Physical Applicability of Self Gravitating Isothermal Sphere Equilibrium Theory V Quantized Dark Matter Mass Densities Gravitational Quantization from Λ Why Einstein's Cosmological Constant is Essential Steady State Stabilised Galactic Halos

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1 Abstract

Using a new isothermal gravity equilibrium theory, the dust universe model together with a cosmological Schrödinger equation are applied to solving the problem of generating mass spectra. The masses generated can range from sub fundamental particle rest masses to masses greater than that of the universe. The ranges all depend on a quantum integer number l, related to the isotropic index n, which can lie between unity and infinity. One such mass obtained is given by $l = 8$ and can represent a small galaxy. The rotation curves for stars, in motion, within this galaxy are examined for flatness and found to have gradients of approximately, -10^{-23}. Examination of the Newtonian gravitation potential associated with these mass quanta reveals that it is, consistent with the dust universe model, based on Einstein's cosmological constant, Λ, rather than on Newton's gravitational constant, G, as this last constant disappears by fractional cancellation within the theory structure. Thus this quantization of gravity is based on the cosmological constant. There is found within this theory structure a simulation of negative mass from suitably geometrically orientated positive mass. It is suggested that this feature could supply an explanation for the character of *dark energy* mass as being due to suitably orientated positive mass. However, this last point needs further study. This paper is a corrected version involving an added section (8) explaining the corrections.

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2 Introduction

This paper is a follow up of papers, [48], [49], [50] and [52] of similar titles on the problem of formulating the equation that describes the equilibrium of a gaseous material in a self gravitational equilibrium condition in the galaxy modelling context, [47], see also, appendix 2 of ([35]). Here I shall examine, in more detail, the quantum set of dark matter density distributions $\varrho(r,n)$ which depend on the radial distance r and the isotropic quantum state index $n(l) = 2l/(2l-1)$ rational number which itself depends on the quantum integer numbers $l : 1.2.3...\infty$. The, mass per unit volume, density solutions of the new isothermal gravitational equilibrium equation as functions of the usual dimensioned radius parameter, r, and of the isotropic index, n, can be written as,

$$\varrho\prime(r,n) = \left(\frac{-2K}{\pi G(1-n)^2}\right)^{\frac{n}{n-1}} r^{\frac{2n}{1-n}}, \tag{2.1}$$

It is easy to show that the function (2.1) can be used as the space dependent part of a steady state solution to a specific Schrödinger equation by the following steps. Firstly, given a function, $E(n)$, for a set of energies dependent on the parameter n, we note that the kinetic energy term of the Schrödinger equation (2.8) will have the form, (2.3), when $\Psi_1(\mathbf{r},t)$ is assumed to have the product form,

$$\begin{aligned} \Psi_1(\mathbf{r},t) &= e^{-\frac{E(n)it}{\hbar}} \varrho\prime^{1/2}(r,n) \\ &= e^{-\frac{E(n)it}{\hbar}} \left(\frac{-2K}{\pi G(1-n)^2}\right)^{\frac{n}{2(n-1)}} r^{\frac{n}{1-n}} \end{aligned} \tag{2.2}$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi_1(\mathbf{r},t) = -\frac{\hbar^2}{2m}\frac{\partial(r^2\partial\Psi_1(\mathbf{r},t))}{r^2\partial r^2} = -\frac{\hbar^2 n\varrho_\prime^{1/2}(r,n)\exp(-\frac{E(n)it}{\hbar})}{2mr^2(1-n)^2} \tag{2.3}$$

$$i\hbar\frac{\partial}{\partial t}\Psi_1(\mathbf{r},t) = E(n)\Psi_1(\mathbf{r},t) = E(n)\exp(-E(n)it/\hbar)\varrho_\prime^{1/2}(r,n) \tag{2.4}$$

The second equation above gives the result of the quantum energy operator acting on $\Psi_1(\mathbf{r},t)$. Thus if we denote and define an external potential $V(\mathbf{r})$ by

$$V(\mathbf{r})\Psi_1(\mathbf{r},t) = E(n)\Psi_1(\mathbf{r},t) + \frac{\hbar^2}{2m}\nabla^2\Psi_1(\mathbf{r},t) \tag{2.5}$$

$$= \left(E(n) + \frac{\hbar^2 n}{2mr^2(1-n)^2}\right)\Psi_1(\mathbf{r},t) \tag{2.6}$$

$$V(\mathbf{r}) = E(n) + \frac{\hbar^2 n}{2mr^2(1-n)^2}, \tag{2.7}$$

we find looking at equations (2.3) $\rightarrow$ (2.7) that the square roots of solutions of the new isothermal equilibrium equation are also solutions of the Schrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\Psi_1(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi_1(\mathbf{r},t) + V(\mathbf{r})\Psi_1(\mathbf{r},t), \tag{2.8}$$

provided the an external potential contribution is defined by (2.7). It follows from this that the mass densities of the new isothermal equilibrium equations, apart from a multiplicative dimensioned constant, coincide with the probability densities of the Schrödinger equation. The formula at lines (2.5) and (2.6) has a well known significance in the quantum regime. It represents as shown at (2.10) the statement that $\Psi_1(\mathbf{r},t)$ is an eigen-function of the operator version of the external potential,

$$\hat{V}(\mathbf{r}) = i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2 \tag{2.9}$$

$$\hat{V}(\mathbf{r})\Psi_1(\mathbf{r},t)) = V(\mathbf{r})\Psi_1(\mathbf{r},t) \tag{2.10}$$

$$V(\mathbf{r}) = E(n) + \frac{\hbar^2 n}{2mr^2(1-n)^2}, \tag{2.11}$$

and in this case the eigen-values of this operator are given by the function $V(\mathbf{r})$ at (2.11), these actual values being determined by whatever the appropriate value of the isotropic index n happens to be. Thus more appropriately, we should make the notation changes at (2.12) and (2.13) with the consequent change in the schrödinger equation (2.8) recorded at (2.14).

$$\Psi_1(\mathbf{r}, t) \rightarrow \Psi_1(\mathbf{r}, t, n) \tag{2.12}$$

$$V(\mathbf{r}) \rightarrow V(\mathbf{r}, n) \tag{2.13}$$

$$i\hbar \frac{\partial}{\partial t}\Psi_1(\mathbf{r}, t, n) = -\frac{\hbar^2}{2m}\nabla^2\Psi_1(\mathbf{r}, t, n) + V(\mathbf{r}, n)\Psi_1(\mathbf{r}, t, n). \tag{2.14}$$

Thus we have, perhaps, the unusual quantum situation, that what might be called an augmented Laplace operator $\hat{V}(\mathbf{r})$, (2.9), has steady state eigen-functions which are solutions of an *eigen*-Schrödinger equation (2.14). The solutions $\Psi_1(\mathbf{r}, t, n)$ of the Schrödinger equation (2.14) can be used to give a space variable character to the spatially constant but epoch time variable solutions of the basic quantum solution, $\rho^{1/2}(t)$, of the dust universe model just by multiplication as below

$$\Psi(\mathbf{r}, t) = \Psi_1(\mathbf{r}, t, n)\Psi_{nl,\rho}(t), \tag{2.15}$$

where

$$\Psi_{nl,\rho}(t) = \rho^{1/2}(t) = A^{1/2}\sinh^{-1}(3ct/(2R_\Lambda)) \tag{2.16}$$

$$A = (3/(8\pi G))(c/R_\Lambda)^2 \tag{2.17}$$

$$R_\Lambda = (3/\Lambda)^{1/2}. \tag{2.18}$$

and

$$\frac{i\hbar\partial\Psi_{nl,\rho}(t)}{\partial t} = V_C(t)\Psi_{nl,\rho}(t) \tag{2.19}$$

$$V_C(t) = -(3i\hbar/2)H(t) \tag{2.20}$$

$$H(t) = (c/R_\Lambda)\coth(3ct/(2R_\Lambda)) \tag{2.21}$$

$$\rho(t) = (3/(8\pi G))(c/(R_\Lambda)^2\sinh^{-2}(3ct/(2R_\Lambda)). \tag{2.22}$$

$H(t)$ above is the epoch time variable Hubble *constant* and $\rho(t)$ is the epoch time variable substratum density both from the dust universe model. The objective of this work so far is to establish that the wave function $\Psi(\mathbf{r}, t)$ defined at (2.15) is the solution of a *cosmological Schrödinger* equation which might be described as a hybrid structure giving a theoretical mixture of general relativity and quantum theory from a new isothermal gas gravity self equilibrium theory. The cosmological Schrödinger equation takes the form,

$$\frac{i\hbar\partial\Psi(\mathbf{r},\mathbf{t})}{\partial t} \quad = \quad -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)+V(\mathbf{r},\mathbf{t})\Psi(\mathbf{r},t)+V_C(t)\Psi(\mathbf{r},t), \tag{2.23}$$

where $V_C(t)$, the feed back potential, is given by equation (2.20). There is a freedom to choose the numerical multiplier both in magnitude and dimensionality that goes along with the solutions of the Schrödinger equation (2.14) because of its linearity. This multiplier will be determined by the way the solutions are to be used. The intention here is to use these solutions to modulate with space variability the otherwise time only dependent substratum quantum solutions from general relativity. Because these wave functions are not complex the mass density solutions from the quantum Hermitian product is just the square of the wave function for the substratum from general relativity, $\rho(t)=\Psi_{nl,\rho}(t)\Psi^*_{nl,\rho}(t)\rightarrow\Psi^2_{nl,\rho}(t)$. This squared quantity has the built in dimensionality of mass per unit volume. Thus if the density solutions of the cosmological Schrodinger equation is to have the dimensions of mass per unit volume then the wave function $\Psi_1(\mathbf{r},t,n)$ used as a multiplier at (2.15) needs to be taken, initially at least, as dimensionless. Recalling the definition of this wave function at (2.2)

$$\Psi_1(\mathbf{r},t)=e^{-\frac{E(n)it}{\hbar}}\left(\frac{-2K}{\pi G(1-n)^2}\right)^{\frac{n}{2(n-1)}} r^{\frac{n}{1-n}} \tag{2.24}$$

it can be seen that a dimensionless version of this is easily obtained if it is written in the form

$$\Psi_1(\mathbf{r},t)=e^{-\frac{E(n)it}{\hbar}}\left(\frac{-2a}{\pi(1-n)^2}\right)^{\frac{n}{2(n-1)}} (r/r_0)^{\frac{n}{1-n}}, \tag{2.25}$$

where a is a dimensionless real number numerically equal to K/G is used to replaces the dimensioned quantity, K/G, and r_0 is a dimensioned length both determined by the physical context of application. Thus finally we can write out in full the solution for the cosmological Schrödinger equation

