THE MAGNETIC FORCE BETWEEN TWO CURRENTS FURTHER ANALYSED USING COULOMB'S LAW AND SPECIAL RELATIVITY THEORY

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February 18, 2009 Final version corrected Feb. 23.

Abstract
In a paper 1997 a model capable of explaining the electromagnetic forces between electric currents in conductors, using only Coulomb's law, was proposed, and the results applied to experiments upon Ampere's bridge. The approach succeeded, due to a rigorous geometric analysis, thereby focusing upon the delay effects thanks to the velocity of light. Simultaneously, the Lorentz force failed completely to explain the behaviour of the force. The special relativity theory (SRT) was not being used and due to the very low velocities involved in the currents in conductors, the need was not felt.

However, since the SRT is widely recognized, a check to what extent that would change the results above seems very urgent to perform. That is also one of the main concerns of this paper. And the result is that the SRT does not affect the results, as far as low charge velocities are involved, as is the case with circuit currents. The effects of propagation delay are of higher order and supersede those of the SRT.

Since efforts have been made by other scientists to explain the results with Ampere's bridge, thereby using Ampere's law, this theory will be further discussed here. A model claiming that Ampere's law is a direct consequence of applying the special relativity theory straightforwardly upon Coulomb's law will also be analysed, but with negative result. What Ampere derived through deduction with respect to experimental results remains empirical. It is being shown that Ampère's law in fact approximates the first term of the Lorentz force, i.e. the 'electric force' term. However, the Lorentz force is unable to explain the measurement results, even though the invariance of Maxwell's equations during Lorentz transformations according to the predominant school supports that claim that Maxwell's equations also are consistent with reality.

1. Introduction.

One aim with the present paper is to update the 1997 paper [1] of this author with respect to the special relativity theory. It is often being claimed that today's electromagnetic formulas within the "Maxwell's equations" school are a consequence of this application upon classical electrostatics[10]
However, in the 1997 paper [1] a strict mathematical analysis of the effects of retardation of propagation of 'light' that this gives credit to the electromagnetic forces felt between two current carrying conductors. It was possible to successfully predict measurement results upon Ampere's bridge performed in the 1980s [14].
Simultaneously, it was shown that the Lorentz force was completely unable to explain even the spatial dependence of the force that was measured. Due to this overall success, the need to analyse the effects of the special relativity theory (SRT) was not felt, especially as the velocities of conductor currents are typically very small compared with the velocity of light.
Nonetheless, since the SRT has been widely successful in explaining physical phenomena, it seems inevitable to finally perform an investigation of its effects.

Since other scientists have been working with other models, a look at their standpoints seems also necessary. Wesley [16] applies Ampere's law upon experiments upon Ampere's bridge performed by Pappas and Moyssides in the 1980s [15] and succeeds to some extent in that effort, too.
It is in that context interesting to make a study of what Ampere himself stated in the 1820s. He performed numerous experiments in order to deduce a theoretical formula, able to predict the forces between any two currents. The observant reader will find that Wesley makes a modification of the original Ampere's law [16], [30], [20].
Keele[31] in another paper states that Ampere's law can be derived by applying the SRT directly upon the original formulation of Coulomb's law and succeeds rather well in doing so, even though he does not draw the full consequences of this by rejecting the "magnetic force" term. He still has the two term thinking, typical for the Maxwell equations model. But this must not imply any vast problem for the reader.
It is the hope that these analyses will bring about a full understanding of the origins of electromagnetism.

2. The application of Coulomb's law upon currents carried by conductors.

2.1 The preliminary analysis, not using the special relativity theory (SRT)

The total electric force between two charges is assumed to obey Coulomb's law, which on differential form may be written [6]:

$$\frac{d^2\vec{F}}{dx_1 dx_2} = \frac{\rho_1\rho_2}{4\pi\varepsilon_0}\frac{\vec{u}_R}{R^2} \qquad (1)$$

The law seems originally to be intended to be used with respect to stationary charges, but this author has introduced its use also with respect to moving charges.
In a paper [1] it has thoroughly been shown, how Coulomb's originally electrostatic law can be applied upon currents, without any involvement of magnetic fields, which is otherwise common practice today. The effort succeeds through the consequent usage of retarded evaluation of the Coulomb force. The method this is being done is similar to that used by Jackson [45]. The steps this was being done will briefly be shown here. The first step was to derive an expression for the charge density of the conductor, taking into account the effects of retardation with respect to the velocity of light.

