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# Electric and magnetic fields: do they need Lorentz covariance?

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**Abstract.** Electric and magnetic fields are relative. They depend not only on a choice of electromagnetic sources via Maxwell equations, but also on a choice of observer, a choice of material reference-system. In 1908 Minkowski defined electric and magnetic fields on a four-dimensional spacetime, as tensorial concomitants of observer. Minkowski defined Lorentz-group-covariance of concomitant tensor field as group-action that commute with contractions. Present-day textbooks interpret Lorentz-group-covariance of concomitant tensor differently than Minkowski in 1908. In 2003-2005 Tomislav Ivezić re-invented Minkowski's group-covariance. Different interpretations of group-covariance, lead to different relativity transformations of electric and magnetic fields.

An objective of present article is to explore third possibility, implicit in [Minkowski 1908, §11.6], where a set of all relativity transformations of all material observers forms a groupoid category, which is *not* a group.

**Keywords:** history of relativity theory, group-covariance, material reference frame, observer, tetrad versus monad, electric and magnetic fields, groupoid category, relativity groupoid as a category

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## 1. Introduction: symmetry group of Maxwell equations

In 1904 Lorentz and Poincaré showed that a system of four Maxwell's differential equations (observer-dependent) has a symmetry group the same as a symmetry group of metric tensor. This was the Lorentz and Poincaré isometry group. In 1909/1910 Bateman and independently Cunningham, showed that Lorentz and Poincaré were wrong, because they discovered only a **subgroup** of actual symmetry group of Maxwell equations. In fact symmetry group of Maxwell equations of electromagnetic field is conformal group of dimension 15, this is actual symmetry group of massless radiation.

What does it mean to be a symmetry of differential equation? Symmetry act on solutions of differential equation and maps solutions into solutions. Therefore a symmetry is iso-morphism of a manifold of solutions.

As we know perfectly today symmetry group of massless Maxwell's equations, the conformal group, was never considered as a group permuting material reference systems. Isometry group of metric tensor was considered incorrectly as the *same* as a symmetry group of Maxwell's equations. If this could be made plausible then a hypothesis, accepted in 1905 by Albert Einstein, and in 1908 explicitly by Hermann Minkowski, that this non-Euclidean metric tensor of empty spacetime and this Lorentz isometry-group is a group that can permute not only solutions of massless Maxwell's equations, but can permute also material reference-systems of relativity theory?

In present paper we argue that light-like massless radiation must not be considered as a reference system, and therefore must be outside of domain of transformations of material reference-systems. From the other hand, isometry of metric, as it is the Lorentz group, must possess in domain *all* vectors, including also light-like vectors in Minkowski space. This implies that a set of all transformations among all material reference systems do not need to be postulated a priori to be the Lorentz isometry-group.

We propose here an alternative for the Einstein isometric special relativity. We suggest that a set of all relativity transformations (between material reference-systems, among the time-like vector fields) could be a groupoid viewed as a *category*, that it is *not* a group<sup>1</sup>. One consequence of groupoid relativity, that it is implicit in Minkowski last publication in 1908, in §11.6, is examined here for groupoid relativity transformation of the electric and magnetic fields, and groupoid transformation of the electromagnetic sources Equation (12.7)–Equation (12.8). This consequence could eventually be tested experimentally.

We wish to show, following Minkowski's implicit idea in §11.6, that Maxwell theory of electromagnetic fields does not need the concept of Lorentz group, nor Lorentz-covariance, and can be formulated alternatively in terms of groupoid relativity, in terms of groupoid category that is not a group.

### 1.1. What is groupoid?

We owe to readers brief explication of the difference among concept of a group, a very special groupoid category, and a groupoid viewed as a category. Groupoid generalize a concept of a group. Every group is a groupoid category, but not every groupoid category is a group. There are two differences interrelated. We refer to:

<http://mathworld.wolfram.com/Groupoid.html>.

The first property of a group distinguishing them from a general groupoid category, is that a group possess *unique* neutral element. In the case of Lorentz relativity group, the zero velocity  $\mathbf{0}_{\text{Earth}}$  of Earth relative to Earth, and zero velocity  $\mathbf{0}_{\text{Sun}}$  of Sun relative to Sun, are identified as the unique neutral Lorentz boost,  $\mathbf{0}_{\text{Sun}} \equiv \mathbf{0}_{\text{Earth}} \equiv \mathbf{0} \in \text{Lorentz group}$ . In case of groupoid relativity, instead, these are different elements,

$$\mathbf{0}_{\text{Sun}} \neq \mathbf{0}_{\text{Earth}} \in \text{Groupoid}. \quad (1.1)$$

Every two elements of a group must be composed, group binary operation is *global*. For example, if  $L_1$  and  $L_2$  are two elements of Lorentz group, hence their group compositions,  $L_1 L_2$  and  $L_2 L_1$ , belong to Lorentz group. This is not the case in a groupoid category, where not every pair of elements of a groupoid (*i.e.* not every pair of morphism of groupoid category) must be composable. The groupoid binary operation is not necessarily global. In our example of relativity groupoid, for a set of four different material reference systems denoted by  $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ ,

<sup>1</sup> Frequently groupoid is understood as binary operation, renamed by Bourbaki as *magma*. Groupoid viewed as a category is not magma.

consider two groupoid transformations,

$$\mathbf{P} \xrightarrow{g_1} \mathbf{Q} \quad \text{and} \quad \mathbf{R} \xrightarrow{g_2} \mathbf{S}. \quad (1.2)$$

Each of these transformations is invertible in the case of a group and also necessarily invertible in a case of groupoid. However, if  $\mathbf{Q} \neq \mathbf{R}$  and  $\mathbf{S} \neq \mathbf{P}$ , these transformations,  $g_1$  and  $g_2$ , can be constructed in such way that a priori (e.g.  $\mathbf{Q}$  not in domain of  $g_2$ , etc) this would *not* allow us to compose them. In this case, a set of all such invertible transformations without of global composition is said to be a *groupoid category*, briefly groupoid.

Einstein's second postulate, Postulate II state: light velocity is independent of motion of emitting source [Einstein 1905]. It is rather known that this second Einstein's postulate is a consequence of **only one** Minkowski's postulate, that a group of all relativity transformations among material reference systems must coincide with Lorentz isometry group. One can deduce the second Einstein's postulate also as a consequence of groupoid relativity [Oziewicz 2005]. Therefore as far as light velocity is concerned, there is **no** difference among Lorentz-group relativity and groupoid relativity. It is, therefore, impossible to infer Lorentz relativity transformations from second Einstein's postulate alone. In fact Lorentz-*group* relativity transformations were silently *assumed* by Einstein. Einstein postulated *group* structure. Equivalently, one can deduce group structure by postulating reciprocity of relative velocity (also postulated by Einstein in 1905), *i.e.* that inverse of relative velocity  $\mathbf{v}$  is  $-\mathbf{v}$ . Reciprocity axiom of relative velocity does not hold in groupoid relativity [Świerk 1988, Matolcsi 1994, Oziewicz 2005].

## 2. Notation: basis-free four-dimensional space-time

It is worth to mention that idea of spacetime was implicit in Galileo's observation of relativity of space (thus absolute spacetime), and that *four-dimensional* space-time, with 'fourth time coordinate', was proposed by Jean Le Rond D'Alembert (1717–1783) in articles about the partial differential wave equation, published in Encyclopedia of Denis Diderot around 1772.

Starting in 1884, Gregorio Ricci-Curbastro developed tensor analysis, and, working jointly with his student Levi Civita, they made clear in 1901 the difference among 'contravariant' and 'covariant' vectors. Élie Cartan (1869–1951) used a name a differential form as a synonym for Ricci's covariant vector field. Category theory born in 1945, clarify this distinction in terms of dual spaces, and in terms of dual pair of vector and covector fields. It was clarified that Ricci's 'covariant' vector must acquire categorical meaning as contravariant vector.

In 1908 Hermann Minkowski introduced the following terminology, a *space vector*, as a synonym of present terminology *space-like vector in a four-dimensional Minkowski spacetime*, and a *space-time vector*, that presently is understood as a vector in Minkowski spacetime.

Minkowski in 1908	Presently
space vector	space-like vector
space-time vector	vector, or '4-vector'

The Minkowski terminology was, and is, misinterpreted, in particular, when considering observer-dependent product-structure or

a splitting of spacetime into: 'space'  $\oplus$  'time'.

The name *spacetime*, introduced in 1908 by Minkowski, is misleading, suggesting incorrectly that this concept is derived from two primitive concepts of 'space' and 'time'. It is just opposite, the very primitive concept is space-time of events, and space is a *derived* concept that needs an artificial *choice* of material body, e.g. Earth or Sun, as a reference system, see Equation (2.1) below. But any such *choice* is irrelevant for physical phenomena, it is no more then for example a convenience for a computer program. If a material reference system is chosen, Earth or Sun,

then corresponding space of *this* material body is *not* a fiber in space-time, but it is rather a quotient-space = spacetime/material-body,

$$\text{Space} \equiv \frac{\text{Spacetime}}{\text{material body}} \quad \text{Time} \equiv \frac{\text{Spacetime}}{\text{Convention of simultaneity}}, \quad (2.1)$$

$$\text{Proper-Time} \equiv \frac{\text{Spacetime}}{\text{Metric simultaneity of material body}}. \quad (2.2)$$

Misinterpretation of Minkowski's terminology grows into three-dimensional and four-dimensional quantities, 3-dimensional vector, 3-space vectors, 3D versus 4D quantities, 4-vectors, four-tensors, etc. (e.g. Rindler 1969, §5.4 Four-vector, §5.10 Three-Force and Four-Force; Landau & Lifshitz, 1975, §6. Four-Vectors). A vector does not have dimension. It is a manifold and a vector-space that possess dimension.

Spacetime splitting does not mean that there then appear some strange '3-dimensional quantities'. Like Minkowski in 1908, we exclusively use **four**-dimensional space-time manifold only. In all our expressions, a boldface electric field **E**, and a boldface magnetic field **B**, denote 'contravariant' vector fields (derivations, see next section) on *four*-dimensional spacetime manifold. All tensor fields, scalar fields, vector fields, bivector fields, differential multiforms, etc., are tensor fields on four-dimensional spacetime manifold only.

Our boldface notation distinguishes 'contravariant' multivector fields, from the 'covariant' differential forms denoted by italics letters, or by Greek alphabet. Thus, **F** denotes a bivector field, whereas *F* denotes a differential biform. A material observer is a time-like vector field, and is denoted by boldface **P** for an observer **Paul**, and, by boldface **R** for an observer **Rose**.

The source of misinterpretation is a concept of a vector field. Many authors denote by **E** time dependent electric field strength, *i.e. not static* electric field, **E**(**r**, *t*), and of course such electric field is a vector field on four-dimensional space-time, independently of a choice of a basis. This notation assume silently that derivative of some time coordinate (a parameter *t*) in direction of **E** is zero, **E***t* ≡ (*dt*)**E** = 0. This means that this 4D vector field **E** on spacetime is tangent to three-dimensional super-surfaces *t* = const. This condition could be equivalent to stressed many times by Minkowski, that electric field **E** must be orthogonal to time-like observer-vector field **P**, *i.e.* **E** · **P** ≡ *g*(**P** ⊗ **E**) = 0, for some Minkowski's metric tensor *g*. To be Minkowski's metric tensor *g* does not means here that curvature and torsion of connection must be absent, we emphasize only that this non-Euclidean spacetime metric was recognized by Minkowski (and early by Henri Poincaré).

$$\text{If } g\mathbf{P} = dt \text{ then: } \mathbf{E} \cdot \mathbf{P} = 0 \iff \mathbf{E}t = 0. \quad (2.3)$$

Nobody call **E** to be 'nonrelativistic' because exists another vector **P** orthogonal to **E** !

How to understand apparently 'nonrelativistic' electric vector field **E**(**r**, *t*) in textbooks by Sommerfeld [1948], or by Jackson [1962] ? Somebody interpret this as infinite-family of vector fields on three-dimensional space, family that is parameterized by time parameter. But of course, such family of vector fields is not a vector field on space, it is not a '3D' vector. Calling time-dependent vector field as 'nonrelativistic' bring objection: nonrelativistic one can associate to limit *c* ⇒ ∞, but this field **E** is a solution of Maxwell differential equations. Authors call **E**(**r**, *t*) incorrectly as '3D' vector, but in fact this is a vector field on space-time, *i.e.* this is '4D' vector field.