(2.23) associated with the isotropic index n as at (2.27) etc

$$\Psi(\mathbf{r},t,n) = \Psi_1(\mathbf{r},t,n)\Psi_{nl,\rho}(t) \quad (2.26)$$

$$= e^{-\frac{E(n)it}{\hbar}}\left(\frac{-2a}{\pi(1-n)^2}\right)^{\frac{n}{2(n-1)}}(r/r_0)^{\frac{n}{1-n}}\Psi_{nl,\rho}(t) \quad (2.27)$$

$$\Psi_{nl,\rho}(t) = \rho^{1/2}(t) = A^{1/2}\sinh^{-1}(3ct/(2R_\Lambda)) \quad (2.28)$$

$$A = \left(\frac{3}{8\pi G}\right)\left(\frac{c}{R_\Lambda}\right)^2 \quad (2.29)$$

$$R_\Lambda = (3/\Lambda)^{1/2}. \quad (2.30)$$

We should note that the mass density per unit volume solutions of the cosmological schrödinger equation are given by the usual Hermitian scalar product,

$$\rho_S(\mathbf{r},t,n) = \Psi(\mathbf{r},t,n)\Psi^\dagger(\mathbf{r},t,n) \quad (2.31)$$

$$= \left(\frac{-2a}{\pi(1-n)^2}\right)^{\frac{n}{n-1}}(r/r_0)^{\frac{2n}{1-n}}\Psi^2_{nl,\rho}(t). \quad (2.32)$$

From here on in this paper, the work will be carried through in terms of the integer quantization parameter, l, rather than in terms of the isotropic index, $n = 2l/(2l-1)$. The quantum number l will be placed as a subscript so that we have

$$\rho_{S,l}(\mathbf{r},t) = \rho_S(\mathbf{r},t,n(l)), \quad n(l) = \frac{2l}{2l-1}, \quad \Psi^2_{nl,\rho}(t) = \rho(t) \quad (2.33)$$

$$\rho_{S,l}(\mathbf{r},t) = \left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}(r/r_0)^{-4l}\Psi^2_{nl,\rho}(t) \quad (2.34)$$

$$= \rho_{1,l}(\mathbf{r})\rho(t) = \rho_{b,l}(\mathbf{r})(\rho(t)/\rho(t_b)) \quad (2.35)$$

$$\rho_{1,l}(\mathbf{r}) = \left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}(r/r_0)^{-4l} \quad (2.36)$$

$$\rho_{b,l}(\mathbf{r}) = \rho(t_b)\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}(r/r_0)^{-4l} \quad (2.37)$$

$$= \sigma_l(r_0)r^{-4l},\ say,\ with \quad (2.38)$$

$$\sigma_l(r_0) = \rho(t_b)\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} r_0^{4l}. \quad (2.39)$$

The formula at (2.34) above gives the density solutions, $\rho_{S,l}(\mathbf{r},t)$, of the cosmological schrödinger in terms of the *integer* quantization parameter l which is now placed as a subscript on the density.

The introduction of the constant $\rho(t_b)$ at that line is self cancelling so that the solution of the cosmological Schrödinger is not changed but in effect the two distinct differential equations are renormalized, if from now on, $\rho_{b,l}(\mathbf{r})$ is taken to be a density solution of the differential equation for $\Psi_1(\mathbf{r},t,n)$. The introduction of the self cancelling function at line (2.35) is important for the physical-philosophical interpretation of the solutions of the cosmological Schrödinger equation. By construction the solutions of this equation take the product form (2.35), one factor of this product is pure quantum mechanics and the other is pure cosmology from the dust universe model. However, as we have seen, without the $\rho(t_b)$, the solutions of the quantum part $\Psi_{l,1}(\mathbf{r})$ have to be dimensionless and so the Hermitian product form cannot represent a mass density. It can however, be regarded as a spatial modulation of the cosmological factor. My suggestion is that the product with or without the $\rho(t_b)$ factor represents two points of observational view. With the $\rho(t_b)$ the density solutions, $\rho_{b,l}(\mathbf{r})$, of the $\Psi_1(\mathbf{r},t,n)$ equation represent the view of an observer within and part of its quantum system with cosmology somewhat sidelined. Without the $\rho(t_b)$, the cosmological Schrödinger equation solutions represent the view of a general observer not particularly interested in any specific galaxy but being conscious that regions within galactic domains are spatially different from the substratum. With this philosophical slant on the meaning of the product solutions of the cosmological Schrödinger equation, the with $\rho(t_b)$ can be explained as follows. It is usually assumed that galaxies have been around for a very long time. Often it is suggested that the milky way is nearly as old as the universe itself. This seems to be a very reasonable idea and along with this idea it seems likely that a galaxy is a large amount of mass conserved within a not expanding volume. Thus galaxies seem to be objects of almost constant mass density over very large epoch times. Obviously a region of astro-space which accommodates a galaxy is greatly distinguished by its mass density from the substratum mass density in which it swims. Now although mass density may be conserved, if this mass density is mean or average mass density in the region of occupation, great changes or evolution of the local distribution of this mass within the galactic region is not precluded from taking place over time. We can envisage the beginning of a new galaxy as a birth process taking place at a definite epoch time, t_b, by a *quantum* process in which a spherical region of the substratum at the time t_b, when the substratum density is $\rho(t_b)$, stops expanding with the substratum by a spatially extended change of state. A region fractures from the substratum at time t_b to retain the mass and volume at its birth to follow its own evolution under the cyclic steady state factor (2.27). Thus for all following times the actual internal mass mean density will retain its birth value $\rho(t_b)$ whilst the environment substratum mass density outside the galactic region will at time t have assumed the much reduced evolved value $\rho(t)$ for $t > t_b$. Thus the birth of a galaxy can be regarded as a random centred and time determined quantum change of state process that effects spherical volumes of the substratum which then evolves scale wise independently of their environment except for their mass centroids which will move with the environment. This is what the wave function $\Psi_1(\mathbf{r}, t, n)$ describes. It is convenient at this point to introduce a useful conceptual radius associated with the birth of a galaxy. The structure is such that we know two physical characteristics involved with a galactic birth. Its mass M_l can be found from the theory given its quantum state l and its uniform mass density $\rho(t_b)$ equal to the substratum mass density at the moment of birth given by the assumed time of birth t_b. Thus we can define a conceptual spherical volume $V_b = \frac{M_l}{\rho(t_b)} = \frac{4\pi r_b^3}{3}$ and the conceptual radius r_b associated with the birth process. I shall interpret this *conceptual* radius as the radius of a sphere of visible material that suddenly appears at time t_b although it is not likely that there will have been any observers to see the creation event. However, this is not mass creation from nothing, it is a visible change of state of the pre-existing substratum mass. Thus I shall call r_b the *visibility* radius of the galaxy and as previously discussed this is a feature that stays with the galaxy for very many following years. This radius is theoretically important because the object that it represents is mathematically an infinitely radially extended material sphere. This can be the recognition of the recently substantiated conclusion that with galaxies what you see is only part of the story.

The n in the subscript nl above at (2.33) which is short for non-linear should not be confused with the isotropic index n. The version at (2.35) gives the density solution of the cosmological schrödinger in terms of the corresponding density solution of the related schrödinger equation (2.14) with (2.38) and (2.39) giving a convenient abbreviation for this function.

So far in this paper, the argument has been developed on the premiss that eigen-values for steady state energies for the mass density distributions (2.1) are given in the form of the function $E(n)$ of the isotropic index n. In the next section, I shall show that there is a natural function within the *quantized* isothermal theory for the dark matter galaxy halos that fits this bill.

