George de Bothezat's Teaching on the Infinitesimal

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This is a review of George de Bothezat's account of the spatial infinitesimal in his work, **Back To Newton: A Challenge to Einstein's Theory of Relativity.** [1]

1. Introduction

I had not even heard of George de Bothezat while I was writing **Absolute Space, Absolute Time, & Absolute Motion** [2] (herein referenced as AAA.) Afterwards, his 1936 book, **Back To Newton: A Challenge To Einstein's Theory of Relativity**, was brought to my attention. [3] I recognize him as a precursor of my work on the infinitesimal. It is with respect to his treatment of the subject of the special infinitesimal in the aforementioned book that this review is written.

2. de Bothezat's Appreciation of the Spatial Infinitesimal

Mr. de Bothezat grasped an important characteristic of the spatial infinitesimal. "To consider points as having zero dimension and line and surfaces having no thickness is but a self-contradiction." (54). . . . . "A straight line must be conceived as made of the juxtaposition, end by end of infinitesimally small dots touching one another. These infinitesimally small dots are the points of the straight line. We can plot divisions upon such straight lines. These divisions will be points of infinitesimally small dimensions." (55).

It is significant that the line be conceived as completely filled with infinitesimals touching each other. To imagine that line is not full, but that there are holes in it would open it to the possibility of an inner infinity at every point, a notion that can be found in Vladimir Lenin, Bertrand Russell, George Cantor, perhaps Einstein, and certain, the Zeitgeist of 20th century intellectuality in general.

de Bothezat seems to have understood that the infinitesimals in the line were not literally dots, small roundish blobs: "In the objectivation of our conceptions in the sensed reality, these divisions will be small discernible dots, that is, dots smaller than the smallest estimable length." (55)

I would not put it that way. The infinitesimal is the basic constituent of the realm of the sub-finite. They are not dots, because a dot has a shape; the infinitesimal is below the level in which shapes can exist -- the level of the finite. Yet, it is precisely because they have no shape that all shapes are possible.

De Bothezat made some mistakes, but he understood an important consequence of the idea of the infinitesimal. For instance he saw that it provides the solution to the mystery of irrational numbers. "The irrational quantities are the quantities represented upon a continuous infinite linear segment, by points which can never be reached by a finite division in equal parts, carried as far as you want, of the considered finite segment." (56)

To this, I add: this exhibits the distinction between the finite and the sub-finite, both of which are evident in any single irrational number.

3. de Bothezat's Phenomenalism

On the other hand, his idea of the so-called complex numbers was so imprecise (43) that he did not realize that a proper understanding of infinitesimals, such as that found in my The Nature of Negative Numbers, [4] would aid in comprehending √-1 and beyond that, the discovery of the veritable number system. It may be possible to explain the veritable number system through set theory of some other inferior method, but the important point is that it was done through the proper understanding of infinitesimals.

Part of the un-clarity of his book is that at the base he accepts a version of Berkeley's "To be is to be perceived." On page 64, he writes " If I am not mistaken it is Poincare who once told the story that if during our sleep everything in the World would double in size . . . the whole or the Universe doubling in size, we would never be able to detect any change,. This is a fallacy, because variation of size means variation in regard to a certain standard of length and if the standard has supposedly also doubled and all other standards, doubled too, this rigorously means that nothing has changed in this World. Because what we call size is but the ratio of one thing to another and if all ratios have remained invariable this just means that nothing has changed."

He was wrong. If everything changed in size, it would still have changed, even if the means of measuring the same did not exist. This is the case, not just with the things which are too deep or too big for us, but also when we are unable to measure things of the work-a-day world. A pencil lost in a landfill still has its size, though we are unaware of its presence; moreover, it s size is not dependent on whether a foot ruler is longer, the same length, or shorter than it -- or whether it or not it may rot. More fundamentally, size is not a ratio, although it may be measured to be so many times another thing, or inversely, some other thing may be found to be so many times it.

