Galilean and Lorenz Transformations

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The Lorenz transformations are so connected with Special Relative theory that, as a rule, their interpretation is possible, only within the limits of this theory. We come to these transformations independently, on the other hand, and as a result we will receive unexpected results.

1. The Galilean Transformation

Let's begin with the Galilean transformations, so named, for the first time, in the work of P. Frank in 1909 [1]. We consider the basic equations for the dynamics of material objects:

$$m\ddot{x} = G_x, \quad m\ddot{y} = G_y, \quad m\ddot{z} = G_z,$$

where m is mass (kg) of the object, and length is in meters (m).

$$\ddot{x} = \frac{d^2x}{dt^2}, \quad \ddot{y} = \frac{d^2y}{dt^2}, \quad \ddot{z} = \frac{d^2z}{dt^2}$$

are the second derivative of length in $\mathrm{m/s^2}$, time t in seconds (s).

G_x, G_y, G_z are projections of force vector $\vec{\mathbf{G}}$ at three Cartesian axes. Then $\vec{\mathbf{G}} = \hat{\mathbf{i}}G_x + \hat{\mathbf{j}}G_y + \hat{\mathbf{k}}G_z$, with $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ unitary vectors. Force is given in Newtons $\left[\mathrm{N = kg\text{-}m/s^2}\right]$.

The formula for the Galilean Transformation is

$$x' = x + ut, \quad y' = y, \quad z' = z, \quad t' = t$$

New Cartesian axes (x', y', z') transfer with constant velocity u with respect to the former Cartesian axes (x, y, z). And new time t' relates to former time t. Then

$$\frac{dx}{dt} = \frac{dx'}{dt} - u, \quad \frac{dy}{dt} = \frac{dy'}{dt}, \quad \frac{dz}{dt} = \frac{dz'}{dt}.$$

And yet one time derivative of length $\frac{d^2x}{dt^2} = \frac{d^2x'}{dt^2} = \ddot{x}'$ and result

$$m\ddot{x}' = G_{x'}, \quad m\ddot{y}' = G_{y'} \quad m\ddot{z}' = G_{z'}.$$

As a result, the Galilean Transformation does not change the equations for the dynamics of material objects or the invariability of the Galilean Transformation equations themselves. Here $G_{x'}, G_{y'}, G_{z'}$ are the projections of force $\vec{\mathbf{G}}$ onto three new Cartesian axes.

2. Lorenz's Transformations

We now consider Maxwell's Electrodynamics equations. In vector representation, there are four equations:

1. **Gauss's Law for Electric Field:** $\vec{\nabla} \bullet \vec{\mathbf{D}} = \rho$.

   Electric charge is the parent of electric induction.

   $\vec{\nabla}$: Vector differential operator $\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$.

   $\vec{\mathbf{D}}$: Vector of electric induction $\left[\mathrm{C/m^2}\right]$.

   ρ: External electric charge density $\left[\mathrm{C/m^3}\right]$.

2. **Gauss's Law for Magnetic Fields:** $\vec{\nabla} \bullet \vec{\mathbf{B}} = 0$.

   Magnetic charge doesn't exist.

   $\vec{\mathbf{B}}$: Vector of magnetic induction $\left[\mathrm{T = Wb/m^2 = kg/C\text{-}s}\right]$.

3. **Faraday's Law:** $\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$.

   Magnetic induction creates a changing electric field.

   $\vec{\mathbf{E}}$: Vector of electric field $\left[\mathrm{V/m = N/C = kg\text{-}m/C\text{-}s^2}\right]$.

4. **Ampere's Law:** $\vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$.

   Electric current/charge/induction creates a changing magnetic field.

   $\vec{\mathbf{H}}$: Vector of magnetic field $\left[\mathrm{A/m = N/Wb}\right]$.

   $\vec{\mathbf{J}}$: Vector of electric current density $\left[\mathrm{A/m^2}\right]$.

We now consider the physical vacuum, where virtual particles develop, with a temperature $2.732°\mathrm{K}$, where $°\mathrm{K}$ represents Kelvin's scale. A physical vacuum does not imply emptiness.

For the physical vacuum, the magnetic constant is

$$\mu_0 = 1.25663706... \times 10^{-6}\ \mathrm{H/m}\left[= \mathrm{V\text{-}s/A\text{-}m}\right],$$

and the electric constant is

$$\varepsilon_0 = 8.85418782.... \times 10^{-12}\ \mathrm{F/m}\left[= \mathrm{A\text{-}s/V\text{-}m}\right],$$

where Farads $[\mathrm{F}]$ measure the electric receptivity of the vacuum. Then $c = 1/\sqrt{\varepsilon_0 \mu_0} = 299792458\ \mathrm{m/s}$ is the velocity of light in the physical vacuum.