3 Steady State Dark Matter Energies, E(n)

In reference, [50], I showed that in general relativity the total gravitationally *effective* mass within a sphere of radius r for a spherically extended source in an isotropic equilibrium state can be written as,

$$M_{GR}(r) \;=\; M^+(r) + M_P(r) - M_\Lambda(r) = \int_{r_\epsilon}^{r} \varrho_g(r')dr' + M_\epsilon. \tag{3.1}$$

The actual mass as opposed to effective mass within the same sphere is

$$M_{gr}(r) \;=\; M^+(r) + M_P(r) + M_\Lambda(r) \tag{3.2}$$

because all masses and mass densities are to be taken as positive. To avoid confusion, I am using the lower case subscript gr for actual mass. The effective mass $M_{GR}(r)$ expression above is a convenient abbreviation for

$$M_{GR}(r) \;=\; (G_+M^+(r) + G_+M_P(r) + G_-M_\Lambda)/G \tag{3.3}$$

$$G_+ \;=\; +G \tag{3.4}$$

$$G_- \;=\; -G. \tag{3.5}$$

The Newtonian gravitational potential at radius r from the centre of a distribution such as (2.34) above is given by

$$V_G(r) = \frac{M_{gr}(r)G}{r}. \tag{3.6}$$

Here, as indicated by lower case subscript gr, the mass should be the actual mass. In earlier versions of this paper, I mistakenly used the effective mass. This change has the consequence that further changes have been made in the following text. This includes the addition of a section (8) in this version of this paper entitled *Explanation of Corrections* in which my mistake is explained and is implications are discussed. If we are to use the potential (3.6) then $M^+(r)$ for example needs to be calculated from the formula,

$$\begin{aligned} M^+(r) &= \int_0^r \rho_{b,l}(\mathbf{r},t)4\pi r^2 dr. & (3.7)\\ &= \sigma_l(r_0)\int_0^r r^{-4l}4\pi r^2 dr & (3.8)\\ &= \sigma_l(r_0)\int_0^r r^{2-4l}4\pi dr & (3.9)\\ &= 4\pi\sigma_l(r_0)\left[\frac{r^{3-4l}}{3-4l}\right]_0^r. & (3.10)\end{aligned}$$

The $4\pi r^2$ factor in the first two integrals above converts the mass density per unit volume to mass per unit radius. The integer quantization parameter l can have the numerical values, $1,2,3,4\ldots\ldots\infty$. It follows that $3-4l$ is always negative.

$$3-4l < 0\ \forall l. \qquad (3.11)$$

Thus the upper value for r in (3.10) can be ∞ when $r^{3-4l}\rightarrow 0$ but at the lower limit when $r\rightarrow 0$, the lower value of r^{3-4l} diverges to ∞. It follows that the raw density functions cannot *comfortably* be used in calculations. In fact, nature comes to the rescue here with the factual existence of galactic cores. What seems to me to be the simplest assumption is to replace the densities $\rho_{b,l}(\mathbf{r})$ with a more physical realistic densities, $\rho_{b,l,\epsilon}(\mathbf{r})$, defined as follows

$$\begin{aligned} \rho_{b,l}(\mathbf{r}) &\rightarrow \rho_{b,l,\epsilon}(\mathbf{r}) = \rho_{b,l}(\mathbf{r}) \quad r\geq r_\epsilon & (3.12)\\ \rho_{b,l}(\mathbf{r}) &\rightarrow \rho_{b,l,\epsilon}(\mathbf{r}) = \rho_{b,l}(\mathbf{r}_\epsilon) \quad r< r_\epsilon & (3.13)\\ \lim_{r_\epsilon\rightarrow 0}\rho_{b,l,\epsilon}(\mathbf{r}) &= \rho_{b,l}(\mathbf{r}) \quad \forall l & (3.14)\end{aligned}$$

with the region within the radius r_ϵ being regarded as the galactic core and having the constant density $\rho_{b,l}(\mathbf{r}_\epsilon)$. The last equation above shows that this modification is reversible by taking the limit $r_\epsilon\rightarrow 0$. Thus for practical calculational purposes we can work with the always finite densities $\rho_{1,l,\epsilon}(\mathbf{r})$ and if needs be take the limit $r_\epsilon\rightarrow 0$ afterwards. However, I shall usually drop the ϵ subscript on these densities and only restore it, if it is really needed in context. Let us now return to calculating the effective mass within a sphere of radius r using the finite everywhere densities. Firstly consider the positively gravitating mass excluding the pressure generated part $M_P(r)$ calculated above,

$$\begin{aligned}
M^+(r) &= \int_0^r \rho_{b,l,\epsilon}(\mathbf{r},t)4\pi r^2 dr && (3.15)\\
&= \sigma_l(r_0)\int_0^{r_\epsilon} {r_\epsilon}^{-4l}4\pi r^2 dr + \sigma_l(r_0)\int_{r_\epsilon}^r r^{-4l}4\pi r^2 dr && (3.16)\\
&= \sigma_l(r_0)\int_0^{r_\epsilon} r_\epsilon^{-4l} r^2 4\pi dr + \sigma_l(r_0)\int_{r_\epsilon}^r r^{-4l}4\pi r^2 dr && (3.17)\\
&= \sigma_l(r_0) r_\epsilon^{-4l}\left[\frac{r^3}{3}\right]_0^{r_\epsilon} 4\pi + 4\pi\sigma_l(r_0)\left[\frac{r^{3-4l}}{3-4l}\right]_{r_\epsilon}^r && (3.18)\\
&= 4\pi\sigma_l(r_0)\left(r_\epsilon^{-4l}\frac{r_\epsilon^3}{3} + \frac{r^{3-4l}}{3-4l} - \frac{r_\epsilon^{3-4l}}{3-4l}\right) && (3.19)\\
&= 4\pi r_0^{4l}\rho(t_b)\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(r_\epsilon^{-4l}\frac{r_\epsilon^3}{3} + \frac{r^{3-4l}}{3-4l} - \frac{r_\epsilon^{3-4l}}{3-4l}\right)\\
&= 4\pi r_0^{4l}\rho(t_b)\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(\frac{r^{3-4l}}{3-4l} - \frac{4l r_\epsilon^{3-4l}}{3(3-4l)}\right) && (3.20)\\
&= A_l\left(\frac{4l r_\epsilon^{3-4l}}{3} - r^{3-4l}\right),\ say && (3.21)\\
A_l &= \frac{4\pi r_0^{4l}\rho(t_b)}{4l-3}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}. && (3.22)
\end{aligned}$$

According to construction here the mass of the core M_ϵ should be given by $r = r_\epsilon$ in equation (3.15) and inspection of (3.19) shows that the core mass is

$$M_\epsilon = M^+(r_\epsilon) = 4\pi\sigma_l(r_0)\frac{r_\epsilon^{3-4l}}{3} = A_l\frac{r_\epsilon^{3-4l}}{3}. \qquad (3.23)$$

Using $s(t)$ to denote the function $\sinh^{-2}(3ct/(2R_\Lambda))$, the total mass of this type is

$$M^+(\infty) = 4\pi r_0^{4l}\rho(t_b)\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(\frac{4lr_\epsilon^{3-4l}}{3(4l-3)}\right) = A_l\left(\frac{4lr_\epsilon^{3-4l}}{3}\right) \tag{3.24}$$

$$= \frac{12\pi r_0^{4l}s(t_b)}{8\pi G}\left(\frac{c}{R_\Lambda}\right)^2\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(\frac{4lr_\epsilon^{3-4l}}{3(4l-3)}\right). \tag{3.25}$$

Thus the ratio of total mass of this type to core mass is

$$\frac{M^+(\infty)}{M^+(r_\epsilon)} = \frac{4l}{4l-3}. \tag{3.26}$$

Formula (3.25) can be regarded a giving the total mass $M_g(r_\epsilon, r_0, l) = M^+(\infty)$ of this type of a galaxy represented as having values for its parameters given by (r_ϵ, r_0), if additionally it is in the gravitational equilibrium quantum state given by the integer, l. Other parameters used in this formula have approximate known numerical values. The parameter a can be taken to be just the non *dimensioned numerical* value of the dimensioned ratio R/G from isotropic gravitation theory. Let us now consider the positively gravitation mass arising from Einstein's pressure term

$$
\begin{aligned}
M_P(r) &= \int_0^r \frac{3P(r')4\pi r'^2}{c^2}dr' = \int_0^r 3K_\prime \varrho^{\frac{4l-1}{2l}}(r')4\pi r'^2 dr' && (3.27)\\
&= \int_0^r 3K_\prime(\rho_{1,l}(\mathbf{r}'))^{\frac{4l-1}{2l}} 4\pi r'^2 dr' \\
&= \int_0^r 3K_\prime \left(\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(\frac{r'}{r_0}\right)^{-4l}\right)^{\frac{4l-1}{2l}} 4\pi r'^2 dr' && (3.28)\\
&= \frac{12\pi K_\prime}{r_0^{2-8l}}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}\int_0^r (r')^{4-8l}\,dr' && (3.29)\\
&= \frac{12\pi K_\prime}{r_0^{2-8l}}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}\left(\frac{r_\epsilon^{2-8l}r_\epsilon^3}{3}+\frac{r^{5-8l}}{5-8l}-\frac{r_\epsilon^{5-8l}}{5-8l}\right)\\
&= \frac{12\pi K_\prime}{r_0^{2-8l}(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3}-r^{5-8l}\right)\\
&= B_l\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3}-r^{5-8l}\right),\ \mathit{say} && (3.30)\\
B_l &= \frac{12\pi K_\prime r_0^{8l-2}}{(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1} && (3.31)\\
P(r) &= c^2K_\prime\rho_{1,l,\epsilon}(\mathbf{r})^{\frac{4l-1}{2l}}. && (3.32)
\end{aligned}
$$

The last equation above is the Lane-Emden type polytropic gas equation used above in a form most suitable for use with this work in terms of the quantum number l. $K_\prime$ is a constant with dimensions of mass per unit volume and $\rho_{1,l,\epsilon}(\mathbf{r})^{\frac{4l-1}{2l}}$ is the dimensionless mass density defined at (2.35) and (2.36) and core modified, see (3.12) etc. The negatively gravitating mass $M_\Lambda(r)$ within a sphere of radius r and volume $4\pi r^3/3$ is the easiest term to obtain. It is

$$
\begin{aligned}
M_\Lambda(r) &= \frac{4\pi r^3}{3}(3/(4\pi G))(c/R_\Lambda)^2 = \frac{c^2\Lambda r^3}{3G} && (3.33)\\
&= C_l r^3,\ \mathit{say} && (3.34)\\
C_l &= \frac{c^2\Lambda}{3G}. && (3.35)
\end{aligned}
$$