Some readers would prefer as more clear an explanation in terms of mathematical bases. When nowhere-vanishing vector field, **E** ≠ **0**, is chosen to be a part of mathematical basis, e.g. if a basis of vector fields (a frame) is, {*e*<sub>0</sub>, **E**, *e*<sub>2</sub>, *e*<sub>3</sub>}, then, this vector field **E** in such basis has exactly one non-vanishing scalar component, *i.e.* **E** ≃ (0, 1, 0, 0). Each non-zero vector field on a four-dimensional spacetime possesses a set of three adopted (or associated) differential one-forms

annihilating this vector field. Each non-zero electric vector field,  $\mathbf{E} \neq \mathbf{0}$ , in such adopted cobasis (coframe), has only one non-zero scalar component. For example, let there exist the differential one-forms,  $\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \neq 0$ , such that,  $\alpha_1 \mathbf{E} = 0$ ,  $\alpha_2 \mathbf{E} = 0$  and  $\alpha_3 \mathbf{E} = 0$ . In a co-frame basis,  $\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \wedge \alpha_4 \neq 0$ , a vector field  $\mathbf{E} \neq \mathbf{0}$  possesses only one non-vanishing scalar component,  $\alpha_4 \mathbf{E} \neq 0$ .

In this basis-dependent meaning, should this vector field  $\mathbf{E}$  *on spacetime* to be ‘1D’ or ‘3D quantity’? Phrases ‘3D  $\mathbf{E}$ ’, and ‘3D  $\mathbf{B}$ ’, as well as three-dimensions, could be understood in several different and misleading ways:

- ‘Time-independent’ tensor on spacetime, i.e. tensor with one Lie-symmetry.
- Tensor on three-dimensional sub-manifold.
- Tensor on three-dimensional quotient-manifold.
- Basis-dependent concept that in an accidental (badly chosen) mathematical basis, ‘3D  $\mathbf{E}$ ’ has at most three non-vanishing scalar components.

In the present paper we do not need terminology like 3D, 3-dimensional vector and no ‘4-vectors’, ‘4-velocities’, - all misleading. We do not accept the distinction between ‘3D’ and ‘4D quantities’, because if ‘4’ means a dimension of a space-time, then *all* time-dependent tensor fields are on spacetime, and in the present paper, and in *all* textbooks, in Jackson’s  $\mathbf{F}$  and  $\mathbf{E}$ , etc., are always ‘4D’, independently of how many components are not vanishing.

Two different material observers, call them **Paul** and **Rose**, each have their own material reference-system. In each material reference-system, concomitant-compound electric and magnetic vector fields are defined in 1908 by Minkowski, see Definition 7.1 and expressions Equation (7.1) below,

$$\mathbf{E}(\mathbf{Paul}), \quad \mathbf{B}(\mathbf{Paul}), \quad \mathbf{E}(\mathbf{Rose}), \quad \mathbf{B}(\mathbf{Rose}). \quad (2.4)$$

In a generic basis of four independent vector fields, each of these vector fields Equation (2.4) possesses *four* non-zero scalar components; never more than four. A differential one-form of an electric field,  $E(F, \mathbf{Paul})$ , is a tensor field on exactly the same manifold on which differential bi-form  $F$ , and an observer-monad field  $\mathbf{Paul}$ , are living.

One-hundred years after duality was realized, and important distinction among a concept of a (coordinate) vector field,  $\partial_x$ , and a concept of a coordinate-free differential form,  $dx$ , was realized, still it is very hard to find this distinction in present University textbooks. Another story is to understand that every vector field is coordinate-free, and that also coordinate-vector-field  $\partial_x$  that is completely fixed by one coordinate co-frame, is also coordinate-free as every tensor field must be.

Differential one-form, known also as Pfaff form, like differential of a scalar field,  $dx$ , is not Leibniz’s small infinitesimal increment; instead, following an idea of Isaac Barrow, it is a real-valued function on vectors (vector is synonym of a process), associating to a vector (process) the directional derivative (of scalar field) along this vector, see for example [Bishop and Goldberg 1968, page 58]. This paper distinguishes contravariant vectors and tensors on four-dimensional spacetime, denoted by boldface fonts, from covariant vectors and tensors, and in particular differential forms, denoted by italic or Greek letters.

A differential one-form is a synonym of covariant vector field, it is a scalar-field-valued evaluation-map on vector fields. For example,  $(dx)\mathbf{P} = \mathbf{P}x$  is a (directional) derivative of a scalar field  $x$  along a vector field  $\mathbf{P}$ . *Every* derivative is directional, however this most important point is missed in all Calculus textbooks. A vector field annihilated by a given differential one-form is said to be in kernel of this differential form. Therefore, a value of differential one-form on vector field is said to be evaluation, and does not need a concept of a scalar product applied to pair of vector fields of the same covariance. For example,  $ev(df \otimes \mathbf{P}) \equiv (df)\mathbf{P} \equiv \mathbf{P}f$ , is evaluation, and *not* scalar product.

The concomitant of compound electric field, Paul-dependent electric field as Pfaff differential form,  $E(F, \mathbf{P}_{\text{Paul}})$ , has a time-like vector field  $\mathbf{P}$  in his kernel,  $\{E(F, \mathbf{P})\}\mathbf{P} = 0$ . For  $E = g\mathbf{E}$ , equivalently,  $\mathbf{E} \cdot \mathbf{P} \equiv 0$ , [Minkowski 1908 §11.6, Eq. (49)]. Hence, whenever a vector field  $\mathbf{P}$  is chosen to be a basis vector field, then in such mathematical basis electric field on spacetime possesses no more than three non-vanishing scalar components.

The last objective of the present note is to advocate a basis-free and coordinate-free approach, and therefore, basis-dependent classification into 4D, 3D, 2D and 1D ‘quantities’, is totally irrelevant for mathematics and for physics.

### 3. Each vector field is a derivation

Most textbooks of XXI century separate Calculus’s concept of a derivative or a derivation, from algebraic concept of a vector field. The source of this separation lay also in sorrowful history of Calculus, a war of Newton against Leibniz. Students learn ancient historical Pierre de Fermat’s definition of derivative of a function (derivative of a scalar field) that do not allow to imagine that derivative of a scalar field in reality is *not* unique, and needs always a choice of a vector field along which such derivative one can calculate. *Every* derivative is directional, however initial chapters of Calculus textbooks insists incorrectly that no one vector field is involved in the concept of derivative. Derivative of a not constant scalar field  $x$  along a coordinate vector field  $\partial_x$ , is, by definition,  $\partial_x x = 1$ , but this is *not* the only derivative possible. The same scalar field  $x$ , has a different derivative along another coordinate vector field, say  $\partial_{(x^2+2)}$ , namely,  $\partial_{(x^2+2)} x = \frac{1}{2x}$ , this is the chain rule. There are so many derivatives of a given not constant scalar field as many there are vector fields. No vector field chosen, it is not possible to calculate derivative of a function! The most frequent Calculus-books problem ‘Calculate derivative of ‘ $2x + 1$ ’, is meaningless, because  $\partial_{(2x+1)}(2x + 1) = 1$ .

Each partial derivative, like  $\partial_x$ , is a coordinate vector field (a derivation of an algebra of the scalar fields), and, as a partial derivative, is not given uniquely by a given scalar field  $x$ . Notation  $\partial_x$  is misleading because this coordinate vector field  $\partial_x$  depends on a choice of coordinate system, on a choice of an integrals of motion, for example we need to choose a scalar field  $t$  for  $\partial_x$  such that  $\partial_x t = 0$ . This information is missing in notation  $\partial_x$ . Textbooks of thermodynamics use a correct notation,

$$(\partial_T)_P \neq (\partial_T)_V. \quad (3.1)$$

When writing  $\partial_x$  it is most important a choice of a coordinate chart to which a scalar field  $x$  belongs; for example,  $x \in \{x, t\}$ . If  $\partial_x t = 0$ , then  $\partial_x \equiv (\partial_x)_t$ .

Each electric field and each magnetic field, is a vector field, therefore these fields are *derivations* of algebra of scalar fields. Therefore coordinate expressions of these vector fields must be as follows

$$\begin{aligned} \mathbf{E} &= (\mathbf{E}x^\mu)\partial_\mu = (\mathbf{E}x^\mu)\frac{\partial}{\partial x^\mu}, \\ \mathbf{B} &= (\mathbf{B}x^\mu)\partial_\mu = (\mathbf{B}x^\mu)\frac{\partial}{\partial x^\mu}, \\ \mathbf{P} &= (\mathbf{P}x^\mu)\partial_\mu = (\mathbf{P}x^\mu)\frac{\partial}{\partial x^\mu}, \\ \mathbf{R} &= (\mathbf{R}x^\mu)\partial_\mu = (\mathbf{R}x^\mu)\frac{\partial}{\partial x^\mu}, \quad \text{etc.} \end{aligned} \quad (3.2)$$

Evidently, in Equation (3.2), scalar components, like  $\mathbf{E}x^\mu$ , is a derivative of a scalar field  $x^\mu$ , along electric field  $\mathbf{E}$ . This scalar component is denoted historically as  $\mathbf{E}^\mu \equiv \mathbf{E}x^\mu$ . In what follows we will not need and not use neither coordinates nor bases. Physics is coordinate-free and bases-free.

#### 4. Herman Minkowski

In 1908 Hermann Minkowski published his last paper, entitled "The foundations for electromagnetic phenomena in the moving bodies" (Die Grundlagen für die electromagnetischen Vorgänge in bewegten Körpern). In 1910, after Minkowski's death, two other papers were published under Minkowski's name. The 1910-paper, of almost the same title, was written by his pupil Max Born, and sometimes is referred as Minkowski and Born paper, although was published under the name of Hermann Minkowski alone. When comparing the Minkowski 1908-paper with Born's 1910-paper, it is clear that Born's 1910-interpretation was different from 1908-paper by Minkowski. Born put full emphasis on Lorentz-group covariance, whereas Minkowski in Part II §11.6 of his 1908-paper defined electric and magnetic fields in a covariance-free way.



**Figure 1.** Hermann Minkowski 1864-1909

Portrait provided at <http://library.thinkquest.org/05aug/01273/whoswho.html>

Minkowski's 1908 paper deserves commemoration more than Einstein's 1905 paper did, for several following reasons.

- Minkowski defined Lorentz group as isometry group of a metric and this definition do *not* involve a concept of reference system. Relativity is formulated by Minkowski as **one axiom-postulate only**: a group of all transformations among material reference systems coincide with Lorentz isometry group. See exactly the same definition for example in [Schrödinger



1931, Bergmann 1942, Einstein 1949, Bargmann 1957]. This postulate (subject of revision in present paper) reduced the relativity principle to Lorentz group-covariance.

- A tensor is said to be a *concomitant* tensor, if it is build-from/dependent-on other primary tensors. Minkowski defined group-covariance, and in particular at the beginning of §11.6, defined Lorentz-covariance of concomitant tensors. Minkowski's definition of Lorentz-covariance does not appears in contemporary University textbooks on electromagnetism. Tomislav Ivezić rediscovered the Minkowski group-covariance in 2003-2005.
- The entire §11.6 of Minkowski's 1908-paper is devoted to ingenious invention of definition of electric and magnetic fields, as concomitants of absolute electromagnetic field. These concepts are covariance-free, and do not need at all Lorentz-isometry group.
- Relativity axiom, **only one** Minkowski's axiom, does not involve a concept of relative velocity among material reference systems. Within Lorentz-group axiom, relative velocity belongs to Lobachevski factor space, see [Varićak 1924], and this factor space is not unique. This implies that relative velocity among reference systems is also not unique. Contrary to relativity axiom (not unique relative velocity), in §11.6 of 1908-paper, Minkowski defined the **unique** relative velocity among material reference systems identified with normalized time-like vectors.

When relative velocity among pair of reference systems is **not** deduced from Lorentz isometry transformation, *i.e.* if it is not an element of Lobachevsky factor space, but it is postulated a priori, then a set of all transformations among material reference-monads is not a group, but it is a groupoid.

- The Minkowski 1908-paper attracted most attention for constitutive equations that include electric permittivity  $\mathbf{D} = \varepsilon \mathbf{E}$ , and magnetic permeability  $\mathbf{H} = \frac{1}{\mu} \mathbf{B}$ , extended by Minkowski to moving material reference system. Rousseaux [2008] derived Galilean limit  $\gamma \Rightarrow 1$ , of constitutive equations, considering that only this limit can be tested experimentally in present day practice.

## 5. Observer: physical material reference system

Each measurement needs a choice of a **material** observer [Brillouin 1970]. What is a precise mathematical model for a physical material reference system?

### 5.1. Einstein in 1905: observer is a coordinate-basis = tetrad

Lorentz transformation is 'defined' frequently as transformation of scalar coordinate system, from  $\{x^\mu\}$  to  $\{x^{\mu'}\}$ . Here  $x^\mu$  is interpreted as a scalar field,  $\mathbb{R}$ -valued function on a manifold of spacetime events. Each coordinate system determine coordinate co-frame,  $\{x^\mu\} \Rightarrow \{dx^\mu\}$ . In 1905 Einstein considered that a physical material reference system can be identified with a mathematical coordinate-basis in spacetime, an observer =  $\{dx^\mu\}$ . Such observer is known also as a (holonomic, integrable) frame, or as a (holonomic) tetrad. Whereas tensor fields are basis-free, *i.e.*, observer-free within identification of an observer with a basis, therefore, every tensor field is absolute (not relative) within observers-bases. When observer is a basis, then, physically meaningful (experimentally measurable) quantities are scalar components of absolute-tensors, relative to a chosen coordinate-basis-observer. This is because these scalar components of absolute-tensors are observer-dependent, being dependent on a choice of an observer-basis.