What would be unchangeable is this fact: An object twice as large as it is would necessarily occupy twice as much space. The spatial infinitesimals would not change in size. To assert otherwise would be to contradict oneself: if the size is doubled, so is the volume occupied by it.

De Bothezat was too close to the phenomenalism, if not the actual positivism of his day to be able to enunciate in clear terms that the spatial infinitesimal is the smallest element of space, that nothing can be smaller. The spatial infinitesimal has location, but no parts. It has neither center nor edge. When a material body moves, the space within it does not move aside. Space as such is empty of matter and (as far as can be discerned, of spirit, as well). And the smallest part of any volume in this emptiness is the immovable infinitesimal.

4. de Bothezat's Treatment of Numbers

Let us turn to his treatment of numbers: He wrote" The product of two unit-quantities of the same kind or different kinds is but the symbol for a third unit-quantity of a kind different from the two first ones and derived from them." (40) To illustrate what he meant: The product of 3 feet time 2 feet is 6 square feet and the product of that and 4 feet is 24 cubic feet. It is well that he understood that a number must be a number something even when it is not specified, as in a school exercise; i.e., it is attributive, rather than substantive.

But this is not a universal. He was unaware of the possibility of linear multiplication. In this kind, that is, multiplication along a line, the unit is the same throughout the multiplication, e. g., 2 feet times 3 feet is simply 6 feet along a line – not 6 square feet.

His handling of zero was defective. He wrote on page 38, that "not to take a unit is a negation. For the uniformity of language, we introduce, however, the number 0, called zero, which expresses a negation by an affirmation. Instead of saying, 'we are not taking a unit,' we say, 'we are taking a unit zero times.'"

This is wrong. The phrase, "not to take a unit' has some meaning in decimal symbolization where its use in a number indicating .002 or two thousandth simply means that a number is not taken in the tenths and hundreds places, but is in the thousandth place. Crucially, it slights two more important uses of zero. Not to take a unit is just neutrality in many instances. In the number line, 0 is simply a point, +1 and -1 on either sides are full units, units which can be further divided into finite and irrational parts. Utter zero, symbolized by **0,** means a total absence of being in some respect.

My recognition of this distinction between the two zeros was indispensable in the writing of **The Nature of Negative Numbers**. Subsequence to the discovery of the veritable number system, the system which is compatible with the existence of both the even and odd roots of a negative number, it was necessary to realize its possibility, i. e., to show how it can be applied to the fundamental operations of arithmetic and beyond. It could not have been completed without the recognition of the distinction between 0 and **0**…

5. de Bothezat on Infinity, Continuity, and the Spacial Infinitesimal

De Bothezat recognized potential or Aristotelian infinity (∞), which he defined as "that number larger than any selected number g, taken as large as we want, which at the same time insures the division of g into numbers smaller than the number p, also selected as small as we want." (45)

Absolute infinity, he defined as "the singular number . . . represented by the symbol Ω, and which, by definition, is equal to the quotient of any number n divided by zero . . . . $\Omega = n / 0$."

This, of course, is wrong. If the attempted division by zero is utter zero, or 0, no quotient is possible, simply because there is nothing with which to divide. If it is the zero of neutrality in the midst of a number line, the quotient would only be extremely large, an actual infinity of a lesser kind, but not the boundless quantity he required. Stated more exactly: It would be *n* times the (unknown) quantity of infinitesimals in the unit selected divided by one infinitesimal.

Absolute infinity cannot be a ratio (except in the most trivial way). Yet, endlessness might exist. Space, for all we humans know, is extended outwardly forever. Within, there may be vast regions, unvisited by light, matter, and energy – not even by the vaunted anti-matter. The same could be the case with Aether: locations of incredible magnitude may exist in which anything which requires Aether for its transmission would not send at all --another kind of black hole, perhaps.

(There might be a case for using a revised form of de Bothezat's equation to indicate the less form of actual infinity, which I term immeasurable infinity, a quantity that is not endlessly great, but which is congenitally impossible for human beings to ascertain – an example being the quantity of infinitesimals in some defined unit length. This revised formula is $\Omega = n / m0$, where the zero is a single infinitesimal, such as in found as the point of indifference in a number line, and the m is a natural number or integer.)