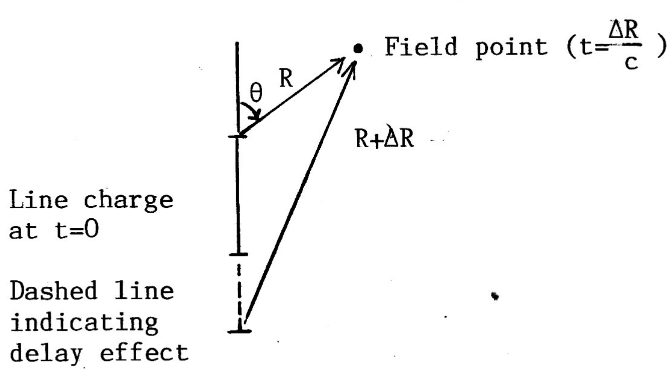

Figure 1. Geometry in order to compute the effects of retarded action with respect to the 'sending' charges [2]

The charge density due to the moving electrons an observer will register will be [8]

$$\rho_1' = \rho_1(1-(\frac{\vec{v}_1 \bullet \vec{R}}{Rc})\cos\omega) \qquad (2)$$

An unexpected term $\cos\omega$ has to be attached, if assuming that the conductors are not necessarily situated in the same plane. In the paper [1], where this analysis was first performed, both the conductors were assumed to be situated in the same plane.
This expression – or its equivalent – has also been attained by others, among others Jackson [45] and is commonplace.
In analogy to what can be said about the 'sending' point may also be said about the point of observation. This means that also the charge density which the fields is interacting with will be changed due to retarded action. This, however, is not being treated by Jackson. Instead, it seems to constitute a discovery by this author. The result for that charge density due to moving electrons will be [4]

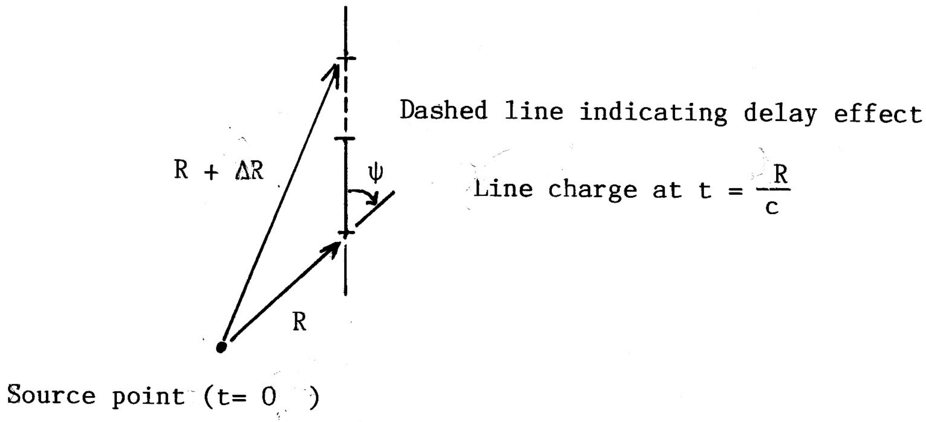

Figure 2. Geometry in order to compute the effects of retarded action with respect to the 'receiving' charges [3]

$$\rho_2' = \rho_2 (1 - (\frac{\vec{v}_2 \bullet \vec{R}}{Rc}) \cos\omega) \qquad (3)$$

An unexpected term $\cos\omega$ has again to be attached, if assuming that the conductors are not necessarily situated in the same plane.

In order to attain the net charge density that will affect other currents one has to subtract the effect by the positive ions. charge densities of the immobile positive ions respectively. This results in the net charge densities [5]:

$$\rho_1'' = -\rho_1 (\frac{\vec{v}_1 \bullet \vec{R}}{Rc}) \cos\omega \qquad (4)$$

and

$$\rho_2'' = -\rho_2 (\frac{\vec{v}_2 \bullet \vec{R}}{Rc}) \cos\omega \qquad (5)$$

If writing Coulomb's law on differential form, thereby using charge densities instead of point charges, one generally will write [6]

$$\frac{d^2 \vec{F}}{dx_1 dx_2} = \frac{\rho_1 \rho_2 \vec{u}_R}{4\pi\varepsilon_0 R^2} \qquad (6)$$

For this particular case with two kinds of charges, positive ions and electrons, and with net charge densities according to Eq. (4) and (5) above, one attains

$$\frac{d^2\vec{F}}{dx_1 dx_2} = \frac{\rho_1''\rho_2''\vec{u}_R}{4\pi\varepsilon_0 R^2} \qquad (7)$$

Inserting the full expressions of Eq. (4) and (5) and using the relation

$$\mu_0\varepsilon_0 = \frac{1}{c^2} \qquad (8)$$

as well as

$$I = \rho v \qquad (9)$$

give altogether the following expression for the Coulomb force between two currents, carried by conductors [7]:

$$\frac{d^2\vec{F}}{dx_1 dx_2} = \frac{((\mu_0 I_1 I_2 \cos\theta\cos\psi)\cos^2\omega)\vec{u}_R}{4\pi R^2} \qquad (10)$$

2.2.Application of the special relativity theory (SRT) upon the above results.