Relativity transformation does not acts on events, does not acts on Minkowski space-time. Therefore does not acts on scalar fields, not on functions, not on coordinates. To say that a scalar field is Lorentz invariant (or *GL*-invariant) is empty phrase because Lorentz group does not acts on scalar fields. A phrase 'Lorentz-invariant parameter' *e. g.* in [Trump and Schieve 1999 p. 13] raise eyebrows. Domain of Lorentz group are all vectors, and this induce transformation of

all tensors, except of scalar fields. Lorentz group commute with contraction of tensors, cf with [Minkowski 1908 §11.6 formula (43) and ff.].

Each observer-frame-tetrad,  $\{dx^\mu\}$ , defines a unique Lorentz metric tensor  $g$ , such that relative to  $g$ , this observer-frame is Lorentz orthonormal basis. Then, Lorentz isometry group is defined as a group permuting orthonormal observer-bases, leaving derived Lorentz metric-tensor invariant. The Lorentz-covariance, then, is identified with transformation of scalar components with respect to change of observers-frames. This is the philosophy shared by all present textbooks, see, for example [Hehl and Obukhov 2003, §D.5.4, pages 296–297]. The transformation of observer-frames and of scalar components, is considered to be only relativity transformations. All tensors that are basis-free are considered to be absolute = Lorentz-invariant.

A transformation is said to be passive if domain of transformation is a non-linear manifold of all bases of a vector space. ‘Passive’ means **active** action on coordinate-basis, but not on individual absolute-vectors. Some authors claim that tensor fields (like for example differential biform of electromagnetic field  $F$ ), are invariant with respect to passive Lorentz transformations. This claim is incorrect, because every tensor field is basis-free and coordinate-free, and can be presented in arbitrary non-orthogonal basis, not necessarily in Lorentz-basis. Therefore, in fact, each tensor is invariant relative to passive general linear group, GL-group, acting on bases.

Of course, every tensor is GL-covariant, and also Lorentz-covariant, relative to GL and Lorentz transformations acting on individual vectors (and not on bases).

## 5.2. Euler’s fluid and Minkowski in 1908: observer = monad

An objection against Einstein’s identification of physical material reference system with mathematical coordinate-basis or with a frame, is that neither coordinates nor bases need physical concept of a material body. How distinguish coordinates of material “point” from not material point? No inertial mass is involved in a mathematical concept of a frame as a basis [Brillouin 1970]. In fact kinematics does not exist without a concept of inertial mass and center of mass, however in textbooks concept of a mass is considered as a part of dynamics, and not of kinematics.

An alternative point of view, different from Einstein’s in 1905, is axiom that physics (observers, measurements, etc.) is coordinate-free, and basis-free. Because several physical concepts are observer-dependent, like electric field, hence an observer, within this axiom, cannot be identified with a mathematical basis.

If physics is coordinate-free and basis-free, then, mass-irrelevant old coordinate kinematics as presented for example in [Whittaker 1952], is useless, and must be replaced by Leonard Euler’s material fluid introduced in 1754 as a vector field in spacetime. Euler’s fluid was reincarnated by Minkowski in 1908: physical material reference system is identified with a time-like fluid vector field in a space-time, known also as a monad, and abbreviated in the present note as observer-monad, or as observer-vector. Within this view mathematical bases are irrelevant for physics. Clearly, a time-like vector field cannot describe a massless radiation, and therefore is related to some non-zero mass-density as in Euler’s approach. Mass-density ‘ $\rho$ ’ enter to energy-momentum tensor of a perfect fluid, and it is indispensable for two-body kinematics of center-of-mass and for reduced mass.

The monad-observers, time-like fluids, were re-invented independently by Eckart in 1940, Ehlers in 1961, and by Abraham Zelmanov (1913-1987) in his PhD Dissertation in 1944, and in his publication in 1976. Minkowski’s first invention of monad-observer in 1908 went to oblivion. For discussion of Einstein’s tetrad, versus Minkowski’s monad, we refer to [Mitskievich 2006, Chapter 2].

Consider two material bodies, reference systems, **P** for **P**aul, and **R** for **R**ose. Each material reference system is a monad, *i.e.* it is  $g$ -normalized time-like vector field. All material bodies

(inertial or non-inertial) in spacetime must be treated on the same footing and must therefore be not-distinguishable by normalization [Minkowski 1908 Part I §4 Eq. (19); Part II §8 Eq. (27) and §11.6 Eq. (46)],

$$\mathbf{P}^2 \equiv \mathbf{P} \cdot \mathbf{P} = \mathbf{R} \cdot \mathbf{R} = -1. \quad (5.1)$$

Vector field is coordinate-free, therefore coordinate expressions, like,  $\mathbf{P} = \mathbf{P}^\mu \partial_\mu = \partial_{ct}$ , are not so important. The scalar magnitude of light velocity in vacuo is denoted by  $c$ .

Let  $\mathbf{w} = \mathbf{w}_x \partial_x + \mathbf{w}_y \partial_y + \mathbf{w}_z \partial_z$ , be space-like velocity of Rose relative to Paul, i.e. a relative velocity as measured by Paul,  $\mathbf{w} \cdot \mathbf{P} = 0$ . Two observers-monads are as follows [Minkowski 1908 §4 before time-like normalization (19); §11.6, before equation (46)],

$$\text{Rose's monad } \mathbf{R} \equiv \gamma \{ \mathbf{w} + \mathbf{P} \}, \quad \text{Paul's monad } \mathbf{P} = \partial_{ct}. \quad (5.2)$$

Minkowski's observer is denoted by a letter ' $w$ ' in [Minkowski 1908 before Eq. (46)], and here is denoted by **Rose R**.

Minkowski in 1908 before Eq. (46)	Our notation in Equation (5.2)
$w \equiv \{w_1, w_2, w_3, w_4\}$	$\mathbf{R}$
$i$	$\mathbf{P}$
$\mathbf{w} \equiv \{\mathbf{w}_x, \mathbf{w}_y, \mathbf{w}_z\}$	$\mathbf{w} = \mathbf{w}_x \partial_x + \mathbf{w}_y \partial_y + \mathbf{w}_z \partial_z$

Minkowski's line before equation (46), is the definition of relative velocity  $\mathbf{w}$ , and can be read as follows

$$w \equiv \{w_1, w_2, w_3, w_4\} \equiv w_1 \partial_x + w_2 \partial_y + w_3 \partial_z + w_4 \partial_{ict} = \gamma \{ \mathbf{w} + i \partial_{ict} \} \quad (5.3)$$

This pair of Minkowski's observers, Equation (5.2),  $\mathbf{P}$  and  $\mathbf{R}$ , is Eq. (8a) in [Hamdan 2006].

Heaviside [1888, 1889] introduced scalar factor  $\gamma$ , and his definition that follows, is equivalent to Minkowski's normalization  $\mathbf{R}^2 = -1$  in [Minkowski Eq. (46)]

$$\mathbf{P} \cdot \mathbf{P} = \mathbf{R} \cdot \mathbf{R} = -1 \implies \mathbf{P} \cdot \mathbf{R} = -\gamma, \quad (5.4)$$

$$\mathbf{R}^2 = -1 \iff \left( \frac{\mathbf{u}}{c} \right)^2 = 1 - \frac{1}{\gamma^2}, \quad (5.5)$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \mathbf{u}^2/c^2}} \simeq 1 + \frac{1}{2} \left| \frac{\mathbf{u}}{c} \right|^2 + \frac{3}{8} \left| \frac{\mathbf{u}}{c} \right|^4 + \frac{5}{16} \left| \frac{\mathbf{u}}{c} \right|^6 + \dots \quad (5.6)$$

## 6. Cross product of vectors in spacetime

Relativity transformation of electric and magnetic fields, see Equation (11.3) and Equation (12.15) below, need orientation-dependent Gibbs-like cross product of vectors,  $\times$ , in spacetime of dimension four. Gibbs's cross product of vectors,  $\times$ , needs Hodge-star isomorphisms (or duality). One Hodge-star acting on Grassmann's multi-vectors, and another Hodge-star acting on Grassmann multi-forms. Hodge star ' $\star$ ' was invented by Hermann Grassmann under the name *Ergänzung*, and denoted by vertical dash  $|$ , [Grassmann 1862, Chapter 3, §4-5]. In 1908 Minkowski denoted dual of  $F$  by  $F^*$ . Star-notation,  $\star$ , dual of differential form  $F$  by  $\star F$ , introduced Hermann Weyl in 1945. Hodge-star isomorphism is a tensorial concomitant of metric tensor-field  $g$ , and depends on a choice of orientation [e.g. duality in Misner, Thorne & Wheeler 1973, §3.5; Kocik 1998 and WEB page; Oziewicz 1994; Cruz & Oziewicz 2003]. Cross-product of vectors gives an orientation-dependent *pseudo*-vector, i.e. strictly speaking this is *not* a binary operation.

In dimension three, binary cross product of vectors,  $\mathbf{u} \times \mathbf{v} \equiv \star(\mathbf{u} \wedge \mathbf{v})$ , was invented by Clifford, and was popularized by Heaviside in monograph *Electromagnetic Theory* [1893], and by Gibbs

in *Vector Analysis* [1901]. However, Maxwell differential equations needs physical intensive and extensive vector fields on **four**-dimensional spacetime, and spacetime was ‘not known’ explicitly before Minkowski in 1908. Clifford-Gibbs cross product in dimension three is totally irrelevant for electromagnetism, have nothing to do with the subject matter of electric and magnetic fields, and it darkens ideas. Electromagnetic laws, four Maxwell’s differential equations, ponderomotive force (called the Lorentz force), and relativity transformation of electric and magnetic fields, with Gibbs cross in *three* dimensions, become a thoughtless mechanical set of strange formulas, and this is mortal for electromagnetism and radiation in spacetime. Relativity transformation of electric and magnetic fields, see Equation (11.3) below in §11, ponderomotive force, **need** cross product of vectors in **four**-dimensional spacetime.

Generalization of cross product in arbitrary dimension was considered by Eckmann in 1942. Plebański with Przanowski in 1988 defined binary cross product of vectors in arbitrary dimension, in terms of augmented quaternion-like algebra of para-vectors. However both these attempts we consider not satisfactory because, among other things, orientation-dependence is either lost or it is not explicit.

Minkowski’s definition of magnetic pseudo-field  $B$  needs Hodge-star isomorphism  $\star$  acting on Grassmann’s multiforms. Hodge-star intertwine the Grassmann exterior multiplication, the exterior wedge product acting on right, denoted by  $e_{\mathbf{P}}\mathbf{R} \equiv \mathbf{P} \wedge \mathbf{R}$ , with interior product that is dual to exterior, denoted by  $i_{\mathbf{P}} \equiv (e_{\mathbf{P}})^*$ ,

$$\star \circ e_P = i_P \circ \star, \quad e_P \circ \star = \star \circ i_P, \quad (6.1)$$

$$\mathbf{P}^2 \cdot \text{id} = e_P \circ i_P + i_P \circ e_P. \quad (6.2)$$

For interior product we use also following abbreviation,  $i_{\mathbf{P}}F \equiv \mathbf{P} \cdot F$ .

**6.1 Definition** (Cross product in spacetime). Let  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{P}$  be vector fields on four-dimensional space-time. Gibbs-like ‘binary’ cross-product,  $\times_{\mathbf{P}}$ , of vector fields,  $\mathbf{A}$  and  $\mathbf{B}$ , is orientation-dependent, and  $\mathbf{P}$ -dependent, and is defined in terms of the Hodge star map as follows,

$$\mathbf{A} \times_{\mathbf{P}} \mathbf{B} \equiv \star(\mathbf{A} \wedge \mathbf{P} \wedge \mathbf{B}) = -\mathbf{B} \times_{\mathbf{P}} \mathbf{A}. \quad (6.3)$$

Hence, cross-product of vectors in dimension four, is a ‘ternary’ operation. The same definition Equation (6.3) applies for covariant differential one-forms. In four-dimensional spacetime, binary cross  $\times_{\mathbf{P}}$  depends on a choice of an auxiliary vector field  $\mathbf{P}$ . This vector-field-dependence of binary-cross in a spacetime, is of crucial importance for understanding. It is either not realized or thoughtlessly suppressed, when presenting Lorentz transformations of electric and magnetic fields, and when presenting ponderomotive (Lorentz) force as a tensorial concomitant of the electromagnetic field and electromagnetic spin-charge density, see subsection 12.1.1 and expression Equation (12.6) below.