If we let $\rho = 0$, $\vec{\mathbf{J}} = 0$, $\vec{\mathbf{H}} = 0$, $\vec{\mathbf{D}} = 0$, $\varepsilon_0 = \mu_0 = 1$, then Maxwell's equations have the view:

$$\vec{\nabla} \bullet \vec{\mathbf{E}} = 0 \qquad \vec{\nabla} \bullet \vec{\mathbf{B}} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \vec{\nabla} \times \vec{\mathbf{B}} = \frac{1}{c^2}\frac{\partial \vec{\mathbf{E}}}{\partial t}$$

The equations have been well known for 150 years. The wave equations generated by them have also been well known:

$$\Delta\vec{\mathbf{E}} - \frac{1}{c^2}\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2} = 0\,, \quad \Delta\vec{\mathbf{B}} - \frac{1}{c^2}\frac{\partial^2\vec{\mathbf{B}}}{\partial t^2} = 0\,,$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the 3D Laplacian operator.

These two vector equations are hyperbolic or wave vector equations. Derived from them are three differential equations. For example:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} = 0\,. \tag{1}$$

New Cartesian axes x', y', z' transfer with constant velocity u with respect to the former Cartesian axes x, y, z. To start with a question.: What transformation of Cartesian axes cause Eq. (1) to remain invariant?

We will look for this transformation in the equation set:

$$x' = \eta x - u\delta t\,, \quad t' = \gamma t - u\xi x\,, \quad y' = y\,, \quad z' = z\,, \tag{2}$$

where $\eta, \delta, \gamma, \xi$ are unknown constants. Then:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial}{\partial t'}\frac{\partial t'}{\partial x} = \eta\frac{\partial}{\partial x'} - u\xi\frac{\partial}{\partial t'}\,,$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial}{\partial x'}\frac{\partial x'}{\partial t} = \gamma\frac{\partial}{\partial t'} - u\delta\frac{\partial}{\partial x'}\,,$$

$$\begin{aligned}\frac{\partial^2}{\partial x^2} &= \eta\frac{\partial}{\partial x'} - u\xi\frac{\partial}{\partial t'}\left(\eta\frac{\partial}{\partial x'} - u\xi\frac{\partial}{\partial t'}\right) \\ &= \eta^2\frac{\partial^2}{\partial x'^2} - 2u\xi\eta\frac{\partial^2}{\partial x'\partial t'} + u^2\xi^2\frac{\partial^2}{\partial t'^2}\,,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2}{\partial t^2} &= \left(\gamma\frac{\partial}{\partial t'} - u\delta\frac{\partial}{\partial x'}\right)\left(\gamma\frac{\partial}{\partial t'} - u\delta\frac{\partial}{\partial x'}\right) \\ &= \gamma^2\frac{\partial^2}{\partial t'^2} - 2u\delta\gamma\frac{\partial^2}{\partial x'\partial t'} + u^2\delta^2\frac{\partial^2}{\partial t'^2}\,.\end{aligned}$$

Thus, Eq. (1) becomes:

$$\begin{aligned}&\eta^2\frac{\partial^2 E_{x'}}{\partial x'^2} - 2u\xi\eta\frac{\partial^2 E_{x'}}{\partial x'\partial t'} + u^2\xi^2\frac{\partial^2 E_{x'}}{\partial t'^2} + \frac{\partial^2 E_{x'}}{\partial y'^2} + \frac{\partial^2 E_{x'}}{\partial z'^2} \\ &-\frac{1}{c^2}\left(\gamma^2\frac{\partial^2 E_{x'}}{\partial t'^2} - 2u\delta\gamma\frac{\partial^2 E_{x'}}{\partial x'\partial t'} + u^2\delta^2\frac{\partial^2 E_{x'}}{\partial t'^2}\right) = 0\,.\end{aligned} \tag{3}$$

If $\xi\eta = \delta\gamma/c^2$, then Eq. (3) becomes:

$$\begin{aligned}&\delta^2\left(\frac{\eta^2}{\delta^2} - \frac{u^2}{c^2}\right)\frac{\partial^2 E_{x'}}{\partial x'^2} - \frac{\gamma^2}{c^2}\left(1 - \frac{u^2}{\gamma^2}c^2\xi^2\right)\frac{\partial^2 E_{x'}}{\partial t'^2} \\ &+\frac{\partial^2 E_{x'}}{\partial y'^2} + \frac{\partial^2 E_{x'}}{\partial z'^2} = 0\,.\end{aligned} \tag{4}$$

Hereafter $\delta = \gamma = \eta = 1/\beta$, $\xi = 1/\beta c^2$, where $\beta = \text{const}$. Then

$$\frac{\partial^2 E_{x'}}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E_{x'}}{\partial t'^2} + \frac{\beta^2}{1 - \frac{u^2}{c^2}}\left(\frac{\partial^2 E_{x'}}{\partial y'^2} + \frac{\partial^2 E_{x'}}{\partial z'^2}\right) = 0\,. \tag{5}$$