Since only spatial functions are involved in the equations above, the Lorentz transformations to be done will not involve time specifically. The spatial variables involved must on their part be specified with respect to each coordinate, x, y and z. For the case with a current I
aligned with the x axis, the Lorentz transformation of the charge density, thereby assuming ρ_0 at rest [12]

$$\rho = \frac{\rho_0}{\sqrt{(1-\frac{u^2}{c^2})}} \qquad (11)$$

which can simpler be written

$$\rho = \rho_0\gamma(u) \qquad (12)$$

That means that the charge density increases with regard to an observer in a "stationary inertial frame", denoted by K. With the "stationary inertial frame" is meant the frame, according to which the electric circuits are at rest. This relativistic effect is an effect that exists independently of the "retardation effects" [21], which can be seen already in the preceding section. The Lorentz transformation implies that the charge density of moving charges physically differs from that when they are at rest, whereas retardation effects mean that the geometry gives rise to different propagation times due to different distances from the observer to different parts of the circuit.

2.2.2. Problems thus far to restrict the applicability of the second postulate.

The Special Relativity Theory (SRT) has now and then been criticized for being unable to explain some classes of phenomena, among others the Sagnac Effect [22]. And the very founder of the SRT, Einstein himself, is hesitating concerning its validity in the cases of non-linear motion. A prerequisite for the second postulate is that inertial systems are being regarded [23], [25],[29]. For the case of non-linear motion, Einstein takes the position of both pro [24] et contra [27].

The second postulate does not originally explicitly specify 'inertial systems' as a prerequisite for being in force [23], even though the first postulate indirectly indicates that [23]. Later interpretations lay the words 'inertial systems' into the mouth of Einstein [25].

Apparently Einstein himself was not fully aware of how precise the postulate had to be formulated and that is also the starting point for much of today's agony around this issue.

Hence, the problem how to solve the question of the dominion of the SRT, is still to be treated.
The Lorentz transformation implies that a "distance" within the "space-time" domain may be written as

$$x^2 + y^2 + z^2 - c^2 t^2 = const. \qquad (13)$$

for all inertial systems [26]

2.2.3. Support for the usage of the SRT and its updating into non-linear movement

The solution to the problems is to realize that one has to regard the Lorentz transformation as being intrinsically of differential nature. That has already been done in a paper about the Sagnac effect [28] and will shortly be recalled here.
By dividing all curves into infinitesimally short pieces, curves can be treated as a sum of short straight lines, just like in integral calculus. The Lorentz transformation can thus be applied by making successive transformations between the line elements.

Generalizing this discovery, the Lorentz transformation might then be applicable to movement along most continuous curves.
The space-time constancy will accordingly apply to the modified formula

$$dx^2 + dy^2 + dz^2 - c^2 dt^2 = const. \qquad (14)$$

of which

$$x^2 + y^2 + z^2 - c^2 t^2 = const. \qquad (13)$$

constitutes a special case.

This had to be said here in order to increase the support for the usage of the Special Relativity Theory (SRT), which would better be called an 'Updated Relativity Theory'(URT) hereafter.

2.2.4. Analysis of the charge densities of the "first conductor"

Equation (12) above was attained for the moving electrons above, when the effects of retardation was taken into account. If assuming that for the case of the movement of the electrons, Coulomb's law remains unchanged only with respect to the inertial system following the moving electrons. For the immobile ions Coulomb's law will not be changed due to velocity. This in turn implies that electrons and ions must be treated separately. Further, the 'net charge densities' which were attained above in Eq.(4), (5) and (7) can no more be used, since there will now be a Lorentz transformed distance vector between the sending charges and the point of observation. Namely, if assuming that Coulomb's law is valid in one coordinate frame, the space variables must obey the Lorentz transformation, and, hence, the moving electrons will give rise to one distant vector, the immobile ions another. This seems at a first glance rather complicated, but it is deeply logical. For example,
the variable x will be transformed into $\gamma(u)x$, provided the movement takes place along the x axis with velocity u.
Since the Lorentz transformation was regarded to be independent of the retardation effects, it should only be needed to multiply the expression with the gamma factor from Equation (12), denoting by SRT that the expression has been derived, taking into account the effect of the Lorentz transformation.