**6.2 Exercise.** Let  $\mathbf{P}^2 = -1$ ,  $\mathbf{w} \cdot \mathbf{P} = 0$ , and  $\mathbf{P} \cdot \mathbf{E} = 0$ . Then

$$\mathbf{w} \times_{\mathbf{P}} \{\mathbf{w} \times_{\mathbf{P}} \mathbf{E}\} = \mathbf{w}^2 \mathbf{E} - (\mathbf{w} \cdot \mathbf{E}) \mathbf{w}. \quad (6.4)$$

## 7. Minkowski in 1908: electric and magnetic fields are concomitants

In 1908 Minkowski introduced electromagnetic field as a differential bi-form  $F$  on spacetime. A differential biform of electromagnetic field,  $F$  (or a bivector field  $\mathbf{F}$ ,  $F \equiv g^\wedge \mathbf{F}$ ), are often called the Faraday tensors, however they were introduced in 1908 by Minkowski (1864-1909), and not by Michael Faraday (1791-1867).

It is convenient to consider electric and magnetic fields also as differential one-forms on four-dimensional spacetime, denoted by italics letters,  $E$  and  $B$ , instead of corresponding vector fields,  $\mathbf{E}$  and  $\mathbf{B}$ , where  $E \equiv g\mathbf{E}$ , etc.

**7.1 Definition** (Electric and magnetic fields). [Minkowski 1908, Part II: Electromagnetic Phenomena, §11.6] Let observer Paul be given by time-like vector field  $\mathbf{P}$ , and can also be given as a time-like differential one-form, denoted by italic letter  $P \equiv g\mathbf{P}$ . These are fields on four-dimensional Lorentzian space-time with a metric tensor  $g$ .

Minkowski defined electric and magnetic fields on *four-dimensional spacetime* as differential one-forms being the following observer-dependent concomitants,

$$\begin{aligned} E(F, \mathbf{P}_{\text{Paul}}) &\equiv \mathbf{P} \cdot F, \\ B(F, \mathbf{P}_{\text{Paul}}) &\equiv \star(P \wedge F) = \mathbf{P} \cdot (\star F), \quad \star B(F, P) = P \wedge F. \end{aligned} \quad (7.1)$$

Electric and magnetic fields are concomitant of two variables, they are  $F$ -dependent (depends on electromagnetic sources via Maxwell equations), and they are observer-dependent (depends on time-like Paul- reference system).

**7.2 Exercise.** Set  $E \equiv E(F, \mathbf{P})$  and  $B \equiv B(F, P)$ . As a corollary we have in particular the following relation,

$$E \mathbf{P} = 0 = \mathbf{E} \cdot \mathbf{P}, \quad B \mathbf{P} = 0 = \mathbf{B} \cdot \mathbf{P}, \quad (7.2)$$

$$(\star F)^2 \simeq -F^2, \quad -F = P \wedge E + \star(P \wedge B), \quad (7.3)$$

$$\begin{aligned} E^2 &= (\mathbf{P} \cdot F)^2 = P^2 F^2 - (P \wedge F)^2, \\ B^2 &= (\mathbf{P} \cdot \star F)^2 = P^2 (\star F)^2 - \{\star(\mathbf{P} \cdot F)\}^2. \end{aligned} \quad (7.4)$$

Minkowski's definition 7.1 was re-invented by number of authors, e.g. [Fecko 1997; Kocik 1998; Ivezić 2003; Cruz and Oziewicz 2003; Hehl and Obukhov 2003, §B.2.2 page 123, Definition (B.2.10)]. Hehl and Obukhov denoted observer vector field by ' $n$ ' in Hehl's (B.1.22), see pages 115-117, and depart from Minkowski Definition 7.1 in two respects. Firstly, Hehl and Obukhov are metric-free except of Chapter E.4. Their observer  $n$  is restricted by transverse condition for a given 'would-be-time  $\sigma$ ' scalar field on page 115. This is similar to an idea that an observer is an idempotent endomorphism field,  $n \otimes d\sigma$ , with  $\text{trace}(n \otimes d\sigma) = 1$  [Świerk 1988; Kocik 1998]. Many-observer, many-body metric-free kinematics generated by trace-class idempotents we considered in [Oziewicz 2007].

Secondly, Hehl and Obukhov stress that electric and magnetic fields are in **three-dimension** and not on spacetime [Hehl and Obukhov 2003, after (B.2.9) on page 123]. In Minkowski definition 7.1 electric field is on four-dimensional spacetime.

We must emphasize essential difference of Minkowski's 1908-Definition 7.1 when compared with other textbooks. According to Minkowski, electric field  $\mathbf{E}$  is  $F$ -dependent and observer-dependent, depends on electromagnetic field,  $\mathbf{E} = \mathbf{E}(F, \mathbf{P})$ . Electromagnetic field  $F$  is absolute (observer-free), and electric field is **derived** concept that is observer-relative.

**7.3 Misleading 'definition'** (Electromagnetic field strength). In some textbooks, one can find the following 'definition'

$$F^{\alpha\beta} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} \quad (7.5)$$

See for example [Sommerfeld 1948, 1964 §26 B and C; Landau and Lifshitz since 1951, edition 1975, formula (23.5) on page 61; Møller 1952 §53 page 141; Fock 1955, 1961 §24; Tonnelat 1959 Chapter 9; Jackson, since 1962, last edition 1999 formula (11.137); Barut 'Electrodynamics' 1964, 1980, page 96].

What means ‘definition’ Equation (7.5)? Authors explain: on the right there are ‘well known time-dependent three-dimensional’ electric and magnetic fields, denoted by  $\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{B}(\mathbf{r}, t)$ , as primary concepts given in terms of basis-dependent components,  $\mathbf{E} = \{E_x, E_y, E_z\}$  and  $\mathbf{B} = \{B_x, B_y, B_z\}$ . Whereas we consider that electric field is basis-free. On the left is new ‘tensor’  $F$  **defined** in terms of ‘well known time-dependent three-dimensional’ electric and magnetic fields, *i.e.*  $F = F(\mathbf{E}, \mathbf{B})$ , so electromagnetic field is defined as  $\mathbf{E}$ -dependent and  $\mathbf{B}$ -dependent.

Time-dependent electric field  $\mathbf{E}$  must be vector field on four-dimensional spacetime, independently of how many basis-dependent components vanishes. Definition 7.5 suggest that time-component  $E_t$  vanishes or it is not known, or maybe  $\mathbf{E}$  is in fact a time-dependent field on three-dimensional space? But in fact this never appeared time-component is nothing but  $E_t = \mathbf{E} \cdot \mathbf{P}$  a scalar product with suppressed observer  $\mathbf{P}$ . We have the Minkowski conditions  $\mathbf{E} \cdot \mathbf{P} = 0 = \mathbf{B} \cdot \mathbf{P}$ , *i.e.* electric and magnetic fields are  $\mathbf{P}$ -constrained. Such observer-constrained vector field on four-dimensional space-time Ivezić call ‘3D’ quantity, and we consider that this name is misleading.

One can try to interpret ‘definition’ Equation (7.5), as well as Sommerfeld’s six-vector, as follows. Primary concept must be observer, electric and magnetic fields needs observer as the constraint, and then electromagnetic field in Equation (7.5) must be also observer-constrained,  $F_{\mathbf{P}}$ ,

$$\text{observer} \quad \leftrightarrow \quad \text{electric and magnetic} \quad \leftrightarrow \quad \text{electromagnetic}, \quad (7.6)$$

$$\mathbf{P} \quad \leftrightarrow \quad \mathbf{E} \cdot \mathbf{P} = 0 = \mathbf{B} \cdot \mathbf{P} \quad \leftrightarrow \quad F_{\mathbf{P}}(\mathbf{E}, \mathbf{B}) \simeq F(\mathbf{P}, \mathbf{E}, \mathbf{B}). \quad (7.7)$$

Let look for explicit  $\mathbf{P}$ -dependence of electromagnetic field  $F_{\mathbf{P}}$  in Equation (7.5). This task has the well known analogy in basis-dependent theorem.

Using identity Equation (6.2) [Minkowski 1908 §11.6 identity (45)] and the Minkowski definitions Equation (7.1), Minkowski deduced the following **theorem** [Minkowski formulas (55)-(56)],

$$F = \text{id } F \simeq E(F) \wedge P - i_{\mathbf{P}} \star B(F). \quad (7.8)$$

Minkowski’s **theorem** Equation (7.8), some textbooks takes naively as the **definition** 7.5 of observer-dependent electromagnetic field,

$$F(P, E, B) \equiv E \wedge P - i_{\mathbf{P}} \star B, \quad \text{with} \quad E \cdot P = 0 = B \cdot P. \quad (7.9)$$

Coordinate-free and basis-free definition Equation (7.9) is equivalent to matrix ‘definition’ Equation (7.5).

In textbooks that accept ‘definition’ Equation (7.5), electric field  $\mathbf{E}$  is observer-constrained vector field on *four*-dimensional space-time  $\mathbf{E} \cdot \mathbf{P} = 0$ . Scalar components,  $E_x \equiv (dx)\mathbf{E} = \mathbf{E}x$ ,  $E_t \equiv (dt)\mathbf{E} = \mathbf{E}t$ , etc, are  $\mathbf{E}$ -dependent. But we must be careful here, because these textbooks consider that vector depends on his scalar components! For example, some authors consider that expression  $\mathbf{E} = E_x \partial_x + \dots$ , is a *definition* (whereas it is a theorem!). These textbooks consider that on the right there are obvious and clear physical basis vector  $\mathbf{i} \equiv \partial_x$  (dogma that basis is physical and not mathematical), and measurable scalar field  $E_x \equiv (dx)\mathbf{E}$ , whereas on the left there is an ‘artificial’ symbol,  $\mathbf{E}$ , that must be understood as a function of  $E_x$ , *i.e.* it is insisting that vector  $\mathbf{E}$  is nothing but a set of components  $\mathbf{E} \equiv \{E_x, E_y, E_z, E_t\}$  and vector  $\mathbf{E} = \mathbf{E}(E_x, \dots)$  is  $E_x$ -dependent. When we change  $E_x$  then such vector  $\mathbf{E}$  must change,  $\mathbf{E}(E_x, \dots)$ . It is hard to find worse case.

In fact, theorem,  $\mathbf{E} = E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k} + \dots$ , must be understand that a vector  $\mathbf{E}$  and a basis  $\{\mathbf{i}, \mathbf{j}, \mathbf{k}, \dots\}$ , are independent concepts, and scalar component  $E_x$  is  $\mathbf{E}$ -dependent, and is basis-dependent,  $E_x(\mathbf{E}, \mathbf{i}, \mathbf{j}, \mathbf{k}, \dots)$ ,

$$\text{Theorem:} \quad \mathbf{E} = E_x(\mathbf{E}, \mathbf{i}, \mathbf{j}, \mathbf{k}, \dots) \mathbf{i} + E_y(\mathbf{E}, \mathbf{i}, \mathbf{j}, \mathbf{k}, \dots) \mathbf{j} + \dots \quad (7.10)$$

The worst textbooks of algebra consider Equation (7.10) as the definition of a vector, without aware that every vector of a basis is also a vector, and is impossible to define a vector  $\mathbf{E}$  in terms of another vector  $\mathbf{i}$ , etc.

What means ‘definition’ Equation (7.5)? In order to define electromagnetic field  $F$ , we need first to have primary electric and magnetic fields, that probably must have never more than three components maybe, because component  $E_t$  is absent in ‘definition’ Equation (7.5). Time-dependence of such electric field is explicit in Maxwell equation, e.g.  $\mathbf{E} = \partial_t \mathbf{A} + \dots$ , that must read as Lie-Ślebodziński derivative of potential in direction of observer field,  $E = \mathcal{L}_{\mathbf{P}} A - d(\mathbf{A}\mathbf{P})$ . Such electric field with strange  $E_t = 0$  or  $E_t \neq 0$  must be known without of knowing  $E_t$ , before we can start to define electromagnetic field  $F$  Equation (7.5). According to some lecturers I knew personally  $F$  is artificial mathematical constructs that does not exists in Nature.

In present paper electromagnetic field  $F$  is primary concept, and it is  $\mathbf{E}$ -independent, as state by Minkowski in 1908.

Textbook presentations of four Maxwell differential equations hidden reference system. Implicit reference system is a source of **wrong** interpretation of Maxwell equations by Dirac suggesting magnetic monopole. The explicit observer-dependence is given in [Minkowski 1908; Fecko 1997; Kocik 1998; Cruz Guzman & Oziewicz 2003].

## 8. Isometric relativity theory (not about gravity)

**8.1 Definition** (Minkowski 1908, Introduction and §11.6). Theory where material reference systems are allowed to be connected by isometric Lorentz-group transformations **only** is said to be the *isometric relativity theory*. Isometric relativity, or Lorentz-group relativity, is defined by this **only one** postulate.

**8.2 Clarification.** The same definition of relativity theory was given by many authors: Schrödinger 1931; Peter Bergmann 1942, page 159; Einstein 1949; V. Bargmann 1957 p. 161; Barut 1964 Chapter I.6; Hehl and Obukhov 2003, §C.2, page 211.

