**Showing how the accepted relativistic addition of velocities and energies is only a two-variable version of a new infinite variables formula**

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The well-known simple formula for adding velocities relativistically is only a 2-variable version of a far more powerful formula which enables an infinite number of relative velocities, energies or any Planck-limited variables to be totalled. That formula is based on the product of those variables, treated as fields, rather than their addition. The interpretation of the formula is of the comparison of the stretching of the fields in opposite directions versus the total stretching involved. The formula for the total value of any variable $i$ with $n$ relative values is $T\_{G}\left(n\right)=(\left\{\prod\_{}^{}(1+i)\right\}-\left\{\prod\_{}^{}\left(1-i\right)\right\})/(\left\{\prod\_{}^{}\left(1+i\right)\right\}+\left\{\prod\_{}^{}\left(1-i\right)\right\})$**.**

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**Addition of Velocities and Energies**

It is generally accepted that the relativistic addition of two velocities follows the form

$T\_{r}=(x+y)/(1+xy)$

This short note sets out to show that this is only a specific example of a general form of treating actions within any number of dimensions or any number of interacting bodies, based on the interaction of, for example, energies or velocities acting as the product of fields thus

$T\_{G}\left(n\right)=\{\prod\_{}^{}\left(1+i\right)-\prod\_{}^{}\left(1-i\right)\}/\{\prod\_{}^{}\left(1+i\right)+\prod\_{}^{}\left(1-i\right)\}$

as will be explained below.

This result can be obtained by starting simply and extending the simple treatment above of $T\_{r}$ to a third velocity, such that

$T\_{r}=\{(x+(y+z)/(1+yz)\}/\{1+x(y+z)/(1+yz)\}$

$T\_{r}=\{(x+y+z)+xyz\}/\{1+xy+xz+yz)\}$

I will now introduce a new short method of describing these variables.

The summation of the variables on their own I will describe as $\sum\_{}^{}\prod\_{}^{}(1)$ meaning the summing of the product of variables of interaction value one, that is they do not interact with any other variables. The summation of the cross products like $xy$, $xz$ and $yz$ I will describe as $\sum\_{}^{}\prod\_{}^{}(2)$ , and here this would mean that

$\sum\_{}^{}\prod\_{}^{}\left(2\right)=(xy+xz+yz)$

The next in the series will be the triple interaction product

$ $ $\sum\_{}^{}\prod\_{}^{}\left(3\right)=xyz$

It is immaterial that in this example there is no actual summation because there is only one value of triple interaction, it standardises the use of this method. We now have the relativistic formula as

$T\_{r}=T\_{r}(3)=\{\sum\_{}^{}\prod\_{}^{}\left(1\right)+\sum\_{}^{}\prod\_{}^{}\left(3\right)\}/\left\{1+\sum\_{}^{}\prod\_{}^{}\left(2\right)\right\}$

What is found is that as the number of variables rises, the number of product parameters increases in line, with actual numbers of cross components following a Fibonacci-like sequence. So, starting at the left and end of the denominator and then up to the left hand end of the numerator, alternating to next along thereafter, and keeping the value 1 in the $T\_{r}(1)$ case, we get

$T\_{r}(1)$ has 2 parameters with components 1, 1

$T\_{r}(2)$ has 3 parameters with components 1, 2, 1

$T\_{r}(3)$ has 4 parameters with components 1, 3, 3, 1

$T\_{r}(4)$ has 5 parameters with components 1, 4, 6, 4, 1

$T\_{r}(5)$ has 6 parameters with components 1, 5, 10, 10, 5 1

$T\_{r}(6)$ has 7 parameters with components 1, 6, 15, 20, 15, 6, 1

and so on. Note that the sum of the numerator components is always equal to the sum of the denominator components, ensuring that the maximum value of any $T\_{r}(n)$ is 1.