(Pp 47-48) He was right to reject Cantor's theory of aggregates.

His concept of continuity is wrong. He wrote: "The concept of continuity is but the direct consequence of the acceptance of the illimited divisibility of magnitudes. "(50) The mistake is made explicit in his definition of the "infinitesimally small:" $n / \infty = \varepsilon$. . . . . "The infinitesimally small ε is by definition a variable number, which is never equal to zero, but which is as many times smaller than any number we want." (51)

This means that the infinitesimal has no definite size. Moreover, that it is merely an intellectual construct, which he tells us elsewhere, must be inexact. "Every measurement can basically be only an approximation, because it only consists in the evaluation of concepts the realization of which in the realm of the sensed reality is itself only approximate."

This takes us back to Leibniz, who held that an approximate could be considered as an exact, and more importantly, that the infinitesimal might just be a useful idea, but not necessarily true.

Which vitiates his definition that a "continuous magnitude is just made up of the juxtaposition end by end, of infinitesimally small magnitudes." (54) If they are themselves inexact, how can they completely fill a number line? And if they are of different sizes, how can one line be interchangeable with another of equal length, unless the same sizes appear in the same relative positions in each line? Assuming this to be true or the sake of the argument, how could he show this?

He explained: "Let us consider a finite linear segment. If we plot this segment g equivalent points, the number g being as large as you want, these points will not touch one another, but if we plot on this same segment an infinite number of equidistant points, these points will touch one another and will , so to say, fill the continuity of the line."

What does he mean by infinity here? If it is absolute infinity, this would mean that line segment A half the length of line segment B would have just as many infinitesimals as B, which is the contradiction embraced by Bertrand Russell and most others. If he means Aristotelian infinity, then no clear statement can be made.

The same is true with his "infinitely small," which he declared to be variable and just smaller than anything we might pick.

As was shown in AAA, the infinitesimal has nothing to do with how small we humans want it to be. To the contrary, the infinitesimal must be the smallest possible element of space. If line segment A is half the length of line segment B, it must contain half as many infinitesimals. The type of actual infinity appropriate to them must be of the immeasurable type, not the endless type like we believe to be the case with outer space.

By this conception, a sub-finite segment of a line must be definite, an integral multiple of the basic infinitesimal; if it consists of only 5 infinitesimals, it is no longer or no shorter than that.

The problem is that his conception of space is inexact. It was his greatest weakness. He wrote on page 30 that "Physical space is nothing but the whole of our sensed reality." Here, he failed to differentiate between space and what is in it

"The notion of the geometrical solid is formed by the contemplation of rigid bodies in our surroundings and abstracting from all of their mechanical and physical properties." "Space is but the solid of infinitely large dimensions." (61) In truth, space is not a solid. Solids are found in space. Space as a whole is a volume extending everywhere. It is not initially discovered by abstracting away mechanical and physical properties, but by observing that objects move from place to place; that they are not where they were. The abstracting away is a refinement.

Furthermore, the fact that we would not be aware of space without the presence of some motion does not mean that it cannot exist without motion. This, he denied when he argued that "configurations are quite conceivable without space participating in them." (62) That is impossible. If the configuration is in space, it exists, regardless of whether we mentally attend to that space. He explained: "If the considered configuration is made up of two points only (space as such excluded), the only possible displacement for them is their coming closer or further apart on the line joining them." (62)

The only way that can make sense is to suppose that he was not talking about spatial infinitesimals as such, but of their relationship to a moving body as it goes past through two near-by locations. Spatial infinitesimals themselves are just locations in absolute space and cannot move.