That is to say $M_\Lambda(r)$ is the volume times twice Einstein's dark energy density term,

$$(3/(8\pi G))(c/R_\Lambda)^2 = \frac{c^2\Lambda}{8\pi G}. \tag{3.36}$$

Thus the total gravitationally *effective* mass within a spherical volume is given by the sum of the three components M^+, M_P and $-M_\Lambda$,

$$\begin{aligned} M_{GR}(r) &= M^+(r) + M_P(r) - M_\Lambda(r) && (3.37)\\ &= A_l\left(\frac{4lr_\epsilon^{3-4l}}{3} - r^{3-4l}\right) + B_l\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3} - r^{5-8l}\right) - C_l r^3 && (3.38)\\ A_l(r_0) &= \frac{4\pi r_0^{4l}\rho(t_b)}{4l-3}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} = \frac{r_0^{4l}s(t_b)c^2\Lambda}{2G(4l-3)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} && (3.39)\\ B_l(r_0) &= \frac{12\pi K_\prime r_0^{8l-2}}{(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}\\ &= \frac{3r_0^{8l-2}s(t_b)c^2\Lambda}{2G(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1} && (3.40)\\ C_l &= \frac{c^2\Lambda}{3G}. && (3.41) \end{aligned}$$

From (3.38), we can find the total core mass is given by,

$$\begin{aligned} M_{GR}(r_\epsilon) &= A_l\left(\frac{4lr_\epsilon^{3-4l}}{3} - r_\epsilon^{3-4l}\right) + B_l\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3} - r_\epsilon^{5-8l}\right) - C_l r_\epsilon^3\\ &= A_l\left(\frac{4l}{3} - 1\right) r_\epsilon^{3-4l} + B_l\left(\frac{(8l-2)}{3} - 1\right) r_\epsilon^{5-8l} - C_l r_\epsilon^3 && (3.42)\\ &= A_l\left(\frac{4l-3}{3}\right) r_\epsilon^{3-4l} + B_l\left(\frac{(8l-5)}{3}\right) r_\epsilon^{5-8l} - C_l r_\epsilon^3. && (3.43) \end{aligned}$$

It follows that

$$\begin{aligned}
M_{GR}(r) - M_{GR}(r_\epsilon) &= A_l\left(r_\epsilon^{3-4l} - r^{3-4l}\right) + B_l\left(r_\epsilon^{5-8l} - r^{5-8l}\right) \\
&\quad -C_l(r^3 - r_\epsilon^3) \qquad (3.44)\\
&= M_{GR,\epsilon} + A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) - C_l(r^3) \qquad (3.45)\\
M_{GR,\epsilon} &= A_l\left(r_\epsilon^{3-4l}\right) + B_l\left(r_\epsilon^{5-8l}\right) - C_l(-r_\epsilon^3) \qquad (3.46)\\
M_{GR}(r) &= M_{GR,\epsilon'} + A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) - C_l(r^3) \qquad (3.47)\\
M^+_{GR}(r) &= M_{GR,\epsilon'} + A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) \qquad (3.48)\\
M_{GR,\epsilon'} &= M_{GR,\epsilon} + M_{GR}(r_\epsilon) \qquad (3.49)\\
M^+_{GR}(r) &= M_{GR}(r) + C_l(r^3). \qquad (3.50)
\end{aligned}$$

Formula (3.48) is a key result for this section, in a suitably simplified form, which can be used to find, the quantum steady state energy values E_l and which I also intend to try out for the special case quantum state $l = 8$ as a generator of galactic rotation curves. The reason for this choice of special case will be explained later. This involves evaluating the coefficients having given the free parameters specific values. This will be carried through in the next subsection after firstly dealing with the steady state energies issue. From formula (3.48) we can obtain the total *positively* gravitating mass M_l associated with each quantum state l by taking the limit $r \to \infty$ with the result

$$M_l = M^+_{GR}(\infty) = M_{GR,\epsilon'} = M_{GR,\epsilon} + M_{GR}(r_\epsilon) \qquad (3.51)$$

as both $3 - 4l$ and $5 - 8l$ are negative. To obtain this result the *negatively* gravitating dark energy mass involved in the term $-C_l(r^3)$ has to be excluded as at (3.48). In more detail

$$\begin{aligned}
M_l = M^+_{GR}(\infty) &= A_l\left(r_\epsilon^{3-4l}\right) + B_l\left(r_\epsilon^{5-8l}\right) - C_l(-r_\epsilon^3) + \\
& \quad A_l\left(\frac{4lr_\epsilon^{3-4l}}{3} - r_\epsilon^{3-4l}\right) + \\
& \quad B_l\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3} - r_\epsilon^{5-8l}\right) - C_l r_\epsilon^3 \qquad (3.52) \\
&= \frac{r_0^{4l}s(t_b)c^2\Lambda}{2G(4l-3)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}\left(\frac{4lr_\epsilon^{3-4l}}{3}\right) + \\
& \quad \frac{3r_0^{8l-2}s(t_b)c^2\Lambda}{2G(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3}\right) \qquad (3.53) \\
&= \frac{r_0^{4l}s(t_b)M_G 2lr_\epsilon^{3-4l}}{R_\Lambda^3(4l-3)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} + \\
& \quad \frac{3r_0^{8l-2}s(t_b)M_G r_\epsilon^{5-8l}(4l-1)}{R_\Lambda^3(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}. \qquad (3.54)
\end{aligned}$$

It is interesting to consider the meaning of this last formula under the factored dimensioned decomposition of the gravitational constant, G, as in the last two lines above

$$G = M_G^{-1}R_\Lambda^3(R_\Lambda/c)^{-2}, \qquad (3.55)$$

$$\rightarrow \frac{c^2\Lambda}{G} = \frac{3M_G}{R_\Lambda^3} \qquad (3.56)$$

where R_Λ is the *de Sitter* radius, and which essentially defines a mass M_G [53] and if we represent the total mass associated with a galaxy in a quantum state l and as defined by the parametric values r_ϵ and r_0 as $M_{g,l}(r_\epsilon, r_0)$, then

$$\begin{aligned}
M_{GR}(\infty) &\rightarrow M_{g,l}(r_\epsilon, r_0) \\
M_{g,l}(r_\epsilon, r_0) &= \frac{r_0^{4l} s(t_b) M_G 2l r_\epsilon^{3-4l}}{R_\Lambda^3(4l-3)} \left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} + \\
&\quad \frac{3r_0^{8l-2} s(t_b) M_G r_\epsilon^{5-8l}(4l-1)}{R_\Lambda^3(8l-5)} \left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1} \qquad (3.57) \\
N_l^{-1}(r_0, r_\epsilon) &= \frac{M_{g,l}(r_\epsilon, r_0)}{M_G} \\
&= \frac{r_0^{4l} s(t_b) 2l r_\epsilon^{3-4l}}{R_\Lambda^3(4l-3)} \left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} + \\
&\quad \frac{3r_0^{8l-2} s(t_b) r_\epsilon^{5-8l}(4l-1)}{R_\Lambda^3(8l-5)} \left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1}, \qquad (3.58)
\end{aligned}$$

where $N_l(r_0, r_\epsilon)$ is the number of galaxies in quantum state l that would be needed to form a universe of total mass M_G.

The objective of this section was to find the steady state energies

$$E_l = E(n(l)) \tag{3.59}$$

to be associated with the sub-factor density solutions of the cosmological Schödinger equation represented as functions of the quantization integer l. From the above discussion, after taking into account the more detailed specification of the solutions by r_ϵ and r_0, a good choice seems to be

$$E_l(r_\epsilon, r_0) = M_{g,l}(r_\epsilon, r_0)c^2. \tag{3.60}$$

The mass M_G from the decomposition of the gravitational constant has an approximate value $2.00789 \times 10^{53} kg$ which is close to estimates of the total mass of the universe that have been made in recent years. This actual *theoretical* value is a possible candidate for an *exact* value for the mass of the universe. Thus the formula (3.57) gives a quantized relation between a possible mass for the universe M_G and how that as a total mass can be additively built from a number N_l, (3.58), of galactic masses of specific type, quantum number l and of parametric form determined by the values given to r_0, r_ϵ. I shall examine this rather unexpected relation between the possible large mass of the universe and galactic sub-masses in the next section.

4 Galactic Masses relation to Universe Mass

I have shown in reference ([53]) that, if it is assumed that the total mass of the universe is given by M_G, then the de Sitter radius R_Λ is the radius of the universe at time, t_c, when the acceleration of the expansion of the universe was exactly zero. The epoch time t_c is much in the past and very roughly about half the age of the universe now. The radius of the universe *now* is also very roughly twice the de Sitter radius R_Λ. It is obvious that the radii of the galactic cores will be many orders of magnitude less than the radius of the universe now and therefore also many orders of magnitude less than R_Λ. In practice, appropriate values for the adjustable constants (r_0, r_ϵ, K) and a may be obtained from the physical context. G, of course, is well known and tabulated by CODATA.

In the last two sections a relation between total mass of the universe, if taken to be M_G, and a possible set of constituent quantum number described galactic masses is given by (3.58). From this relation the number N_l of such constituent masses involved, if all in the same quantum state, is given by (3.58). Of course, the type of galaxy involved in the actual universe from the usual or quantum point of view ranges over many different forms or quantum states. However, using this theory formulation we can raise the idea of spatially uniform cosmology to a new superior level of galactic identity uniformity, a collection of galaxies all with the same quantum number l. In the next section I shall examine an *internal to dark matter* quantized version of Newton's law of gravitation.