It is worthy to emphasize that Albert Einstein in 1949 accepted Minkowski **one-axiom** definition, and forget about his two 1905-postulates including constancy of light velocity. Einstein separated special from general relativity in terms of properties of metric tensor. However the very concept of *relativity of observers* has nothing to do with properties of metric tensor, observers are *relative* independently of covariant derivative  $\nabla g$ , independently of connections  $\{\nabla\}$  and their curvatures and torsions.

**8.3 Clarification.** Often it is stressed incorrectly that relativity theory deals with *inertial* material systems only. This dogma insist that between non-inertial material bodies does not exist physically equivalent reference systems subject of relativity theory. Do Nature offer inertial reference systems at all? or they are not-natural mathematical idealizations only? After all condition to be inertial is *differential* condition on material system and it is hard to find in abundant ‘special relativity’ bibliography at least one paper where this differential condition is assumed and used. A material system  $\mathbf{P}$  is said to be inertial if covariant derivative vanishes,  $\nabla \mathbf{P} = 0$ . Minkowski definition 8.1 hold for all material reference systems, including also non-inertial ones, as already noted in [Logunov 1990, 2004]. Logunov define (special) relativity theory as spacetime with absolute pseudo-Euclidean metric tensor  $g$ , and this implicitly implies that only allowed transformations between material reference systems are  $g$ -isometries, as in Minkowski Definition 8.1

**8.4 Clarification.** Definition 8.1 does **not** use a concept of relative velocity. Within such axiomatic isometric relativity theory, relative velocity must lay in Lobachevsky factor space. It was Vladimir Varičák in 1924 who developed non-Euclidean geometry as most natural structure for relativity theory. We stress here important fact (overlooked by Varičák) that Lobachevsky factor space is not unique, and this imply that relative velocity between given reference systems

is also not unique, because every such relative velocity needs a priori a choice of Lobachevsky factor space, and each choice is equivalent to privileged  $\mathcal{A}$ ether.

Lorentz group is in fact equivalent to reciprocity principle: each velocity of reference system  $\mathbf{R}$  relative to an observer  $\mathbf{P}$ , not unique  $\mathbf{v}_{\mathbf{P},\mathbf{R}}$  in Lobachevsky factor-space, satisfies reciprocity condition,  $\mathbf{v}_{\mathbf{P},\mathbf{R}} = -\mathbf{v}_{\mathbf{R},\mathbf{P}}$ . For not uniqueness of relative velocity  $\mathbf{v}_{\mathbf{P},\mathbf{R}}$  we refer to [Oziewicz 2007]. One can assume reciprocity of relative velocity, and deduce Lorentz group transformations, see, for example [Silagadze 2008].

**8.5 Clarification.** One can define Lorentz isometry group either as:

- Acting on observers-frames-tetrads with Lorentz ‘passive’ action on scalar components [Einstein 1905], for postulated absolute-vectors.
- Acting on individual covariant-vectors in Minkowski spacetime (when a basis is physically meaningless), with a Lorentz ‘active’-action on the Lorentz-covariant tensor fields.
- Lorentz isometry group, as every isometry, does not act on manifold of events. This means that *all* scalar fields are  $GL$ -invariant, and in particular they are Lorentz-invariant.

**8.6 Clarification.** In Section 12 we define an alternative to above group-relativity. This alternative relativity theory is based on **only one** axiom that each ordered pair of material reference system possesses **only one unique** relative velocity. This unique relative velocity was introduced by Minkowski [Minkowski 1908 §11.6, before equation (46)], as the velocity of matter. As the consequence of this axiom a set of all relativity transformations among material reference systems is a groupoid that is not a group. This alternative relativity theory we call the *groupoid relativity*. Domain of groupoid transformation does not include light-like and space-like vectors. Therefore groupoid transformation do not extends to tensor algebra: it can not be neither isometry nor violate isometry. Metric tensor need not to be postulated as absolute.

## 9. Minkowski in 1908: Lorentz-covariance

Hermann Minkowski in Part II Electromagnetic Phenomena [Minkowski 1908, §11.6, after expression (44)], explains what Lorentz-covariance of a concomitant vector field, like Equation (7.1), means on a four-dimensional spacetime manifold. Let  $\mathbf{F}$  denotes bi-vector field, and  $\alpha$  is a differential Pfaff one-form (all tensor fields are on spacetime). Hence inner product can be interpreted in many ways,

$$(\mathbf{F}_*)\alpha \equiv \mathbf{F} \circ e_\alpha \equiv \alpha \cdot \mathbf{F} \equiv i_\alpha \mathbf{F}. \quad (9.1)$$

We repeat Minkowski’s text in our notation. Let  $GL$  denote general-linear group, and let  $L \in GL$ . This means that  $L$  is an invertible endomorphism.  $GL$ -covariance, according to Minkowski, means the following set of transformations,

$$\begin{aligned} i_\alpha \mathbf{F} &\longmapsto L(i_\alpha \mathbf{F}), \\ \alpha &\longmapsto L^{*-1} \alpha, \\ \mathbf{F} &\longmapsto L^\wedge \mathbf{F}. \end{aligned} \quad (9.2)$$

Reader could ask why transformation of form must be ‘contragradient’? We assume that every vector transform as  $\mathbf{P} \longmapsto L\mathbf{P}$ , and that  $GL$ -action commute with contraction. All scalar fields are  $GL$ -invariant, therefore

$$\alpha \mathbf{P} \longmapsto L(\alpha \mathbf{P}) = (L^\flat \alpha)(L\mathbf{P}) = (L^\flat \alpha \circ L)\mathbf{P} = \{(L^* \circ L^\flat)\alpha\}\mathbf{P} = \alpha \mathbf{P}. \quad (9.3)$$

$$\begin{aligned} (\mathbf{F}_*)\alpha \equiv i_\alpha \mathbf{F} &\xrightarrow{\text{transformation}} L(i_\alpha \mathbf{F}) = i_{L^{*-1}\alpha} \{L^\wedge \mathbf{F}\} = (L^\wedge \mathbf{F})_*(L^{*-1}\alpha) \\ &= (L \circ \mathbf{F}_* \circ L^*) \circ (L^{*-1}\alpha), \end{aligned} \quad (9.4)$$



$$(\mathbf{F}_*)\alpha \longmapsto \{(\mathbf{F}_*)\alpha\}' \equiv L((\mathbf{F}_*)\alpha) = (\mathbf{F}'_*)\alpha'. \quad (9.5)$$

This is, up to notation, Minkowski's definition of  $GL$ -covariance [Minkowski 1908, §11.6, after expressions (44)]. Minkowski is considering Lorentz-covariance (and notation  $A$  for our  $L^{*-1}$ ), however, a concept of group-covariance is exactly the same for general linear group  $GL$ , as well as for any subgroup of  $GL$ , for example for Lorentz-isometry subgroup,  $O_g \subset GL$ ,  $O(3,1) \subset GL(4)$ .

In 1908 Minkowski realized 'pure mathematical formality' [quotation from the first page of his Introduction], that Lorentz transformation is an isomorphism of a vector space, then entire algebra of tensor fields must be Lorentz-covariant. Every vector is Lorentz-covariant, and an observer time-like vector field, also must be Lorentz-covariant. All tensor fields,  $F$  and  $\mathbf{P}$ , must be Lorentz-covariant. Lorentz transformation must act on all tensor fields, including time-like vector fields. Hence electromagnetic field  $F$ , potential  $A$ , and **Paul**  $\mathbf{P}$ , must be Lorentz-covariant.

Lorentz-covariance of concomitant tensor fields that are observer-dependent, like Lorentz covariance of electric and magnetic fields, Equation (7.1), Lorentz covariance of charge and current densities, Equation (12.7)-Equation (12.8), is misunderstood in textbooks on electromagnetism. First definition of Lorentz-covariance of electric and magnetic fields, is Einstein's definition by means of transformation of *coordinate basis* in a system of four differential Maxwell equations,  $\partial_t \mathbf{E} = \text{rot } \mathbf{B}$ , etc., [Einstein 1905, Part II. Elektrodynamischer Teil, pages 907-909; Bergmann 1942, page 106; Hamdan 2006]. A fact that four Maxwell equations depend on a choice of observer, depend on a choice of a product structure (space  $\oplus$  time)-split, was realized by Fecko in 1997 and by Kocik in 1998, see also [Cruz Guzman and Oziewicz 2003].

For how Lorentz-covariance and Lorentz-invariance is understood one hundred years after Minkowski we refer to [Arunasalam 2001].

## 10. Ivezić in 2005: Lorentz-covariance

In 2005 Ivezić re-discovered Minkowski's 'pure mathematical formality' of Lorentz-covariance. Ivezić's version of relativity is coined invariant special relativity = ISR, where invariant is synonym to be basis-free, as in [Misner, Thorne and Wheeler 1973, end of Chapter 2]. A tensor field is basis-free, therefore with respect to a 'passive' action of Lorentz group on non-linear manifold of bases-frames-tetrads, every tensor field is Lorentz-invariant. Such Lorentz-invariance is misleading, if a mathematical basis has no physical interpretation: basis-free tensor is trivially  $GL$ -invariant passively, and Lorentz isometry subgroup is here irrelevant. An alternative is to try to interpret a mathematical basis as a physical-experimental concept; however, this is outside of the philosophy of the present discussion.

Ivezić defined Lorentz-covariance of compound electric and magnetic fields, Equation (7.1), exactly as defined by Minkowski in 1908 in §11.6, just before formula (46). We stress that Minkowski in 1908 does **not** use in practice his definition of Lorentz-covariance. Lorentz transformation of electric and magnetic concomitant vector fields, according to Ivezić definition of Lorentz-covariance, is as follows.

Consider two reference systems, labeled  $\mathbf{P}$  for **Paul**, and  $\mathbf{R}$  for **Rose**, identified, following Minkowski in 1908, with time-like vector fields on four-dimensional Lorentzian space-time with a metric tensor  $g$ .

Space-like electric and magnetic fields (time dependent) as measured by **Paul** are denoted respectively by,  $\mathbf{E}(\mathbf{P})$  and  $\mathbf{B}(\mathbf{P})$ . Analogously, we denote electric and magnetic fields as measured by **Rose**, by  $\mathbf{E}(\mathbf{R})$  and  $\mathbf{B}(\mathbf{R})$ . It was *assumed* by Minkowski in [1908, §11.6], see Exercise 7.2 above, that

$$\mathbf{P} \cdot \mathbf{E}(\mathbf{P}) = 0 = \mathbf{P} \cdot \mathbf{B}(\mathbf{P}) \quad \text{and} \quad \mathbf{R} \cdot \mathbf{E}(\mathbf{R}) = 0 = \mathbf{R} \cdot \mathbf{B}(\mathbf{R}). \quad (10.1)$$

In Eq. Equation (10.1), all vector fields are on a **four**-dimensional spacetime manifold: in order to satisfy the differential Maxwell equations, time-dependence must be allowed.

The question is: how are these electric and magnetic fields, Equation (10.1), which are due to the same sources, but are measured by two *different* observers in relative motion, how they are related?

The electric and magnetic fields are *relative*, they depend not only on electromagnetic sources, via the Maxwell differential equations, but also on the choice of a material reference system, e.g. the transformation Equation (10.4) or Equation (11.3) or Equation (12.15), that we are going to discuss in details.

Let  $\mathbf{u}$  be space-like velocity of Rose relative to Paul,  $\mathbf{u} \cdot \mathbf{P} = 0$ , [Minkowski 1908, §11.6, before Eq. (46)].

**10.1 Assumption.**  $\mathbf{u} \cdot \mathbf{P} = 0$ ,

$$\left. \begin{array}{l} \mathbf{E} \equiv \mathbf{E}(F, \mathbf{P}_{\text{Paul}}) \\ \mathbf{B} \equiv \mathbf{B}(F, \mathbf{P}_{\text{Paul}}) \end{array} \right\} \xrightarrow{\text{Lorentz-boost}} \left\{ \begin{array}{l} \mathbf{E}^I \equiv \mathbf{E}(F^J, \mathbf{R}_{\text{Rose}}) \\ \mathbf{B}^I \equiv \mathbf{B}(F^J, \mathbf{R}_{\text{Rose}}) \end{array} \right. \quad (10.2)$$

$$\mathbf{E} \cdot \mathbf{P} = 0 = \mathbf{B} \cdot \mathbf{P}, \quad \mathbf{E}^I \cdot \mathbf{R} = 0 = \mathbf{B}^I \cdot \mathbf{R}. \quad (10.3)$$

**10.2 Theorem** (Ivezić 2005 page 307, formulas (8-9-10).

$$\begin{aligned} \mathbf{E}^I &= \mathbf{E} + \gamma \left( \mathbf{E} \cdot \frac{\mathbf{u}}{c} \right) \left\{ \mathbf{P} + \frac{\gamma}{\gamma + 1} \frac{\mathbf{u}}{c} \right\}, \\ \mathbf{B}^I &= \mathbf{B} + \gamma \left( \mathbf{B} \cdot \frac{\mathbf{u}}{c} \right) \left\{ \mathbf{P} + \frac{\gamma}{\gamma + 1} \frac{\mathbf{u}}{c} \right\}. \end{aligned} \quad (10.4)$$

There are **no** formulae like, Equation (10.2)-Equation (10.4), in Minkowski's paper in 1908. In Equation (10.2)-Equation (10.16), a superscript  $I$  is for Ivezić. Minkowski never used his definition of group-covariance in practice. Ivezić-transformed electric field is no longer orthogonal to first observer,  $\mathbf{E}^I \cdot \mathbf{P} = -\gamma \mathbf{E} \cdot \mathbf{u}/c$ , according to relativity condition Equation (10.3).

