property is all the TEM wave is ever moving through as it progresses at c_M in the third, propagation dimension of x. Multiplying Equ.(19) by the length of the capacitor plates, x, called 'Distance" in Fig 1., yields a lumped-element model for capacitance, C

$$C = xC_L = \frac{x}{c_M Z} \tag{24}$$

but since

$$C = \frac{Q}{V} \tag{25}$$

is the master equation that defines capacitance in terms of electric charge and voltage, then

$$\frac{Q}{V} = \frac{x}{c_M Z} \tag{26}$$

This eliminates the ratio of charge to voltage in favor of the measured values. By doing so, electric charge and voltage are shown to be auxiliary variables of a mathematical nature. Similarly for lumped-element inductance, L, multiplying Equ. (21) by x gives

$$L = \frac{xZ}{c_M} \tag{27}$$

but the master equation for inductance in terms of electric current and magnetic flux is

$$L = \frac{\Phi}{i} \tag{28}$$

so

$$\frac{\Phi}{i} = \frac{xZ}{c_M} \tag{29}$$

Now both electric current and magnetic flux have been removed from further consideration as well.
By the arguments above, permittivity and permeability are removed from consideration as separate and disjointed physical constants. Charge, electric current [9], voltage [17], and magnetic flux are also unneeded actors. They are all of the nature of mathematical costumery for a Greco-Roman theatrical production. If they were not, then things like the speed of light, the measured forces, and the dimensions of the electrical device would be the mathematical constructs. Only one or the other can refer to something real; the others must dwell somewhere on Olympus.

Conclusion

By unearthing and applying a known relationship between the dielectric constant and the index of refraction to the common electrical expressions it has been shown that the propagation of electricity has everything to do with the type of insulating material between the two wires of an electric circuit, taken together with the geometry of the two wires, and almost nothing to do with the material the wires are made of, aside from resistive losses. Electric charge, voltage, electric current, magnetic flux, permittivity, and permeability were all shown to be merely mathematical play actors. Nor was any bit part available for the static electric field, nor one for the static magnetic field to play in describing electricity. The delivered power was shown in the references, and reinforced herein, to be exclusively accounted for by wave propagation in the dielectric, leaving no starring roles for the "humors" inside the wires called electric currents to play- what Ivor Catt calls "the last of the Medieval fluids".

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