5 Quantized Newtonian Law of Gravitation

We note the decomposition of $M_l(r)$ and $M'_l(r)$ into positive and negative parts,

$$\begin{aligned}
M_l(r) &= M_{l+} + A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) - C_l r^3 && (5.1)\\
&= M_{l+} + M_{l-}(r),\ say, && (5.2)\\
M_{l-}(r) &= A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) - C_l(r^3) && (5.3)\\
M'_l(r) &= M_{l+} + A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) + C_l r^3 && (5.4)\\
&= M'_{l+}(r) + M'_{l-}(r),\ say, && (5.5)\\
M'_{l-}(r) &= A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) && (5.6)\\
M'_{l+}(r) &= M_{l+}(r) + C_l r^3. && (5.7)
\end{aligned}$$

The last formulae with the primes on the Ms is the actual rather than effective mass version. I am here using primes instead of the equivalent gr subscripts to maintain simplicity of notation. Given the detailed formula for $M_l(r)$ (5.1), the total amount of mass within spheres of radius r, the Newtonian gravitational potential felt at radius r can be written down as,

$$V_l(r) = \frac{M'_l(r)G}{r} = \frac{M'_{l+}G}{r} + \frac{M'_{l-}G}{r}. \tag{5.8}$$

The prime on the M_{l-} is here being used to indicate that the actual mass version is being used in this definition rather than the effective mass version. If we now substitute $M'_{l+}(r)$ from equation (5.7) into the above equation we get

$$\begin{aligned}
V_l(r) = \frac{M'_l(r)G}{r} &= \frac{(M_{l+}(r) + C_l r^3)G}{r} + \frac{M'_{l-}G}{r} && (5.9)\\
&= \frac{(M_{l+}(r) + C_l r^3)G}{r} + \frac{M'_{l-}G}{r}. && (5.10)
\end{aligned}$$

The emergent feature here is that the Newtonian gravitational constant G appears, as usual, here in the numerator with the Ms but an inspection of lines (3.39), (3.40) and (3.41) show that it also appears in the denominator of the same formulae. Thus it cancels out and makes no contribution to the *dark matter* gravitational potential. On first encounter, this seems a very startling result. However, in can be explained within the structure of the dust universe model as follows. All the mass functions at line (5.1) have an initial coefficient, (5.11), which becomes, they having been multiplied by G, an initial coefficient, (5.12) and (5.13), for the gravitational potential terms at line (5.8).

$$\begin{aligned} & 4\pi r_0^{4l}(3/(8\pi G))(c/R_\Lambda)^2 & & (5.11) \\ \rightarrow\ & 4\pi r_0^{4l}(3G/(8\pi G))(c/R_\Lambda)^2 = 4\pi r_0^{4l}(3/(8\pi))(c/R_\Lambda)^2 & & (5.12) \\ & = r_0^{4l}(c^2\Lambda/2). & & (5.13) \end{aligned}$$

The result (5.13) follows from the definition of R_Λ in terms of Λ. Thus according to (5.13) the usual gravitational coupling constant G as a multiplier effectively converts to the cosmological constant Λ as the coupling constant for describing the distant gravitational effect of dark matter. Hence the gravitational field for dark matter is not quite the usual Newtonian result in spite of the fact that it looks superficially identical to it. On reflection this is not surprising as in this theory *dark matter* density, $\rho(t)$, the dominant type of matter in the universe before dark energy at the present epoch, appears as *some sort* of time variable *disturbance*, $\sinh^{-2}(3ct/(2R_\Lambda))$, of the *dark energy* space time constant density field, $(3/(8\pi G))(c/(R_\Lambda)^2$, by the formula

$$\begin{aligned} \rho(t) &= (3/(8\pi G))(c/(R_\Lambda)^2 \sinh^{-2}(3ct/(2R_\Lambda)) & (5.14) \\ &= M_G/V_U(t). & (5.15) \end{aligned}$$

The last formula being valid if the mass of the universe $M_U = M_G$ and $V_U(t)$ is the volume of the universe at epoch time t. The constant density

$$(3/(8\pi G))(c/(R_\Lambda)^2 = (\Lambda c^2/(8\pi G)) \qquad (5.16)$$

is Einstein's *dark energy* density introduced to explain his mathematical cosmological constant Λ as being due to an actual *physical* dark energy mass density but appearing here as the basis of all energy density, particularly *dark matter* density. From lines (5.14) and (5.15), it is very clear that the time variable disturbance I referred to earlier is no mystery at all in this cosmological model. The disturbance is just a consequence of the fact that a constant amount of dark matter, M_G, remains within the expanding with epoch time volume of the universe $V_U(t)$. Consequently, the mass *density* of the universe decreases with time and it is this process that accounts for the $\sinh^{-2}(3ct/(2R_\Lambda))$ factor at (5.14). However, the point I wish to emphasise is that the constant amount of *dark matter* within the expanding spherical universe is intimately related by (5.14) to Einstein's universally constant *dark energy density*, a constancy and existence for which that extends to outside the expanding spherical sphere of the universe. Dark energy is hyper-universal. The remarks above are perhaps digressional, but the point I am making about the Newtonian gravitational potential within the dark matter halos discussed above is that it differs *fundamentally* from the usual gravitational Newtonian potential in that its source is the disturbed density for *dark energy*. Notably, the gravitational potential for dark matter (5.8) is, as has been shown above, distinctly not of the usual G coupled Newtonian type. Further more, it is quantized with a quantum state number l associated with the mass sources involved being constructed from solutions of a cosmological Schrödinger equation for dark matter halo wave functions. A *quantization* of gravity is here coming from the cosmological constant, Λ. This remark is reinforced by the unexpected result of the decomposition of the *total* gravitation mass M_l for a galaxy into the two parts M_{l+} and M_{l-} at (5.2). The positive part M_{l+} within the local gravitational potential generates the usual attraction to the centre Newtonian field whilst the negative part generates negative gravity repulsion. This is additional to the repulsive field arising directly from Einstein's cosmological constant. Thus negatively gravitating material is greatly involved with *dark matter* additionally to its involvement arising from the existence of Λ. The explanation for the existence of the negative term in the gravitating mass source or its gravitating potential is very clear from the mathematics and the way it has been obtained from this theory. The formula for the gravitating mass $M(r)$ within a sphere of radius r is basic to this work. However, the total object mass involved $M(\infty)$ extends to infinite distance. Thus the mass outside a sphere of radius r centered on a galaxy, $M_{out}(r)$, is

$$M_{out}(r) = M(\infty) - M(r) = M_{l+} - M_l(r) = -M_{l-}(r) > 0, \tag{5.17}$$

by (5.2). Thus the positive mass $-M_{l-}(r)$ outside the sphere of radius r on which the galaxy is centred contributes negatively to the gravitational potential at r. Thus clearly in this case the negative mass within the sphere is fictional and simply is a reflection of actual positive mass outside the sphere. This result at (5.17) can be seen to be an explanation for the essential need for a cosmological constant in the Einstein field equations by the following considerations. There are obviously very large numbers of galaxies spread throughout the cosmos. According to this theory each galaxy is an infinitely radially extended structure. Hence if we consider any location distance r from the centre of a galaxy at such a position its gravitation potential will be felt included in which will be the influence from its mass outside radius r. This location will also be usually outside all the other infinitely extended galaxies in the universe. Thus all the rest of the galaxies will make a resultant but usually very small negative gravitation contribution. This small negative gravity component is supplied by the Einstein additional ρ_Λ input density contribution in the form of an extra a component of the stress energy momentum tensor so that in this case this mass within the sphere is actually negatively gravitational and not a reflection from actual positively gravitating mass outside the sphere. It seems that Einstein's field equations without Λ only describe a local object in isolation from the rest of the universe. Thus negatively gravitating material is not quite so weird as it has seemed for the past decade since Λ was reinstated. Negative gravitation is just a collective very small attraction felt at any point towards all the rest of the rest of universe but usually masked by the existence of local positively gravitating material towards that point and a directional uniformity. However, this issue is philosophically deep and complicated and I intend to discuss it in more detail in future publications.