As a result of such tergiversations, he ended up defeating his best argument that the infinitesimals in a line segment fill it up; he wandered into the notion of an inner infinity. He did this through his concept of infinitesimals of higher orders. "The passage to infinitesimals of higher orders corresponds to the conception of a set of increasing approximations fitting so to say one into the other. The infinitesimally small of the second order fill any infinitesimally small interval of the first order and so on. One should only, when having recourse to a set of approximations, not confuse them among themselves or imagine that they contradict each other. The conception of such a system of approximations does not exclude the properties of any one of them and each of these stages of approximations is fully self consistent." (57)

What it leads to is refuted in general terms in Chapter VII of AAA. But let us attend to his specific version. "Consider an infinitesimally small length εU – which you can, if you want, consider as temporarily magnified – and let us divide this length in ∞ equal parts. With $\varepsilon = 1 / \varepsilon$ we will have $\varepsilon U / \infty = U / \infty^2$ The symbol ∞^2 being called infinity of the second order. " (56)

This weakens his declaration about irrational numbers. Take the difference between the square root of two and the nearest rational number on a number line with units of a given length. It would mean that inside this difference consisting of an unknown number of infinitesimals, there would be a second difference of higher order infinitesimals, and inside that an even higher order, and so on. The exponent(s) on ∞ suggest(s) that each higher order is greater than its predecessor

How many levels are there in his theory? I submit that it would be endless, which adds up to an inner infinity, thereby basically destroying his whole conception of the infinitesimal. With an inner infinity, every point would contain within it holes where an infinitesimal of a greater order could be inserted, etc. Yet, even the rims of these holes would have holes. And these would contain holes, ad infinitum. His "space" and the objects within it would together amount to a fallacious sum – utter nothing. – just like those of Lenin, Russell, and Einstein. Their mistake was an amplification of the same one that Kant made in his 2nd antinomy of reason.

But perhaps de Bothezat might answer that that he did not mean quite that; he meant only that it would be like a stack of cannon balls of various sizes with chinks left after they were placed next to and on each other. This too would be endless, unless the one fitting the chinks at the final order were of different shape than the one of the next lower order. But to do any of that would be to return to the notion of shape. This would contradict the nature of space, which, being shapeless, makes shape possible. De Bothezat was not far enough from Einstein's conception of curved space.

Elsewhere, he tried to say that two parallel lines would intersect at an infinite (∞) distance. He did not understand that a contemporary of his had corrected the well-known difficulties and defined parallel lines as equidistant at all corresponding points. [5].

6. Conclusion

The question about the existence of the special infinitesimal comes to this: Is there a stop to the division of space? If there is , then that stop is the special infinitesimal. If there is no stop, then there is an inner infinity, endlessly extended forever. This question is not mere arcane. It touches matter , light, energy – even Aether. Consider a small material body. If there second alternative were true, it would be, as Lenin, put it, "infinite in depth."[6] Since a body twice as large would also be capable of endless division, the quantity of potential parts would be the same in both cases an evident contradiction. Or, as de Bothezat airily put it: $\Omega = n = n\Omega = \Omega^n$.

For, for all of his brave desire to return to Newton, George de Bothezat was a man of his time. He took a middle position. "But things are quite different for the variable infinity ∞, which is but a number larger than g / p and thus can only have the properties larger than g / p. Thus, in general, . . . $g / p < \infty > \Omega$" (47) But this $< >$ variation is based upon ignorance. It is subjective. With ∞, whether there is an end or not cannot be discerned. Hegel was right. ∞ is spurious.

References

[1] George de Bothezat, **Back to Newton: A Challenge to Einstein's Theory of Relativity** (New York: G. E. Stechert & Co, 1936), 153pp.

[2] Peter F. Erickson, **Absolute Space, Absolute Time, & Absolute Motion** (Philadelphia, PA: Xlibris, 2006), 267 pp.

[3] By NPA member Greg Volk.

[4] Peter F. Erickson, **The Nature of Negative Numbers** (Vancouver, WA: Fluxion Press, 2011), 166 pp.

[5] J. J. Callahan, **Euclid or Einstein** (New York: Devin-Adair, 1931), 303 pp.

[6] Vladimir Lenin, quoted in Gustav wetter's **Dialectical Materialism: A Historical and Systematic Survey of Philosophy in the Soviet union**, (New York: Frederick A. Praeger, 1960), p. 294.