Taking the $T\_{r}(3)$ formula as a simple example, which can be generalised, we can rearrange this to form

$\{1-T\_{r}\left(3\right)\}=\{\left(1-x\right)\left(1-y\right)\left(1-z\right)\}/\{1+(xy+xz+yz)\}$

$\{1-T\_{r}\left(3\right)\}=\{\prod\_{}^{}(1:3)\left(1-i\right)\}/\{1+(xy+xz+yz)\}$

where $\prod\_{}^{}(1:3)$ is used for brevity and means the product of $\left(1-i\right)$ on each variable over three single variables $i$. However, we can do the same for the denominator, where we get the result

$\left\{1+\left(xy+xz+yz\right)\right\}=\left\{\prod\_{}^{}\left(1:3\right)\left(1+i\right)\right\}-\{\sum\_{}^{}\prod\_{}^{}\left(1\right)+\sum\_{}^{}\prod\_{}^{}\left(3\right)\}$

but it is also the case that

$\left\{\prod\_{}^{}\left(1:3\right)\left(1+i\right)\right\}-\left\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\right\}=2\{\sum\_{}^{}\prod\_{}^{}\left(1\right)+\sum\_{}^{}\prod\_{}^{}\left(3\right)\}$

so that we now have

$\{1-T\_{r}\left(3\right)\}=2\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\}/(\left\{\prod\_{}^{}\left(1:3\right)\left(1+i\right)\right\}-\left\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\right\})$

or

$\{1-T\_{r}\left(3\right)\}=2/(1+(\{\prod\_{}^{}(1:3)\left(1+i\right)\}/\left\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\right\}))$

or

$T\_{r}\left(3\right)=(\left\{\prod\_{}^{}\left(1:3\right)\left(1+i\right)\right\}-\left\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\right\})/(\left\{\prod\_{}^{}\left(1:3\right)\left(1+i\right)\right\}+\left\{\prod\_{}^{}\left(1:3\right)\left(1-i\right)\right\})$

which loses no strength when generalised to $n$ variables $i$ as

$T\_{G}\left(n\right)=(\left\{\prod\_{}^{}(1+i)\right\}-\left\{\prod\_{}^{}\left(1-i\right)\right\})/(\left\{\prod\_{}^{}\left(1+i\right)\right\}+\left\{\prod\_{}^{}\left(1-i\right)\right\})$

What this formula says is that the total velocity (energy) is the difference be­tween the products of the positive and negative velocity (energy) fields present divided by their sum. The use of the term ‘field’ here is deliberately done to emphasise that any type of energy can be totalled in this way when presented as a fraction of its Planck maximum value, and the multiple fields can be acting on a single particle or can be the properties of multiple particles acting on one point. And that a relative velocity acts just like a field when being represented as a fraction of the speed of light $c$. For each different energy/field type, the field value should be totalled using the product formula $T\_{G}\left(n\right)$. Then the totals of each energy/field type present should be totalled to produce the overall total energy/field value on a particle or at a point.

The formula also says that the value of $T\_{G}\left(n\right)$ will always be less than or equal to 1. A simple check for $n=2$ reveals

$T\_{G}\left(2\right)=(\left\{(1+x)(1+y)\right\}-\left\{(1-x)(1-y)\right\})/(\left\{(1+x)(1+y)\right\}+\left\{(1-x)(1-y)\right\})$

$T\_{G}\left(2\right)=\left\{2x+2y\right\}/\left\{2+2xy\right\}=(x+y)/(1+xy)$

which is where we started.

It is also clear that this can be extended to the square of total velocity or energy $T\_{G}^{2}\left(n\right)$ and that the result is very different to that obtained by substituting $x^{2}$ for $x$ in the well-known formula, often done to move between velocity and energy calculations. Reverting to simple variables using $T\_{G}\left(3\right)$ where the variables $x$, $y$ and$ z$ could be velocities or energies and by considering negative variables, we get

$\{1-(-T\_{G}\left(3\right))\}=\left\{\left(1-\left(-x\right)\right)\left(1-\left(-y\right)\right)\left(1-\left(-z\right)\right)\right\}/\{1+\left(xy+xz+yz\right)\}$