5.1 Galactic Rotation Curves

In order to study the galactic rotation curves as a function of r generated by a mass function of r such as (5.1),(5.2)and (5.3) we can look at the simplest case and also use a reasonable set of values for the free parameters r_0, r_ϵ. We need the values,

$$\Lambda = 1.35 \times 10^{-52} \tag{5.18}$$

$$c = 299792458. \tag{5.19}$$

$$\begin{aligned}
A_l(r_0) &= \frac{r_0^{4l}s(t_b)c^2\Lambda}{2G(4l-3)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l} = \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2G(4l-3)} \qquad (5.20)\\
B_l(r_0) &= \frac{3r_0^{8l-2}s(t_b)c^2\Lambda}{2G(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1} = \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2G(8l-5)} \qquad (5.21)\\
C_l &= \frac{c^2\Lambda}{3G} \to \frac{299792458^2\times 1.35\times 10^{-52}}{3G} = \frac{4.0444\times 10^{-36}}{G}. \qquad (5.22)\\
M_{l+} &= M_{GR,\epsilon} + M_{GR}(r_\epsilon) = \\
&\quad A_l\left(r_\epsilon^{3-4l}\right) + B_l\left(r_\epsilon^{5-8l}\right) - C_l(-r_\epsilon^3) + \\
&\quad A_l\left(\frac{4l-3}{3}\right)r_\epsilon^{3-4l} + B_l\left(\frac{(8l-5)}{3}\right)r_\epsilon^{5-8l} - C_l r_\epsilon^3. \qquad (5.23)
\end{aligned}$$

The quantity $\beta = \frac{2ar_0^2}{\pi}$ introduced above at (5.20) has the dimensions length squared, m^2, is a useful simplifier as it is arbitrary because r_0 is arbitrary it can be given the value unity when convenient. Thus largely we can ignore r_0 and sideline a. All the mass contributions combined for the quantum state l are given at (5.25)

$$\begin{aligned}
M_l(r) &= M^+(r) + M_P(r) - M_\Lambda(r) \qquad (5.24)\\
&= A_l\left(\frac{4lr_\epsilon^{3-4l}}{3} - r^{3-4l}\right) + B_l\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3} - r^{5-8l}\right) - C_l r^3. \qquad (5.25)
\end{aligned}$$

In more detail, we have

$$\begin{aligned}
M_l(r) &= \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2G(4l-3)}\left(\frac{4lr_\epsilon^{3-4l}}{3} - r^{3-4l}\right) + \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2G(8l-5)}\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3} - r^{5-8l}\right) - \frac{c^2\Lambda}{3G}r^3. \qquad (5.26)
\end{aligned}$$

This can be separated into positive, $M_{l+}(r)$, and negative, $M_{l-}(r)$, signed terms as follows

$$\begin{aligned}
M_{l+}(r) &= \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2G(4l-3)}\left(\frac{4lr_\epsilon^{3-4l}}{3}\right)+ \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2G(8l-5)}\left(\frac{r_\epsilon^{5-8l}(8l-2)}{3}\right)= \\
M_{l+} &= \textit{a constant with respect to } r \textit{ variation.} \qquad (5.27)
\end{aligned}$$

$$\begin{aligned}
M_{l-}(r) &= \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2G(4l-3)}\left(-r^{3-4l}\right)+ \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2G(8l-5)}\left(-r^{5-8l}\right)-\frac{c^2\Lambda}{3G}r^3 \qquad (5.28)\\
M_{l-,0}(r) &= \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2G(4l-3)}\left(-r^{3-4l}\right)+ \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2G(8l-5)}\left(-r^{5-8l}\right), \qquad (5.29)
\end{aligned}$$

the last version above not including Einstein's dark energy term. Thus we have

$$\begin{aligned}
M_l(r) &= M_{l+}+M_{l-}(r) \qquad (5.30)\\
M_{l,0}(r) &= M_{l+}+M_{l-,0}(r) \qquad (5.31)
\end{aligned}$$

and, using the actual masses case, the Newtonian gravitational potential at radius r is

$$\begin{aligned}
V_l(r) &= \frac{M_{l+}G}{r}+\frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2(4l-3)}\left(-r^{2-4l}\right)+ \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2(8l-5)}\left(-r^{4-8l}\right)+\frac{c^2\Lambda}{3}r^2. \qquad (5.32)
\end{aligned}$$

It follows that the galactic rotation curves given as transverse velocity squared as a function of r have the equation

$$\begin{aligned}
v_l^2(r) &= \frac{M_{l+}G}{r}+\frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}}{2(4l-3)}\left(-r^{2-4l}\right)+ \\
&\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}}{2(8l-5)}\left(-r^{4-8l}\right)+\frac{c^2\Lambda}{3}r^2. \qquad (5.33)
\end{aligned}$$

The gradients of these curves with respect to r are

$$\begin{aligned}\frac{\partial v_l^2(r)}{\partial r} &= -\frac{M_{l+}G}{r^2} + \frac{c^2\Lambda s(t_b)\beta^{2l}(2l-1)^{4l}(2-4l)}{2(4l-3)}\left(-r^{1-4l}\right) + \\ &\quad \frac{3c^2\Lambda s(t_b)\beta^{4l-1}(2l-1)^{8l-2}(4-8l)}{2(8l-5)}\left(-r^{3-8l}\right) + \frac{c^2\Lambda 2}{3}r \qquad (5.34)\end{aligned}$$

5.2 Galactic Curves for a Small Galaxy

I have decided to check out the galactic curve kinematics that this theory delivers for a small galaxy which will be identified below. However, it was initially and in fact still remains unclear how to identify galaxies within the quantum set off galaxies derivable from this new theory. Thus to get going with the use of this theory some trial and error was required which I will now briefly explain. The theory can deliver an infinite discrete set of quantized mass values. However, the actual numerical values involved with this set is determined by the free input parameters which are r_ϵ, β and t_b. It seems to me that these three parameters can take on arbitrary values. However, it is desirable that physically reasonable values are chosen. If one takes the view, among other trial possibilities, that we use a set of quantum states determined by the quantum parameter $l = 1, 2, 3 \ldots 9$ associated with some definite value, to be explained later, of $r_\epsilon = 1.3213133\ m$, in meters say, $\beta = 1\ m^2$ in meters squared, with t_c the approximate epoch time when the universe has zero radial acceleration,

$$\begin{aligned} t_c &= \frac{2R_\lambda \sinh^{-1}(2^{-1/2})}{3c} &(5.35)\\ &= 2.18285 \times 10^{17}\ s, &(5.36)\end{aligned}$$

the function M_{l+} at (5.27) generates in kilograms the nine values,

$$\begin{aligned}&3.4114 \times 10^{-25},\ 5.18055 \times 10^{-20},\ 2.52104 \times 10^{-12},\\ &0.00246163 \times 10^{0},\ 2.09608 \times 10^{7},\ 9.76751 \times 10^{17},\\ &1.84565 \times 10^{29},\ 1.14693 \times 10^{41},\ 2.00789 \times 10^{53}. \qquad (5.37)\end{aligned}$$

I have only taken values for the quantum number up to $l = 9$ because the masses generated beyond 9 with this value for r_ϵ are substantially greater than the usual ideas of what the mass of the universe is likely to be. The last mass displayed above for $l = 9$ coincides with the value of M_G, the value that could be taken to be the mass of the universe. This last value was deliberately achieved by choosing $r_\epsilon = 1.3213133$. The incredibly wide range of mass values generated for ranges of the integer quantum number l is to my mind very striking. The rest mass of the top quark is 3.11966×10^{-25} and the rest mass of the Higgs boson is thought to be approximately 2.22833×10^{-25} kilograms so that all the masses in this range are astro-physically interesting. 0bjects in the range 10^{-12} to 10^{29} above are very common and could include large molecules through planets to objects as heavy as small stars and lastly, the last but one entry above 10^{41}, could be a small galaxy in relation to the milky which probably has a mass of about 10^{42} kilograms. With a different values of r_ϵ the usually assumed value of the milky way can be obtained but my choice of r_ϵ was to include exactly the mass M_G in the hope that the other masses generated would somehow acquire special significance from its inclusion. Clearly the route forward is uncertain and deserving of much more investigation. I have examined the galactic rotation curves of the small galaxy above, quantum state $l = 8$, with the formula for velocity gradient with the formula (5.38) below that can be obtained in detail using (5.33) and (5.34).

$$\frac{\partial v(r)}{\partial r} = \frac{\partial v^2(r)}{2v(r)\partial r}. \tag{5.38}$$

The following list of values of tangential rotation velocity curve gradients just before, $0.8r_{SM}$, the visibility boundary at $r = r_c = r_{SM}$ and then extending out further to $1.2r_{SM}$, the results are shown below,

$$\begin{array}{rl} r\ in\ meters & Gradient\ of\ v(r) \\ 0.8 \times 1.24 \times 10^{22} & -5.04 \times 10^{-23} \\ 0.9 \times 1.24 \times 10^{22} & -4.48 \times 10^{-23} \\ r_{SM} = 1.0 \times 1.24 \times 10^{22} & -4.032 \times 10^{-23} \\ 1.1 \times 1.24 \times 10^{22} & -3.67 \times 10^{-23} \\ 1.2 \times 1.24 \times 10^{22} & -3.36 \times 10^{-23}. \end{array} \tag{5.39}$$

From the second list above, these curves are decisively flat.