*Proof-Hint.* Ivezić's transformation Equation (10.4) is specified case of well known Lorentz isometry transformation of vector, see for example [Fock 1955, 1961 §24], with only one notable difference, that in contrast to textbooks presentations here time-like observer  $\mathbf{P}$  is explicit *variable*, Lorentz-covariant.

In 1937 Élie Cartan noted that isometry can be generated by Grassmann bivector. Consider the following bivector,

$$b \equiv \mathbf{P} \wedge \gamma \frac{\mathbf{u}}{c}, \quad \text{with } \mathbf{P} \cdot \mathbf{u} = 0 \quad \text{and} \quad \mathbf{P}^2 = -1. \quad (10.5)$$

Let  $\mathbf{E}$  be arbitrary vector. Isometry transformation of  $\mathbf{E}$ , generated by above bivector,  $L_b$ , is as follows [Oziewicz 2005, 2006, 2007, 2009],

$$L_b \mathbf{E} = \mathbf{E} + \gamma \left( \mathbf{E} \cdot \frac{\mathbf{u}}{c} \right) \left\{ \mathbf{P} + \frac{\gamma}{\gamma + 1} \frac{\mathbf{u}}{c} \right\} - (\mathbf{E} \cdot \mathbf{P}) \left\{ \gamma \frac{\mathbf{u}}{c} + (\gamma - 1) \mathbf{P} \right\} \quad (10.6)$$

$$\implies \mathbf{R} \equiv L_b \mathbf{P} = \gamma \left( \frac{\mathbf{u}}{c} + \mathbf{P} \right). \quad (10.7)$$

Lorentz transformation of vector Equation (10.6) is nothing but, for example, Fock's transformation in [Fock 1955 or 1961, formulas (24.39)-(24.40)], or Jackson's formulas (11.19) in [Jackson 1962]. Namely, vector  $\mathbf{E}$  can be  $\mathbf{P}$ -decomposed as the sum of  $\mathbf{E}^\perp$  that is orthogonal to time-like observer  $\mathbf{P}$ , and  $\mathbf{E}^\parallel$ , that is parallel to time-like observer,

$$\begin{aligned} \mathbf{E} &\simeq \mathbf{E}^\perp + \mathbf{E}^\parallel, & \mathbf{E}^\perp \cdot \mathbf{P} &\equiv 0, \\ (L_b \mathbf{E})^\parallel &= \gamma \left\{ \mathbf{E}^\parallel + \left( \mathbf{E}^\perp \cdot \frac{\mathbf{u}}{c} \right) \mathbf{P} \right\}, \\ (L_b \mathbf{E})^\perp &= \mathbf{E}^\perp + \frac{\gamma^2}{\gamma + 1} \left( \mathbf{E}^\perp \cdot \frac{\mathbf{u}}{c} \right) \frac{\mathbf{u}}{c} - \gamma (\mathbf{E}^\parallel \cdot \mathbf{P}) \frac{\mathbf{u}}{c}. \end{aligned} \quad (10.8)$$

Let compare Equation (10.4) with derivation by Ivezić in [2005, page 307]. Ivezić used rotor introduced in Clifford algebra by David Hestenes, for a simple bivector Equation (10.5),

$$R \equiv \frac{\gamma + 1 + b}{\sqrt{2(\gamma + 1)}}, \quad b \equiv \mathbf{P} \wedge \gamma \frac{\mathbf{u}}{c}. \quad (10.9)$$

Denoting space-like relative velocity by,  $\beta \equiv \mathbf{u}/c$ , and time-like observer by,  $\gamma_0 \equiv \mathbf{P}$ , Ivezić in 2005 arrived to following expression

$$\mathbf{E}^I = \frac{1}{2}(\gamma + 1)\mathbf{E} + \gamma(\mathbf{E} \cdot \beta) \mathbf{P} \pm \frac{1}{2} \frac{\gamma^2}{\gamma + 1} \beta \mathbf{E} \beta. \quad (10.10)$$

Last term in Equation (10.10) is Clifford product of three one-vectors,

$$\beta \mathbf{E} \beta = 2(\mathbf{E} \cdot \beta) \beta - \beta^2 \mathbf{E}, \quad \beta^2 = \frac{(\gamma + 1)(\gamma - 1)}{\gamma^2}. \quad (10.11)$$

Plus sign in Equation (10.10) leads to Ivezić's relativity transformation Equation (10.4).  $\square$

### 10.3 Corollary.

$$(\mathbf{E}^I)^2 = \mathbf{E}^2, \quad (\mathbf{B}^I)^2 = \mathbf{B}^2, \quad \mathbf{E}^I \cdot \mathbf{B}^I = \mathbf{E} \cdot \mathbf{B}, \quad (10.12)$$

$$(\mathbf{P} + \mathbf{u}/c) \cdot \mathbf{E}^I = 0, \quad (\mathbf{P} + \mathbf{u}/c) \cdot \mathbf{B}^I = 0, \quad (10.13)$$

$$\mathbf{P} \cdot \mathbf{E}^I = -\mathbf{E} \cdot \gamma \mathbf{u}/c, \quad \mathbf{P} \cdot \mathbf{B}^I = -\mathbf{B} \cdot \gamma \mathbf{u}/c. \quad (10.14)$$

### 10.4 Corollary.

$$\mathbf{R} \equiv \gamma \left( \mathbf{P} + \frac{\mathbf{u}}{c} \right) \implies \mathbf{R} \cdot \mathbf{E}^I \equiv 0. \quad (10.15)$$

### 10.5 Corollary.

$$\begin{aligned} \mathbf{u} \cdot \mathbf{E} = 0 &\iff \mathbf{E}^I = \mathbf{E}, \\ \mathbf{u} \cdot \mathbf{B} = 0 &\iff \mathbf{B}^I = \mathbf{B}. \end{aligned} \quad (10.16)$$

## 11. Absolute observer

Different electromagnetic fields, variable electromagnetic fields can be registered in the same fixed reference system. Transformation of electromagnetic field does not imply that an observer must also be transformed.

**11.1 Side remark.** In 1905 and again in 1907 Albert Einstein derived relativity transformation of electric and magnetic fields, transformations Equation (11.3) below, applying Lorentz isometry-group to a system of four differential Maxwell equations. Landau and Lifshitz re-derived transformations, Equation (11.3) below, without Maxwell equations [Landau and Lifshitz 1975, §24]. Whereas Hamdan re-deduced transformations Equation (11.3) using Maxwell equations as Einstein did [Hamdan 2006]. We note that four Maxwell's equations have already a fixed observer assumed a priori [Fecko 1997; Kocik 1998].

Observer need not to be inertial, and isometry need not commute with differential  $d$ , therefore Lorentz-covariance of **four** Maxwell differential equations that are observer-dependent is not obvious.

In this section we assume that differential biform of electromagnetic field  $F$  ‘would to be Lorentz-covariant’. Lorentz group is subgroup of symmetry group of massless Maxwell equations, thus maps solutions to solutions  $F \mapsto F^L$ . But we will depart here from Lorentz-covariance that action of Lorentz group on tensor algebra and on Grassmann algebra, and in particular Lorentz-action on differential biforms, must be induced from action on vector fields. We assume that Lorentz group does **not acts** on vector fields. Observers are assumed **neither** Lorentz-covariant **nor** Lorentz-invariant. One can fix observer for variable electromagnetic fields and variable electromagnetic sources. No such group-covariance is logically allowed. In this meaning we do not need here Lorentz-covariance at all. After all one can consider a priori transformations of electromagnetic fields not induced by isometries, but induced by transformations of electromagnetic sources.

A change of electromagnetic field,  $F \mapsto F^L$ , is interpreted as due to variable sources, due to moving electric charges and magnetic spins, a source motion relative to a **fixed** observer or relative to a fixed source. Electromagnetic source, charge-current density, need not to be time-like, can be light-like as well as space-like, therefore it is a different concept then a material time-like observer.

We accept Minkowski’s definition of concomitant electric and magnetic fields Equation (7.1), where we assume that observer-monad  $\mathbf{P}$  is fixed. Be fixed is not the same as to be Lorentz-invariant. We depart from Lorentz-covariance, because presented below transformation of biforms is independent of a choice of a fixed observers. Lorentz group is assumed do not acts on observers.

### 11.2 Assumption.

$$\begin{aligned} F &\xrightarrow{\text{transformation}} F^L \equiv L \circ F \circ L^*, \\ \left. \begin{array}{l} \mathbf{E}(F, \mathbf{P}_{\text{Paul}}) \\ \mathbf{B}(F, \mathbf{P}_{\text{Paul}}) \end{array} \right\} &\xrightarrow{\text{transformation}} \left\{ \begin{array}{l} \mathbf{E}^L \equiv \mathbf{E}(F^L, \mathbf{P}_{\text{Paul}}) \\ \mathbf{B}^L \equiv \mathbf{B}(F^L, \mathbf{P}_{\text{Paul}}) \end{array} \right\}, \end{aligned} \quad (11.1)$$

$$\mathbf{P} \cdot \mathbf{E} = 0 = \mathbf{P} \cdot \mathbf{B}, \quad \text{and} \quad \mathbf{P} \cdot \mathbf{E}^L = 0 = \mathbf{P} \cdot \mathbf{B}^L. \quad (11.2)$$

In present section  $\mathbf{u}$  is interpreted as a space-like velocity of moving charge-current relative to Paul; i.e., a relative velocity as measured by a fixed time-like Paul,  $\mathbf{u} \cdot \mathbf{P} = 0$ .

**11.3 Theorem.** *Above assumptions leads to following transformation of electric and magnetic fields.*

$$\begin{aligned} \mathbf{E}^L &= \gamma \left\{ \mathbf{E}(F) + \frac{\mathbf{u}}{c} \times_P \mathbf{B}(F) \right\} - \frac{\gamma^2}{\gamma+1} \left\{ \frac{\mathbf{u}}{c} \cdot \mathbf{E}(F) \right\} \frac{\mathbf{u}}{c}, \\ \mathbf{B}^L &= \gamma \left\{ \mathbf{B}(F) - \frac{\mathbf{u}}{c} \times_P \mathbf{E}(F) \right\} - \frac{\gamma^2}{\gamma+1} \left\{ \frac{\mathbf{u}}{c} \cdot \mathbf{B}(F) \right\} \frac{\mathbf{u}}{c}, \end{aligned} \quad (11.3)$$

**11.4 Clarification.** Transformation Equation (11.3) must not be called the *relativity* transformation because reference system, an observer vector field  $\mathbf{P}$  is here fixed. There is **only one** material reference system  $P = g\mathbf{P}$  and two different massless electromagnetic fields  $F^L \neq F$ ,

$$P \wedge E + \star(P \wedge B) = -F \quad \neq \quad -F^L = P \wedge E^L + \star(P \wedge B^L). \quad (11.4)$$

Transformed electric field with absolute (not covariant) observer is necessarily orthogonal to fixed Æther-like observer,  $\mathbf{E}^L \cdot \mathbf{P} \equiv 0$ . Absolute Æther-observer  $\mathbf{P}$  and his proper-time are intact.

**11.5 Corollary.**  $(\mathbf{E}^L)^2 - (\mathbf{B}^L)^2 = \mathbf{E}^2 - \mathbf{B}^2$ .

**11.6 Clarification.** Transformation Equation (11.1)-Equation (11.3) coincides *formally* with given by observers-tetrads: Pauli 1921; Bergmann 1942, 1976; Sommerfeld 1948 §28; Fock 1955, 1961 §24, formulas 24.37-24.38 on page 102; Tonnelat 1959 Chapter 9; Jackson 1962, 1999, formula (11.149); Landau and Lifshitz 1975 §24; Tisza 1976, Chapter 4 (4.1.31); Ingarden and Jamiołkowski 1979, 1985 §18.2; Dvoeglazov & Quintanar Gonzalez 2006; Rousseaux 2008.

Jammer in monograph [1961, Chapter XI], interpreted relativity transformation as coordinate-change that transform the D'Alembert wave differential equation on **four**-dimensional spacetime into Poisson equation in **three**-dimensional space. We disagree with such interpretation [Oziewicz 2008].