$$\rho_1{'}_e(SRT) = \rho_1(1-(\frac{\vec{v}_1 \bullet \vec{R}}{Rc})\cos\omega)\ \gamma(v_1) \qquad (15)$$

For the immobile positive ions one may simply write

$$\rho_1{'}_p(SRT) = -\rho_1 \qquad (16)$$

2.2.5. Analysis of the spatial terms of the force

Coulomb's law on differential form was derived in the preceding section (2.1). The changes that have to be made with respect to the charge density have now already been attained. Remains to rewrite $\frac{\vec{u}_R}{R^2}$ with respect to the relativistic effects, since if this vector can be related to the Coulomb field due to the emitting charges, if they are moving with a velocity $\vec{v}_1$, the spatial variables therein must be transformed according to the Lorentz transformation. One may therefore dissolve $\frac{\vec{u}_R}{R^2}$ into its respective components,

$$\frac{\vec{u}_R}{R^2} = \frac{(x,y,z)}{(x^2+y^2+z^2)^{\frac{3}{2}}} \qquad (17)$$

Making the Lorentz transformation, assuming the electrons moving along the x axis, one attains:

$$(\frac{\vec{u}_R}{R^2})(SRT) = \frac{(\gamma(v_1), y, z)}{(\gamma^2(v_1)x^2+y^2+z^2)^{\frac{3}{2}}} \qquad (18)$$

2.2.6. Combining charge densities of the sending point with respective length vector.

In order to write the total field correctly, one must 'pair' Eq.(15) with Eq. (17) and Eq.(16) with Eq.(18) so that respective charge density (moving or not-moving) is attached to a relevant length vector, due to respective SRT effect. Immobile ions, of course, are thence attached to a 'normal' length vector, with no length contraction effect. But still it is not possible to write the total force, since the charge densities of the second conductor must first be derived.

2.2.7. Analysis of the charge densities of the "second conductor"

For the conductor electrons of the second conductor ('observation point') it is possible to attain an expression that takes the SRT into account, all in analogy with the treatment of the electrons of the first conductor ('sending point') above.

Generally, the second conductor may have any direction. Having chosen the direction of the x axis in K along the first conductor, this is of no use with respect to the second conductor. It is impossible to repeat the procedure above leading to Eq.(15) for the first conductor. Instead, there will appear components dependent on the γ in all three directions. A transformation matrix must be used in order to attain the result.

However, one may use the simplistic case with two parallel conductor currents in order to show the order with which the Lorentz transformation will affect the charge densities of the second conductor. If then the order of v/c is larger for the Lorentz transformation than for the retardation effect that order will not change while changing the direction of that current. A physical property does not change if changing the directions of the coordinate axes.

For the simplistic case above, the charge density of the second conductor will – in analogy with the derivation of Eq.(15) and Eq.(16) be written as

$$\rho_2{}'_e(SRT) = \rho_2(1-(\frac{\vec{v}_1 \bullet \vec{R}}{Rc})\cos\omega)\ \gamma(v_2) \qquad (19)$$

and

$$\rho_2{}'_p(SRT) = -\rho_2 \qquad (20)$$

2.2.8. Combining the results above.

Now it remains to put all relevant parts together in order to attain a new expression for the Coulomb force between the two currents, carried by two conductors respectively. It is Equation (7) that has to been rewritten, using the results of the relativistic analysis above, in sections 2.2.1 to 2.2.7:

Using Equations(7), (17), (18) and applying the results to Equation (10) leads to the following result:

$$\frac{d^2\vec{F}}{dx_1dx_2} =$$

$$(\frac{(\gamma(v_1)x, y, z)}{4\pi\varepsilon_0(\gamma^2(v_1)x^2+y^2+z^2)^{\frac{3}{2}}}\rho_1(1-\frac{\vec{v}_1 \bullet \vec{R}}{Rc}\cos\omega)\gamma(v_1) - \frac{(x,y,z)}{(x^2+y^2+z^2)^{\frac{3}{2}}})(\rho_2((1-\frac{\vec{v}_2 \bullet \vec{R}}{Rc}\cos\omega)\gamma(v_2) -$$

(21)