$\{1+T\_{G}\left(3\right)\}=\left\{\left(1+x\right)\left(1+y\right)\left(1+z\right)\right\}/\{1+\left(xy+xz+yz\right)\}$

and from this, multiplying by $\{1-T\_{G}\left(3\right)\}$ , can be found

$\{1-T\_{G}^{2}\left(3\right)\}=\left\{\left(1-x^{2}\right)\left(1-y^{2}\right)\left(1-z^{2}\right)\right\}/\{1+\left(xy+xz+yz\right)\}^{2}$

If this is simplified by making $z=0$,

$\{1-T\_{G}^{2}\left(2\right)\}=\left\{\left(1-x^{2}\right)\left(1-y^{2}\right)\right\}/\{1+xy\}^{2}$

or

$T\_{G}^{2}\left(2\right)=\left\{x^{2}+y^{2}+2xy\right\}/\{1+xy\}^{2}$

and then compared with the 'usual' method of straight substitution of $x^{2}$for $x$, $y^{2}$for $y$ and $z^{2}$for $z$ which would instead give

$T^{2}(2)=\left\{x^{2}+y^{2}\right\}/\{1+x^{2}y^{2}\}$

It can be seen that there is a significant difference in both the numerators and denominators, even though at an extreme where $x=1$, both expressions result in $T^{2}=1$.

The curve of each formula produce by varying $x$ and $y$ may start and end at the same points, but the curves are different between these. The new formula works in all dimensions, that is with any number of velocities or energies acting. It is in its use for considering multiple overlapping fields that the utility of the $T\_{G}\left(n\right)$ formula becomes apparent because rather than having to add fields, it is easier to multiply them and to understand what the interactions mean in terms of the cross components.

The meaning of the $T\_{G}\left(n\right)$ formula is that the overall total energy (velocity) of a body in multiple energy fields (travelling at multiple relative velocities) is the difference between the product of the sizes of those fields over and under the maximum field size (1 in Planck units or velocity $c$) divided by the sum of those products.

In a universe where energy can be considered to warp space, this is like considering any energy to be both positive and negative simultaneously, stretching space both 'upwards' and 'downwards' and comparing the difference between these stretches to the total energy involved in the stretching. Any variable that reaches 1 ensures that the total will also be 1 regardless of the size of any other variable. Thus space has a maximum stretch of 1 in any 'direction' and cannot be ripped. There is also a distinct echo of the space-stretching in the formula for the rela­tivistic factor $γ=v/c$, where

$1/γ^{2} =\left(1-(v/c)^{2}\right)=(1-v/c)(1+v/c)$

being the product of both 'upward' and 'downward' velocity fields simultane­ously. Here the value of $v$ would be the outcome $T\_{G}\left(n\right)$ where there are multiple relative velocities to consider, so that

$(1-v/c)(1+v/c)=(1+T\_{G}\left(n\right)$)(1-$T\_{G}\left(n\right))$

Giving, now for $γ(n)$,

$T\_{G}^{2}\left(n\right)=\left(1-γ\left(n\right)^{-2}\right)=((\left\{\prod\_{}^{}\left(1+i\right)\right\}-\left\{\prod\_{}^{}\left(1-i\right)\right\})/(\left\{\prod\_{}^{}\left(1+i\right)\right\}+\left\{\prod\_{}^{}\left(1-i\right)\right\}))^{2}$

and

$γ(n)=(\left\{\prod\_{}^{}\left(1+i\right)\right\}+\left\{\prod\_{}^{}\left(1-i\right)\right\})/\sqrt{4\left\{\prod\_{}^{}\left(1-i^{2}\right)\right\}}$

which continues to have the upward and downward products symmetrically included. And for $γ(1)$ we obtain

$γ\left(1\right)=\left\{\left(1+i\right)+\left(1-i\right)\right\}$/$\sqrt{4\left(1-i^{2}\right)}=1/\sqrt{\left(1-i^{2}\right)}$

as expected.

**Conclusions**

The well-known simple formula for adding velocities relativistically is only a 2-variable version of a far more powerful formula which enables an infinite number of relative velocities, energies or any Planck-limited variables to be totalled. That formula is based on the product of those variables, treated as fields, rather than their addition. The veracity of the formula can be tested at high energies or velocities where the new formula departs from the existing well-known version when adding energies based on the square of the velocities.

**References**

None

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