6 Stability of Dark Matter Galactic Halos

There are some deep and complicated issues about the stability of particle distributions assembled under the mutual Newtonian gravitation attraction of the component particles ([54]). This problem does not impact on the theory for the dark matter galactic halos discussed in this paper, as I shall now explain. It is widely believed nowadays that the *missing* matter referred to as *dark matter* exists within a spherical halo that engulfs the visible parts of a galaxy and usually extends greatly beyond the visible parts. I think there is very little evidence that this assumption is correct but it does seem to be a plausible working assumption. Thus let us assume that this view of the situation is correct then it also seems likely that the dark matter is not necessarily rotating with the galaxy for otherwise it would be flattened and not spherical as is usually its parent visible galaxy. Further, remaining spherical and not having clumped into a flattened form in its evolution suggests its compulsive dynamics is not usual. If it is in fact spherical and not rotating then what keeps it in its extended state? The obvious answer to this question is that it is a gaseous structure and in equilibrium caused by an outwards pressure from the gas and an inwards pull from gravitation effectively from its center. The fact that such equilibrium conditions can likely occur at some time, using Newtonian gravitation theory with gas dynamics theory, the forms of possible equilibrium mass densities can be found. However, such solutions are not necessarily time wise stable any more than there is necessarily zero motion when acceleration is zero in general. An absolutely static structure is clearly not appropriate for the description of a galaxy as complex rotational motion is in fact observed. The main static aspect is the requirement that there should be no *overall radial* motion and that the galaxy should not be expanding with the substratum. Clearly, the idea of *steady state* motion in the quantum context is just right for galaxy description. In quantum theory, systems with very complex internal motions are successfully described under this tag and usually such systems have quantum state numbers attached to a range of discrete quantum states. However, such quantum systems play out their motion under the influence of some central potential energy, such as the coulomb potential for example. The quantized dark matter densities with integer quantum number l that I found from isothermal gravitational equilibrium theory are just crying out to be Schrödinger densities formed from the Hermitian scalar product from Schrödinger wave functions. I have shown above and elsewhere [52] that the Schrödnger equation needed in this context involves a non-gravitational potential, $V_1(\mathbf{r})$ if it is to play the part of supplying steady state solutions that space wise coincide exactly with the solutions from the new isothermal equilibrium theory. However, each solution $\Psi_1(\mathbf{r},\mathbf{t})$ has its own Schrödinger equation and potential function $V_l(\mathbf{r})$ as given below in terms of the quantum integer l instead of the isotropic index $n(l)$.

$$\begin{aligned}
\Psi_{1,l}(\mathbf{r},t) &= e^{-\frac{E_l(r_\epsilon,r_0)it}{\hbar}}\left(\frac{-2a(1-2l)^2}{\pi}\right)^l (r/r_0)^{-2l} &(6.1)\\
E_l(r_\epsilon,r_0) &= M_{g,l}(r_\epsilon,r_0)c^2 &(6.2)\\
M_{g,l}(r_\epsilon,r_0) &= \frac{r_0^{4l}s(t_b)M_G 2lr_\epsilon^{3-4l}}{R_\Lambda^3(4l-3)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{2l}+ \\
&\quad \frac{3r_0^{8l-2}s(t_b)M_G r_\epsilon^{5-8l}(4l-1)}{R_\Lambda^3(8l-5)}\left(\frac{2a(2l-1)^2}{\pi}\right)^{4l-1} &(6.3)\\
V_l(\mathbf{r}) &= E_l(r_\epsilon,r_0)+\frac{\hbar^2 l(2l-1)}{mr^2} &(6.4)\\
i\hbar\frac{\partial}{\partial t}\Psi_{1,l}(\mathbf{r},t) &= -\frac{\hbar^2}{2m}\nabla^2\Psi_{1,l}(\mathbf{r},t)+V_l(\mathbf{r})\Psi_{1,l}(\mathbf{r},t). &(6.5)
\end{aligned}$$

The equations above summarise the basic results from a *quantum view* of the type of wave functions and their parametric dependants that needs pertain if the new isothermal gravitation theory solutions are also solutions, $\Psi_{1,l}$, of a Schrödinger equation with a potential function $V_l(\mathbf{r})$. Mathematically, they can in fact be regarded as the solution of an *unusual* classical eigen-value problem expressed as follows. Find the *eigen-potentials* $V_l(\mathbf{r})$ and steady state energy wave functions $\Psi_{1.l}$ that must be operative if the classical Newtonian energy equation is replaced by what might be called a potential function operator version of Schrödinger shape $\hat{V}(r)$ with eigen-values $V_l(\mathbf{r})$ and eigen-wave functions $\Psi_1(\mathbf{r},t)$ as below

$$\begin{aligned}
\hat{V}(\mathbf{r}) &= i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla^2 &(6.6)\\
\hat{V}(\mathbf{r})\Psi_{1,l}(\mathbf{r},t)) &= V_l(\mathbf{r})\Psi_{1.l}(\mathbf{r},t). &(6.7)
\end{aligned}$$

Of course, given only the last two equation, the solutions could not be found from them alone, but they do correctly describe the basis of the problem to be in classical *eigen-value theory* and, importantly for this papers, emphasise the conclusion that galactic halos exist under a special quantized stabilising internal potential in addition to their actual formative gravitational potential structure which itself is also not usual, but rather is Λ orientated as has been shown in section 5. The Schrödinger equation for $\Psi_{1.l}(\mathbf{r}, t)$ does not involve gravity at all. It is a purely quantum structure that represents a discrete infinity of endpoints to cosmological clumping and so the potential involved that conditions steady state motion is a representation of the variety of possible forces involved in clumped mass stability, [52]. Gravitation comes into the picture through the cosmological Schrödinger equation and its solutions $\Psi_S(\mathbf{r}, t)$ of which the $\Psi_{1.l}(\mathbf{r}, t)$ solutions are possible quantized modulating factors that supplies the space variability with radial position r and also importantly ensures physical stability via *steady state motion*.

7 Conclusions

The cosmological dust universe model is applied to the problem of galactic modelling using the quantized mass density solutions of a new theory of gravitational isothermal equilibrium. These solutions depend on a key *integer* state determining parametric pure number l related to the isotropic index n and three other physically adjustable parameters. Assuming the adjustable parameters fixed in value, it is shown that all these density solutions are derivable from *amplitude* solutions of a Schrödinger equation with a special quantized inverse square law eigen-potential. To adequately define these density solutions and then use them to describe dark matter galaxy halos it is necessary to redefine these initially space origin divergent solutions by replacing a small region at the origin with a constant section of radius r_ϵ, which then becomes one of the input adjustable parameters and the core radius of the galaxy. The density solution are all infinitely extended in space but having finite mass core radii they can be integrated over all space to generate mass spectra dependent on ranges of the quantum number l. One such spectrum is calculated so as to terminate at quantum number $l = 9$ giving a theoretical total mass of the universe, M_G. The last but one value $l = 8$ generates a possible small galaxy mass the galactic star rotation curves of which are derived. They are shown to be very flat. The mass spectrum in this case for values near 10^{-25} could possibly represent the most fundamental particle of them all, the Higgs boson. In using the quantized mass values to form the usual Newtonian gravitation potential in the study of galactic rotation curves involving the usual G, coupling constant, it is found that G becomes replaced with Einstein's cosmological constant Λ by fraction cancellation. This implies that consistent with the dust universe model basis on Λ, the *quantization* of gravity implied by this model is Λ dependent rather than G dependent. The structure of the gravitation potential $V(r)$ reveals that a *simulation of dark energy* is involved in its form. For any value of r *apparent* negative mass nearer to the radial origin than r is *actual* positive mass at positions further from the radial origin than r. This has the effect that the apparent negative mass is actual positive mass outside the radius of reference. Thus suitably orientated positive mass can appear elsewhere to be negative. It is suggested that this rather unexpected structure in this formalism could be used to explain *actual* dark energy mass in terms of positively gravitating mass. This last point is an idea *under construction* and will need be examined to see if it reinforces or conflicts with my earlier work on dark energy. However, Einstein's cosmological constant is absolutely essential to all aspects of this physical theory.

Added section on corrections, 21st June 2012

8 Explanation of Corrections

The mistake I made and have corrected in this paper was real and regrettable. However, it has turned out to be useful for the understanding of what seems to me to be a rather subtle power associated with using the *gravitational potential function*. It seems that the process of taking the gradient of that function is mathematically rather subtle. In writing down the gravitational potential to use in deriving the equivalent of Newton's inverse square law formula for force on a particle for the case when dark energy mass was involved as a source of the gravity, I assumed that the mass should be proceeded with a minus sign. Consequently, I chose what I have called in this paper the effective mass version of the total mass for constructing the potential function. This has turned out to be a wrong choice. The process of taking the gradient of a gravitation potential function, operating with ∇, distinguishes between ordinary gravitational mass and negatively gravitating mass on purely analytical geometrical properties of the mass distributions and thus generates the correct sign automatically. Thus by adding the negative sign to the mass simply undid the built in cleverness of the standard ∇ operation on the potential function. The correction to this problem was obviously to start with the actual mass as opposed to the effective mass and let the standard procedure do the work. Thus the corrections involved just changing the use of the effective mass to the actual mass wherever appropriate in the paper. I am writing this explanation of my mistake because I think it reveals something significant about the dark energy concept generally. This dependence of the dark energy concept on geometrical orientation has, as the reader will have seen, has come up strongly in interpreting what the dark energy concept means. Thus my mistake has, I think, had very positive consequences. I can translate these remarks onto a definite mathematical explanation as follows. Consider the total actual gravitating mass and its potential

$$\begin{aligned} V_l(r) = \frac{M'_l(r)G}{r} &= \frac{M'_{l+}G}{r} + \frac{M'_{l-}G}{r}. && (8.1)\\ M'_{l-}(r) &= A_l\left(-r^{3-4l}\right) + B_l\left(-r^{5-8l}\right) && (8.2)\\ M'_{l+}(r) &= M_{l+}(r) + C_l r^3. && (8.3) \end{aligned}$$