John Field in 2006, considering a **moving** charge, derived another scalar factor at  $\mathbf{u} \cdot \mathbf{E}$ , just  $\gamma$ , instead of  $\frac{\gamma^2}{\gamma+1}$  as it is in Equation (11.3):

$$\begin{aligned} \mathbf{B}(\text{static charge}) &\equiv 0, \quad \mathbf{E} \equiv \mathbf{E}(\text{static charge}), \\ \mathbf{E}(\text{moving charge}) &= \gamma \mathbf{E} - \gamma \left\{ \frac{\mathbf{u}}{c} \cdot \mathbf{E} \right\} \frac{\mathbf{u}}{c}. \end{aligned} \quad (11.5)$$

Group-free derivation of transformation by Heaviside, by Thomson, and by Field in 2006, need not be the same as due to isometry.

**11.7 Clarification.** In 2005 Ivezić observed logical and mathematical inconsistency of textbook treatments of Lorentz-covariance. He noted that it is illogical to consider a closed differential biform  $F$  to be Lorentz-covariant, and at the same time, keep observer's time-like vector field, a '4-velocity',  $\mathbf{P} \simeq (1, 0, 0, 0) \simeq \gamma_0$ , to be Lorentz-absolute (not Lorentz-invariant and not Lorentz-covariant). For example, compare how absolute observer is hidden in calculations presented in [Misner, Thorne and Wheeler 1973, Chapter 3].

*Proof of Theorem 11.3.* Our 'proof' is not conceptually correct, because we use covariance as Grassmann algebra map, Equation (11.7)-Equation (11.8) below, and we only hope that such transformation of differential biform of electromagnetic field  $F \mapsto F^L$  (without transforming vectors) can be deduced from transformation of electromagnetic sources.

Let  $L$  be endomorphism of a module of vector fields.  $L$  extends to Grassmann algebra morphism  $L^\wedge$ .  $L$ -covariance means exactly that  $L$  extends to tensor-algebra morphism, Grassmann-algebra morphism and that  $L$  commute with evaluation. Let ' $e$ ' denotes left regular (adjoint) representation of Grassmann algebra, ' $e$ ' is for Grassmann's historical extension, exterior, creator. Grassmann algebra morphism means the following covariance-rule

$$\forall \text{ multivectors } \mathbf{P}, \mathbf{R}, \quad e_{\mathbf{P}} \mathbf{R} \equiv \mathbf{P} \wedge \mathbf{R}, \quad (11.6)$$

$$L^\wedge \circ e_{\mathbf{P}} = e_{L^\wedge \mathbf{P}} \circ L^\wedge \iff i_{\mathbf{P}} \circ L^{*\wedge} = L^{*\wedge} \circ i_{L^\wedge \mathbf{P}}. \quad (11.7)$$

Consider electric field as defined by Minkowski in 1908, Definition 7.1,

$$E \equiv i_{\mathbf{P}} F \mapsto E^L \equiv i_{\mathbf{P}} F^L = i_{\mathbf{P}} L^{*\wedge} F = L^* \{i_{L^\wedge \mathbf{P}} F\}. \quad (11.8)$$

The decomposition Equation (11.4) for  $\mathbf{u} \cdot \mathbf{P} = 0$  gives

$$i_{\mathbf{u}} F = i_{\mathbf{u}} B(P) + \{E(P)\mathbf{u}\} g\mathbf{P}. \quad (11.9)$$

We now use Equation (10.5)-Equation (10.7). This gives transformation Equation (11.3).  $\square$

## 12. Groupoid relativity is isometry-free

Since 2005 we are trying oppose dogmatic trend, that set of all relativity transformations acting on material observers, must coincide with Lorentz isometry group. Lorentz isometry group must act on *all* vectors, also on light-like vector, that can not be reference system. We argue, hence, that material reference systems need not to be necessarily connected by isometry [Oziewicz 2005, 2006, 2007]. Among others, Poincaré group is symmetry group of metric tensor of *empty* energy-less space-time.

**12.1 Definition** (Groupoid category). A category consists of family of objects and family of arrows/morphisms. Category with every morphism being isomorphism, with two-sided inverse, is said to be *groupoid category*. In particular, a group is a groupoid one-object-category.

**12.2 Definition** (Relativity groupoid). Let each object of a groupoid be a material body, not necessarily inertial, given in terms of future directed  $g$ -normalized time-like vector field, for instance as in Minkowski's Definition Equation (5.2). Two material bodies in a relative rest are considered to be the same one body - one reference system. Let each morphism be **unique** binary relative velocity as follows,

$$\text{If } \mathbf{P}^2 = -1 \quad \text{and} \quad \mathbf{P} \cdot \mathbf{u} = 0 \quad \text{then:} \quad \mathbf{P} \xrightarrow{\mathbf{u}} \mathbf{R} = \gamma \left( \mathbf{P} + \frac{\mathbf{u}}{c} \right), \quad (12.1)$$

$$\frac{\mathbf{u}(\mathbf{P} \mapsto \mathbf{R})}{c} \equiv -\mathbf{P} - \frac{\mathbf{R}}{\mathbf{P} \cdot \mathbf{R}}, \quad (12.2)$$

This groupoid category is said to be the *relativity groupoid*.

Each pair of elements of a group is composable. In contrast, this is not the case for morphisms/arrows in a groupoid with more than one object. Consider four massive observers-monads, objects,  $\mathbf{P}, \mathbf{R}, \mathbf{Q}$ , and  $\mathbf{S}$ , with  $\mathbf{R} \neq \mathbf{Q}$  and  $\mathbf{S} \neq \mathbf{P}$ . The groupoid arrows,  $\mathbf{P} \mapsto \mathbf{R}$ , and,  $\mathbf{Q} \mapsto \mathbf{S}$ , are not composable.

Minkowski's basis-dependent expressions [Minkowski 1908 Part II §8 (27)] and [Minkowski 1908 Part II §11.6 (46)], see Equation (5.1), must be understand as basis-free groupoid transformation Equation (12.1). Groupoid transformation, Equation (12.1)-Equation (12.2), we propose in Thesis by Świerk [1988]; and these expressions are foundation of monograph by Matolcsi [1994, Part II, §4.2, page 191]. Hehl and Obukhov's expression (E.4.10) in [2003, page 349], is essentially the same as Equation (12.1)-Equation (12.2). See also [Oziewicz since 2005].

Groupoid kinematics is basis-free, and postulate that electromagnetic field is absolute, i.e. that closed differential biform  $F$ , is observer-free for arbitrary non-inertial observer [Minkowski 1908, §11.6; Cruz & Oziewicz 2003; Oziewicz 2005, 2006, 2007]. Axiom of absolute  $F$ , is within the principle of absolute reality by Giovanni P. Gregori [2005, page 12], stating that physical reality of Nature is observer-free, that laws of Physics and Mathematics are absolute and do not need existence of observers = humans. Postulate of absolute electromagnetic field  $F$  is contrary to basis-free relativity, where closed differential bi-form of electromagnetic field,  $F$ , must be Lorentz-covariant (not absolute).

In a groupoid kinematics electromagnetic field is postulated to be absolute, *i.e.* observer-free, and we have,

$$\left. \begin{array}{l} \mathbf{E}(F, \text{Paul}) \\ \mathbf{B}(F, \text{Paul}) \end{array} \right\} \xrightarrow[\text{not isometry}]{\text{groupoid action}} \left\{ \begin{array}{l} \mathbf{E}^M \equiv \mathbf{E}(F, \text{Rose}) \\ \mathbf{B}^M \equiv \mathbf{B}(F, \text{Rose}) \end{array} \right. . \quad (12.3)$$

In Equation (12.3)-Equation (12.15), a superscript 'M' is for Minkowski.

Minkowski's implicit assumption, that  $F$  is observer-free, implies that relativity transformation of observer-dependent electric and magnetic fields is different when compared with the following cases:

- Lorentz-transformation of electromagnetic field with a fixed observer, Theorem 11.3.
- Minkowski's Lorentz-covariance in Ivezić's 'invariant special relativity' ISR-theory Equation (10.2)-Equation (10.16), [Ivezić 2003, 2005]. Transformation of electromagnetic field is induced from transformation of observer. Both transformations are related. One can chose any observer for a given electromagnetic field, but **group-covariance** require that every group action on observer must be together with induced action on electromagnetic field Equation (9.2).

### 12.1. Sources of electric and magnetic fields: spin

12.1.1. *Ponderomotive force as a sum of three terms* Observer-independent source vector field  $\mathbf{J}$  is a sum of observer-dependent vector magnetic spin  $\mathbf{s}$  and a scalar punctual charge density  $\rho$  as follows

$$\mathbf{J} = \mathbf{s}(\mathbf{R}) - \mathbf{R} \wedge \rho(\mathbf{R}) = \mathbf{s}(\mathbf{P}) - \mathbf{P} \wedge \rho(\mathbf{P}). \quad (12.4)$$

Analogously observer-free electromagnetic field, a differential bi-form  $F$  is a sum of observer-dependent magnetic field  $B$  and electric field  $E$  [Minkowski 1908 §11.6 identity (45)],

$$P \equiv g\mathbf{P} \quad \text{and} \quad R \equiv g\mathbf{R} \quad \implies \quad F = B(\mathbf{R}) - R \wedge E(\mathbf{R}) = B(\mathbf{P}) - P \wedge E(\mathbf{P}). \quad (12.5)$$

Therefore observer-free ponderomotive differential form (identified with the Lorentz force) consists of **three** terms, because magnetic field does not interact with a scalar charge density,  $\rho i_{\mathbf{P}} B \equiv 0$ , whereas an electric field  $E$  interact both with charge and with magnetic spin giving also time-like contribution

$$\mathbf{J} \cdot F = -\rho E + \mathbf{s} \cdot B + (E \mathbf{s}) P. \quad (12.6)$$

12.1.2. *Convection weaker within groupoid.* Let  $\mathbf{u}$  be a space-like velocity of  $\mathbf{R}$ ose relative to  $\mathbf{P}$ aul, *i.e.* a relative velocity as measured by  $\mathbf{P}$ aul,  $\mathbf{u} \cdot \mathbf{P} = 0$ .

**12.3 Theorem.** *Electric punctual charge-densities, scalar fields  $\rho(\mathbf{P})$  and  $\rho(\mathbf{R})$ , and vectors magnetic spin- and current-densities,  $\mathbf{s}(\mathbf{P})$  and  $\mathbf{s}(\mathbf{R})$ , as measured by these two observers are related by means of groupoid transformation as follows*

$$\rho(\mathbf{R}) = \gamma \left\{ \rho(\mathbf{P}) + \frac{\mathbf{u}}{c} \cdot \mathbf{s}(\mathbf{P}) \right\}, \quad (12.7)$$

$$\mathbf{s}(\mathbf{R}) = \mathbf{s}(\mathbf{P}) + \rho(\mathbf{P}) \left\{ \gamma^2 \frac{\mathbf{u}}{c} + (\gamma^2 - 1) \mathbf{P} \right\} + \gamma^2 \left( \frac{\mathbf{u}}{c} \cdot \mathbf{s}(\mathbf{P}) \right) \left( \mathbf{P} + \frac{\mathbf{u}}{c} \right). \quad (12.8)$$

The relativity transformation of the scalar charge-densities Equation (12.7), coincide with the Lorentz-group covariant transformation, compare for example with [Tonnelat 1959, Chapter 9, formulas (9.3) and (9.4).] Whereas the transformation Equation (12.8) differs from the Lorentz-group transformation, and follows from the groupoid kinematics considered here.