If $\beta = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$, we get:

$$\frac{\partial^2 E_{x'}}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E_{x'}}{\partial t'^2} + \frac{\partial^2 E_{x'}}{\partial y'^2} + \frac{\partial^2 E_{x'}}{\partial z'^2} = 0\,. \tag{6}$$

Eq. (6) agrees with Eq. (1), because $E_{x'} = E_x$. Therefore Eq. (1) is invariant to the transformations:

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}\,, \quad t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - \frac{u^2}{c^2}}}\,, \quad y' = y\,, \quad z' = z \tag{7}$$

These are the Lorenz Transformation equations, derived without presumptions about a finite speed of interaction in Nature. In fact, they were derived as early as 1887 by V. Fogt [2]. They were further utilized by J. Larmor [3] in the year 1900 and only afterwards by Lorenz [2] in the year 1904. These prior investigators did not invest physical sense in these transformation equations. Lorenz later moved to another point of view under the influence of the works of H. Poincare [2] and A. Einstein [2]. One thing is, do not eliminate the correct investment of a physical sense in these transformations, because, besides components of the vector of the electric field wave, Eq. (1) also satisfies the velocity of sound in continual aggregate. Instead of Eq. (1), we have:

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} - \frac{1}{C_f^2}\frac{\partial^2 W}{\partial t^2} = 0\,, \tag{8}$$

where sound wave W characterizes continuum parameters

C_f: Velocity of sound in continual aggregate, that for uniform liquid or gas calculate according to the formula:

$$C_f = \sqrt{\frac{1}{\beta_c\,\rho}}\ \left[\text{m/s}\right]. \tag{9}$$

β_c: Aggregate adiabatic of contraction $\left[\text{m}^2/\text{N} = \text{m-s}^2/\text{kg}\right]$.

ρ: Mass density $\left[\text{kg}/\text{m}^3\right]$.

As a result, we have, instead of the Lorenz's Transformation, the analogous transformation:

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{C_f^2}}}\,, \quad t' = \frac{t - \frac{u}{C_f^2}x}{\sqrt{1 - \frac{u^2}{C_f^2}}}\,, \quad y' = y\,, \quad z' = z\,, \tag{10}$$

which differ from Lorenz's Transformation equations in only one symbol: C_f instead of c.

But these transformations never consider anything other than the formal mathematics, which is devoid of all physics sense. Today, all rockets, several planes, and even some cars operate with speeds greater than the velocity of sound in air. We have yet to see a real world example in the case of time dilation and the Twin Paradox. Those adept at Special Relativity Theory, can remonstrate, that transformations in Eq. (7) substantially differ from those in Eq. (10), because the velocity of light in a vacuum, according to the classical Michelson-Morley experiment, is the maximal velocity allowed in Nature.

Actually, the fundamental results of the Michelson-Morley experiment for Einstein and Poincare created the presumption that space is a vacuum or emptiness and the velocity of light in a vacuum is the maximal velocity allowed in Nature, independent of direction. This postulate, named Einstein's postulate, allowed a plausible interpretation of the results of the Michelson-Morley experiment and Lorenz's Transformation acquiring physical sense. Now, there are factors, which suggest hesitance in such a conclusion. Space, which is now referred to as a physical vacuum, is not necessarily empty. The space vacuum has a temperature of $2.732°K$. This vacuum creates and annihilates virtual particles. Here the photons great energy begets vapors: neutrino-antineutrino, electron-positron, proton-antiproton and etc., etc. Thereby, with the increased energy of the photons, mass is generated. This process permits the derivation of the fundamental formula $E = mc^2$ without recourse to Special Relative Theory [4].

The existence of the physical vacuum allows the fundamental static and dynamic Casimir's effects [5,6] and also generates the increased mass of particles with the increase in their velocity [7]. There is a well-known effect in aerodynamics and hydrodynamics of an increase in mass associated with an increase in velocity [8]. However, a series of experiments determined that the velocity of light in a vacuum exceeded the theoretical limit [9,10].

I do not have a convincing answer to the question: What will explain the well founded results of the experiment of Michelson-Morley? Perhaps the experiment can be repeated for sound with the help of the acoustic interferometer.

3. Conclusion

The Galilean and Lorenz transformations are so connected with Special Relative theory that, as a rule, their interpretation is possible, only within the limits of this theory. In this article we demonstrated that these transformations can be developed without the use of Einstein's postulates. With our approach, the Lorenz Transformation acquires physical sense. By ignoring our approach, the Lorenz Transformation is a mathematical formula devoid of all physical sense.

The above derivation can account for the well-known effects of Special Relative theory:

1. The mass of particles increases with an increase in their velocity, and
2. The equation $E = mc^2$ can be obtained by other means.

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