The acceleration per unit mass caused by this potential at distance r from the origin is

$$\begin{aligned} \hat{\mathbf{r}}\cdot\nabla V_l(r) &= \frac{c^2\Lambda s(t_b)\theta^{2l}r_\epsilon^3(2l-1)^{4l}}{2(4l-3)}\left(-\frac{4l}{3r^2} + \frac{(4l-2)r^{1-4l}}{r_\epsilon^{3-4l}}\right) + \\ &\quad \frac{3c^2\Lambda s(t_b)\theta^{4l-1}r_\epsilon^3(2l-1)^{8l-2}}{2(8l-5)}\left(-\frac{(8l-2)}{3r^2} + \frac{(8l-4)r^{3-8l}}{r_\epsilon^{5-8l}}\right) + \\ &\quad \frac{c^2\Lambda r_\epsilon^3}{3}\left(\frac{2r}{r_\epsilon^3}\right), \qquad (8.4) \end{aligned}$$

where the dimensioned parameter β has been replaced by the dimensionless parameter $\theta = \beta/r_\epsilon^2$ to clarify the dimensionality of the various contributions. Thus all the last bracketed quantities become dimensionally inverse square but not all variably inverse square. All the coefficients of the large brackets have dimensions m^3s^{-1}. Thus all the terms are accelerations. Notably, Newton's gravitation constant G does not occur. In fact, G is replaced by Λ. This quantized gravitational expression is clearly a substantial generalisation of Newton's law of gravitation. However, we can identify main inverse square law forms as the first terms in the first two large brackets. Both of these terms have minus signs and so represent the usual Newtonian gravitational law of attraction towards the origin. However, both of the large brackets contain also many possible positive signed terms of inverse form determined by the quantum state parameter l. They thus represent repulsions from the origin. Clearly the last positive term above represents the repulsive effect of twice Einstein's dark energy term. The two first large brackets originate in the galactic context, from the galactic mass density and the Einstein pressure term mass density from general relativity respectively. The inverse repulsive terms in the first two brackets with their positive signs appear to go along with the negative gravity of the last term. They are the terms which simulate negative mass by contributing repulsion and actually exist outside the reference sphere of radius r. I mention one more effect from the correction. The negative gravitating term contributed by Einstein's dark energy, the last term above, was left out when I calculated the rotation curve for the small galaxy on the grounds that for a small galaxy it would only make a negligible contribution on account of the smallness of Λ. However, if is used in such calculations under the corrected version of this theory it would contribute a small positive addition to the rotation curve gradient formula for large galaxies. For sufficiently large galaxies the rotation curves would eventually curve up from their flat condition at very large distances from the origin. There has been mention of observations to this effect.

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References

[1] R. A. Knop et al. arxiv.org/abs/astro-ph/0309368
New Constraints on Ω_M, Ω_Λ and ω from
an independent Set (Hubble) of Eleven High-Redshift
Supernovae, Observed with HST

[2] Adam G. Riess et al xxx.lanl.gov/abs/astro-ph/0402512
Type 1a Supernovae Discoveries at $z > 1$
From The Hubble Space Telescope: Evidence for Past
Deceleration and constraints on Dark energy Evolution

[3] Berry 1978, Principles of cosmology and gravitation, CUP

[4] Gilson, J.G. 1991, Oscillations of a Polarizable Vacuum, Journal of Applied Mathematics and Stochastic Analysis, **4**, 11, 95–110.

[5] Gilson, J.G. 1994, Vacuum Polarisation and The Fine Structure Constant, Speculations in Science and Technology , **17**, 3 , 201-204.

[6] Gilson, J.G. 1996, Calculating the fine structure constant, Physics Essays, **9** , 2 June, 342-353.

[7] Eddington, A.S. 1946, Fundamental Theory, Cambridge University Press.

[8] Kilmister, C.W. 1992, Philosophica, **50**, 55.

[9] Bastin, T., Kilmister, C. W. 1995, Combinatorial Physics World Scientific Ltd.

[10] Kilmister, C. W. 1994 , Eddington's search for a Fundamental Theory, CUP.

[11] Peter, J. Mohr, Barry, N. Taylor, 1998, Recommended Values of the fundamental Physical Constants, Journal of Physical and Chemical Reference Data, AIP

[12] Gilson, J. G. 1997, Relativistic Wave Packing and Quantization, Speculations in Science and Technology, **20** Number 1, March, 21-31

[13] Dirac, P. A. M. 1931, Proc. R. Soc. London, **A133**, 60.

[14] Gilson, J.G. 2007, www.fine-structure-constant.org The fine structure constant

[15] McPherson R., Stoney Scale and Large Number Coincidences, Apeiron, Vol. 14, No. 3, July, 2007

[16] Rindler, W. 2006, Relativity: Special, General and Cosmological, Second Edition, Oxford University Press

[17] Misner, C. W.; Thorne, K. S.; and Wheeler, J. A. 1973, Gravitation, Boston, San Francisco, CA: W. H. Freeman

[18] J. G. Gilson, 2004, Physical Interpretations of
Relativity Theory Conference IX
London, Imperial College, September, 2004
Mach's Principle II

[19] J. G. Gilson, A Sketch for a Quantum Theory of Gravity:
Rest Mass Induced by Graviton Motion, May/June 2006,
Vol. 17, No. 3, Galilean Electrodynamics

[20] J. G. Gilson, arxiv.org/PS_cache/physics/pdf/0411/0411085v2.pdf
A Sketch for a Quantum Theory of Gravity:
Rest Mass Induced by Graviton Motion

[21] J. G. Gilson, arxiv.org/PS_cache/physics/pdf/0504/0504106v1.pdf
Dirac's Large Number Hypothesis
and Quantized Friedman Cosmologies

[22] Narlikar, J. V., 1993, Introduction to Cosmology, CUP

[23] Gilson, J.G. 2005, A Dust Universe Solution to the Dark Energy
Problem, Vol. 1, *Aether, Spacetime and Cosmology*,
PIRT publications, 2007,
arxiv.org/PS_cache/physics/pdf/0512/0512166v2.pdf

[24] Gilson, PIRT Conference 2006, Existence of Negative Gravity
Material, Identification of Dark Energy,
arxiv.org/abs/physics/0603226

[25] G. Lemaître, Ann. Soc. Sci. de Bruxelles
Vol. A47, 49, 1927

[26] Ronald J. Adler, James D. Bjorken and James M. Overduin 2005,
Finite cosmology and a CMB cold spot, SLAC-PUB-11778

[27] Mandl, F., 1980, Statistical Physics, John Wiley

[28] McVittie, G. C.; 1952, Cosmological Theory, Methuen Monographs:
John Wiley

[29] Rizvi 2005, Lecture 25, PHY-302,
http://hepwww.ph.qmw.ac.uk/∼rizvi/npa/NPA-25.pdf

[30] Nicolay J. Hammer, 2006
www.mpa-garching.mpg.de/lectures/ADSEM/SS06_Hammer.pdf

[31] E. M. Purcell, R. V. Pound, 1951, Phys. Rev.,**81, 279**

[32] Gilson J. G., 2006, www.maths.qmul.ac.uk/∼ jgg/darkenergy.pdf
Presentation to PIRT Conference 2006

[33] Gilson J. G., 2007, Thermodynamics of a Dust Universe,
Energy density, Temperature, Pressure and Entropy
for Cosmic Microwave Background http://arxiv.org/abs/0704.2998

[34] Beck, C., Mackey, M. C. http://xxx.arxiv.org/abs/astro-ph/0406504

[35] Gilson J. G., 2007, Reconciliation of Zero-Point and Dark Energies
in a Friedman Dust Universe with
Einstein's Lambda, http://arxiv.org/abs/0704.2998

[36] Rudnick L. et al, 2007, WMP Cold Spot, Apj in press

[37] Gilson J. G., 2007, Cosmological Coincidence Problem in
an Einstein Universe and in a Friedman Dust Universe with
Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*,
PIRT publications, 2008

[38] Freedman W. L. and Turner N. S., 2008, Observatories of the Carnegie
Institute Washington, Measuring and Understanding the Universe

[39] Gilson J. G., 2007, Expanding Boundary Pressure Process.
All pervading Dark Energy Aether in a Friedman Dust Universe
with Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*,
PIRT publications, 2008

[40] Gilson J. G., 2007, Fundamental Dark Mass, Dark Energy Time
Relation in a Friedman Dust Universe and in a Newtonian Universe
with Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*,
PIRT publications, 2008

[41] Gilson J. G., 2008, . A quantum Theory Friendly Cosmology
Exact Gravitational Waves in a Friedman Dust Universe
with Einstein's Lambda, PIRT Conference, 2008

[42] Bondi, H., 1957, July . Negative mass in general relativity Reviews of Modern Physics, **29** (3), 423-428

[43] Ramsey, A. S. 1943, Dynamics Part 1, page 157, Cambridge UP.

[44] Dragan Slavkov Hajdukovic, 2008, Dark matter, dark energy and gravitational properties of antimatter Hajdukovic

[45] Gilson J. G., 2009, Dark Energy and its Possible Existence in Particulate Form in a Friedman Dust Universe with Einstein's Lambda, QMUL, 2008

[46] Zwicky, Fritz Catalogues, Compact Galaxies

[47] Weinberg, David H. Radiative Gas Dynamics, Hydrostatic Equilibrium

[48] Gilson J. G., 2011, Physical Applicability of Self Gravitating Isothermal Thermal Sphere Equilibrium Theory Dark Matter Galactic Halos QMUL, 2011

[49] Gilson J. G., 2011, Physical Applicability of Self Gravitating Isothermal Thermal Sphere Equilibrium Theory II Modified Lane Emden Theory QMUL, 2011

[50] Gilson J. G., 2011, Physical Applicability of Self Gravitating Isothermal Thermal Sphere Equilibrium Theory III Dark Matter Galactic Mass Densities QMUL, 2011

[51] Michael Fowler, 2/28/07, Van der Waals Forces Between Atoms galileo.phys

[52] Gilson J. G., 2011, Physical Applicability of Self Gravitating Isothermal Thermal Sphere Equilibrium Theory IV Gravitational Polytropic Schrodinger Equation QMUL, 2011

[53] Gilson J. G., 2011, A Possible Theoretical Mass of the Universe Calculated using The Friedman Dust Universe Model QMUL, 2010

[54] Pierre-Henri Chavanis, 2011, Instability of a uniformly collapsing cloud of classical and self-gravitating Brownian particles Universite de Toulouse, France