When  $\mathbf{s}(\mathbf{P}) = 0$ , then  $\mathbf{s}(\mathbf{R})$  is purely convection current density,

$$\text{Electric convection} = \rho \left\{ \gamma^2 \frac{\mathbf{u}}{c} + (\gamma^2 - 1) \mathbf{P} \right\}, \quad (12.9)$$

$$|\text{Electric convection}| = \begin{cases} (\gamma^2 - 1) \rho & \text{within groupoid,} \\ \sqrt{\gamma^2 - 1} \rho & \text{within Lorentz group.} \end{cases} \quad (12.10)$$

Formula Equation (12.9) is within groupoid relativity prediction. We see that the electric convection current due to the charge in the motion, within groupoid relativity, contains only

even powers of relative velocity, Equation (12.12). This must be compared with the Lorentz transformations within the isometric special relativity, where the electric convection is predicted to be stronger

for  $|\frac{\mathbf{u}}{c}| \ll 1$ ,

$$\sqrt{\gamma^2 - 1} = \left| \frac{\mathbf{u}}{c} \right| \gamma, \quad (12.11)$$

$$(\gamma^2 - 1) \simeq \left| \frac{\mathbf{u}}{c} \right|^2 + \left| \frac{\mathbf{u}}{c} \right|^4 + \dots, \quad \sqrt{\gamma^2 - 1} \simeq \left| \frac{\mathbf{u}}{c} \right| + \frac{1}{2} \left| \frac{\mathbf{u}}{c} \right|^3 + \dots \quad (12.12)$$

$$\sqrt{\gamma^2 - 1} = \gamma^2 - 1 \iff \left| \frac{\mathbf{u}}{c} \right| = 0 \quad \text{or} \quad \left| \frac{\mathbf{u}}{c} \right| = \frac{1}{\sqrt{2}}. \quad (12.13)$$

### 12.2. Lorentz-covariance-free magnetic and electric fields

**12.4 Theorem** (Minkowski 1908, §11.6). *Let  $\mathbf{E}(\mathbf{R})$  and  $\mathbf{B}(\mathbf{R})$  be electric and magnetic fields measured by Rose =  $\gamma(\mathbf{P} + \mathbf{u}/c)$ , Definition Equation (5.2). Let,  $\mathbf{E} \equiv \mathbf{E}(\mathbf{P})$ , and  $\mathbf{B} \equiv \mathbf{B}(\mathbf{P})$ , be the electric and magnetic fields as measured by Paul. According to Minkowski definition Equation (7.1)*

$$\mathbf{P} \cdot \mathbf{E}(\mathbf{P}) = 0 = \mathbf{P} \cdot \mathbf{B}(\mathbf{P}) \implies \mathbf{R} \cdot \mathbf{E}(\mathbf{R}) = 0 = \mathbf{R} \cdot \mathbf{B}(\mathbf{R}). \quad (12.14)$$

Then, these fields are related by means of the following covariance-free transformation,

$$\begin{aligned} \mathbf{E}^M \equiv \mathbf{E}(\mathbf{R}) &= \gamma \left\{ \mathbf{E}(\mathbf{P}) + \frac{\mathbf{u}}{c} \times_P \mathbf{B}(\mathbf{P}) \right\} + \gamma \left\{ \frac{\mathbf{u}}{c} \cdot \mathbf{E}(\mathbf{P}) \right\} \mathbf{P}, \\ \mathbf{B}^M \equiv \mathbf{B}(\mathbf{R}) &= \gamma \left\{ \mathbf{B}(\mathbf{P}) - \frac{\mathbf{u}}{c} \times_P \mathbf{E}(\mathbf{P}) \right\} + \gamma \left\{ \frac{\mathbf{u}}{c} \cdot \mathbf{B}(\mathbf{P}) \right\} \mathbf{P}. \end{aligned} \quad (12.15)$$

Note that transformations of electromagnetic sources, electric charge and spin-current densities, as given early in Equation (12.7)-Equation (12.8), are covariance-free, in the same manner as Equation (12.15).

The groupoid transformation, Equation (12.2)  $\rightarrow$  Equation (12.3)  $\rightarrow$  Equation (12.15), is induced on concomitants from the primary action on massive observers-monads, on time-like vector-fields only, Equation (12.2). The transformation of concomitant-fields Equation (12.15) were derived by Minkowski in [1908, §11.6, his Eqs. (47-48) and (51-52)]. Minkowski did not make it clear that such transformation among massive observers, Equation (12.2)-Equation (12.15), is *not* an isometry, because, among other, the domain do not include light-like vectors. The Minkowski groupoid transformations, Equation (12.2)-Equation (12.15), are *not* the Lorentz transformations.

Hamdan, University of Aleppo in Syria, died tragically in 2008. Hamdan similarly, considered in 2006 that a transformation of monads, Equation (12.2), is a Lorentz transformation, [Hamdan 2006, Eqs. (8a-8b)]. In fact this is a groupoid transformation, because a vector  $\mathbf{P}$  *must* be time-like, and not all such transformations are composable. Besides, one can show directly that the groupoid transformation is not an isometry. For example, consider a pair of monads,  $\mathbf{P} \neq \mathbf{Q}$ , as follows,

$$\mathbf{P} \cdot \mathbf{u} = 0 = \mathbf{Q} \cdot \mathbf{u}, \quad \text{together with the groupoid isomorphisms,}$$

$$\mathbf{P} \xrightarrow{\mathbf{u}} \gamma \left( \mathbf{P} + \frac{\mathbf{u}}{c} \right), \quad \text{and,} \quad \mathbf{Q} \xrightarrow{\mathbf{u}} \gamma \left( \mathbf{Q} + \frac{\mathbf{u}}{c} \right), \quad (12.16)$$

$$\mathbf{P} \cdot \mathbf{Q} \xrightarrow{\mathbf{u}} \gamma^2 \left( \mathbf{P} + \frac{\mathbf{u}}{c} \right) \cdot \left( \mathbf{Q} + \frac{\mathbf{u}}{c} \right) = \gamma^2 \mathbf{P} \cdot \mathbf{Q} + \gamma^2 - 1 \neq \mathbf{P} \cdot \mathbf{Q}. \quad (12.17)$$

This shows that a groupoid transformation of monads is not an isometry.



Let's compare the Lorentz-covariant electromagnetic field,  $F \mapsto F^L$ , with **fixed** observer-monad-Æther, Equation (11.1)-Equation (11.3), and non-isometric groupoid transformation of monads,  $P \xrightarrow{\mathbf{u}} R$ , with an observer-free electromagnetic field  $F$ , Equation (12.2)  $\rightarrow$  Equation (12.3)  $\rightarrow$  Equation (12.15), where transformation of observers is not related with transformation of electromagnetic fields. The only visual difference are terms containing the scalar products,  $\mathbf{u} \cdot \mathbf{E}$  and  $\mathbf{u} \cdot \mathbf{B}$ . These terms within 'Æther' Equation (11.3), are proportional to space-like relative velocity  $\mathbf{u}$ . Within relativity groupoid these terms give time-like contributions.

If spacelike relative velocity  $\mathbf{u}$  is orthogonal in spacetime to electric and magnetic vector fields, then Equation (11.3) coincides with prediction of groupoid relativity in Theorem 12.4,

$$\text{if, } \mathbf{u} \cdot \mathbf{E}(F) = 0 = \mathbf{u} \cdot \mathbf{B}(F) \quad \implies \quad \mathbf{u} \cdot \mathbf{E}(F^J) = 0 = \mathbf{u} \cdot \mathbf{B}(F^J). \quad (12.18)$$

### 12.3. Experimental consequences

There are the following consequences,

$$\mathbf{u} \times_P \mathbf{B} = 0 \quad \implies \quad (12.19)$$

$$\text{For fixed 'Æther':} \quad \mathbf{E}^L \cdot \mathbf{E} - \gamma \mathbf{E}^2 = -\frac{\gamma^2}{\gamma + 1} \left( \frac{\mathbf{u}}{c} \cdot \mathbf{E} \right)^2. \quad (12.20)$$

$$\text{Within relativity groupoid: } \mathbf{E}^M \cdot \mathbf{E} - \gamma \mathbf{E}^2 = 0. \quad (12.21)$$

Whereas within Lorentz-covariance á la Minkowski 1908,  $\forall \mathbf{u}$ , and  $\forall \mathbf{B}$ , Equation (10.4), there is [Ivezić 2005],

$$\mathbf{E}^I \cdot \mathbf{E} - \mathbf{E}^2 = +\frac{\gamma^2}{\gamma + 1} \left( \frac{\mathbf{u}}{c} \cdot \mathbf{E} \right)^2. \quad (12.22)$$

The differences among three theories are of the second order  $\beta^2$ , for  $\beta \equiv \frac{\mathbf{u}}{c}$ ,

$$\gamma \simeq 1 + \frac{1}{2}\beta^2 + \dots, \quad \frac{\gamma^2}{\gamma+1} \simeq \frac{1}{2} + \frac{3}{8}\beta^2 + \dots, \quad (12.23)$$

$$\mathbf{E}(\mathbf{u}) \cdot \mathbf{E} - \mathbf{E}^2 = \frac{1}{2} \begin{cases} \beta^2 \mathbf{E}^2 - (\mathbf{E} \cdot \beta)^2 & \text{fixed 'Æther' Equation (11.3)} \\ (\mathbf{E} \cdot \beta)^2 & \text{Ivezić's group-covariance} \\ \beta^2 \mathbf{E}^2 & \text{groupoid relativity.} \end{cases} \quad (12.24)$$

We need to stress again that in expressions Equation (12.20)-Equation (12.22), the electric field measured by an observer 'at rest', and electric field measured by a moving observer-monad, within the three different relativity theories, all are the vector fields on **four**-dimensional space-time manifold,

$$\mathbf{E}, \quad \mathbf{E}^J, \quad \mathbf{E}^I, \quad \mathbf{E}^M \equiv \mathbf{E}(\text{Rose}), \quad \text{and} \quad \mathbf{u}. \quad (12.25)$$

In a randomly chosen not-adopted mathematical basis, each of these vector fields Equation (12.25) possesses **four non-zero** scalar components. In terminology used by Ivezić, the fields,  $\mathbf{E}$  and  $\mathbf{E}^J, \mathbf{E}^I, \mathbf{E}^M$ , etc, in Equation (12.20)-Equation (12.22) and in Equation (12.25), are '4D-quantities'.

Note that there are the following implications

$$\begin{aligned} \left\{ \begin{array}{l} \mathbf{u} \cdot \mathbf{E}(P) = 0 \\ \mathbf{u} \cdot \mathbf{B}(P) = 0 \end{array} \right\} & \implies \left\{ \begin{array}{l} \mathbf{u} \cdot \mathbf{E}(R) = 0 \\ \mathbf{u} \cdot \mathbf{B}(R) = 0 \end{array} \right\} \implies \left\{ \begin{array}{l} P \cdot \mathbf{E}(R) = 0 \\ P \cdot \mathbf{B}(R) = 0 \end{array} \right\} \\ & \implies \mathbf{E}(P) \wedge \mathbf{B}(P) \wedge \mathbf{E}(R) \wedge \mathbf{B}(R) = 0. \end{aligned} \quad (12.26)$$

### 13. Conclusion

We propose an alternative for Einstein's special relativity. We suggest that the set of all relativity transformations between material reference systems (between time-like vector fields) could be a groupoid that it is *not* a group. One consequence of this groupoid-relativity, that is implicit in the Minkowski last publication in 1908 [Minkowski 1908 §11.6], is examined here for groupoid transformation of electric and magnetic fields, Theorem 12.4 and Equation (12.15), and for groupoid transformation of electromagnetic sources Equation (12.7)–Equation (12.8). This consequence could eventually be tested experimentally.

In the present paper we also repeated essentially what in 1908 Hermann Minkowski explained on Lorentz isometry group acting on vectors and on vector fields on space-time. The domain of the Lorentz isometry are vectors, and Lorentz isometry transformation of vectors induce Lorentz transformation of all tensor algebra except of scalar fields. The electric field is a vector field on space-time (is time-dependent), and under Lorentz isometry must transform as every vector field. This (trivial) fact was independently re-discovered by Tomislav Ivezić [Ivezić 2003, 2005].

According to Minkowski and Ivezić, and according to present author, an electric field, as a vector field on space-time is defined as dependent on the electromagnetic field tensor  $F$ , *i.e.*  $\mathbf{E} = \mathbf{E}(F)$ . In 1908 Minkowski explained how Lorentz transformation of vectors induce transformation of tensors, and how transformation of a tensor  $F$  induce the transformation of a vectors built from other tensors.

Some present-day textbooks consider electric and magnetic fields as *primary* 'time-dependent three-dimensional vectors', versus the primary electromagnetic field  $F$  introduced in 1908 by Minkowski, *i.e.* some textbooks consider that  $F \equiv F(\mathbf{E}, \mathbf{B})$ , versus Minkowski's  $\mathbf{E} = \mathbf{E}(F)$  and  $\mathbf{B} = \mathbf{B}(F)$ .

There are lecturers of electromagnetism (under name 'electrodynamics' with 'magnetism' removed by Ampère in 1828), convinced by definition of  $F(\mathbf{E}, \mathbf{B})$  Equation (7.5) that electromagnetic field  $F$  is artificial mathematical construction without physical contents, and that in physical reality there are only electric  $\mathbf{E}$  and magnetic  $\mathbf{B}$  fields. Even this  $\mathbf{E}$  must be a strange vector field on space-time with permanent amputee component  $E_t$  for not known scalar field  $t$ . Such strange 'amputee' vector field **on space-time** is called by many textbooks a '3D' quantity. This was called up by Tomislav Ivezić [Ivezić 2003, 2005, among other].

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The present version was essentially inspired by extensive Skype and email discussions during many last years with Tomislav Ivezić from Zagreb. I am most cordially thankful to Tomislav for these important discussions. We agree with Tomislav about the meaning of Lorentz covariance and if Lorentz-group covariance is postulated then the only mathematically correct Lorentz-covariant transformations of electric and magnetic fields are transformations derived by Tomislav in 2003-2005. On the other hand we do not agree with interpretation of a concept of a vector in physical literature. I do not like 3D- or 4D-vector unlucky terminology, whereas Tomislav is constantly stressing importance of these names and the hidden meaning they keep.

We also disagree about interpretation of Minkowski publication in 1908. Minkowski's well known short 14-pages lecture in Cologne *Raum und Zeit* was published in 1909, and in fact it is a short introduction to and summary of 1908 long paper of 59 pages we are referring. With Tomislav we agree that Minkowski defined Lorentz-covariance as Tomislav is advocating since 2005, however Tomislav vigorously disagree that Minkowski introduced the modern concept of observer as a time-like vector field and a relative velocity among two such observers.

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