This expression may be simplified in two separate steps, the part beginning with ρ_2 relating to another velocity and angle than the first part of the equation.
In both parts the terms containing scalar products may for convenience be written, thereby using angles and the absolute of the velocities according to [9]

$$1-\frac{\vec{v}_1 \bullet \vec{R}}{Rc} = 1-\frac{v_1}{c}\cos\theta \qquad (22)$$

and

$$1-\frac{\vec{v}_2 \bullet \vec{R}}{Rc} = 1-\frac{v_2}{c}\cos\psi \qquad (23)$$

Hence, one may rewrite Eq.(21) according to:

$$\frac{d^2\vec{F}}{dx_1dx_2} =$$

$$(\frac{(\gamma(v_1)x, y, z)}{4\pi\varepsilon_0(\gamma^2(v_1)x^2+y^2+z^2)^{\frac{3}{2}}}\rho_1(1-\frac{v_1}{c}\cos\theta)\cos\omega\gamma(v_1) - \frac{(x,y,z)}{(x^2+y^2+z^2)^{\frac{3}{2}}})(\rho_2((1-\frac{v_2}{c}\cos\psi)\cos\omega\gamma(v_2$$

(24)

Neglecting again terms above the lowest order of $\frac{v}{c}$ and using Equation (8) and (9) makes the Equation (24) to simplify to

$$\frac{d^2\vec{F}}{dx_1 dx_2} = \frac{((\mu_0 I_1 I_2 \cos\theta\cos\psi)\cos^2\omega)\vec{u}_R}{4\pi R^2} \qquad (10)$$

Hence, the implementation of the SRT does not imply any change of the expression for the Coulomb force, as far as the velocities remain low, i.e. v<<c
A figure depicting the prediction of measurement results upon a set of Ampère’s bridge follows later, in connection with the predictions by Wesley, section 3.2.2.

3. Ampère’s law explaining the force between currents.

3.1. Ampère deduces a law for the forces between electrical currents.

Already from the title of Ampère’s paper [30] it becomes very clear that his intention is to deduce a theory for the interaction between electrical currents based upon experimental evidence. One experiment series described in this paper was performed in Geneva 1822 [31], [38]. It consists of two mercury troughs, two conductors lead from an external voltage source into respective trough at one end. A “boat” made of conducting copper wires in a catamaran shape is resting upon the surface of respective basin, with a bridge connecting them to each other, going through the air. Thereby a closed electrical circuit is created and as the voltage is switched on, it can be observed that a repulsive force is pushing the boat ahead, away from the electric poles at the boarder of the basin.

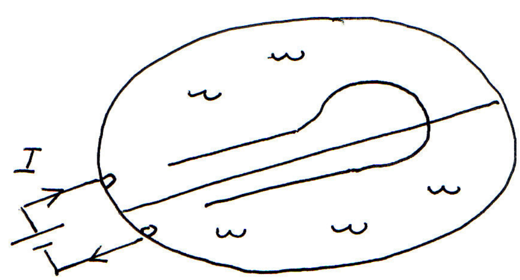

Figure 3. Illustration of Ampère’s experiment with a copper boat, floating on troughs filled with mercury [31]

Earlier in the paper he discusses more rudimentary experimental situations and deduces a formula, step by step, [32], [34] for the forces between two electrical currents, beginning with a discussion of the very way currents may be measured [33]. According to Blondel [37] that formula was first presented in 1820 [20]He thereafter assumes that the force must be proportional to both currents involved, to the length of the conductors and he assumes also an angular dependence and a spatial dependence proportional to the inverse distance to the nth potency. Due to experimental evidence he seems to be forced to assume a sum of two angular functions. Short-to-speak he defines the reasons for both according to a principle that there are parallel components of respective current that affect the force and since each current can be separated into two perpendicular components, there are the two parallel component pairs thus derived which must be multiplied to each others respectively. That sounds rather complicated. But for the sake of clarity it can be said that there are first the components of both currents along a line at right angles to the mid point of respective current that have to be multiplied with each others, i.e. their cosines, thereafter the components of respective current along a line perpendicular to that connecting line (i.e. sines), times the cosine (i.e. component) betweens the planes the currents lie within respectively and which also contains the distance vector. [32], [37]

$$Force = \frac{ii'dsds'(\sin\theta\sin\theta'\cos\omega + k\cos\theta\cos\theta')}{r^n} \quad (25)$$

Tedious derivations lead Ampère to the conclusion that
$n = 2$ and $k = -½$
and hence, the force between the two currents can be written

$$Force = \frac{ii'dsds'(\sin\theta\sin\theta'\cos\omega - ½\cos\theta\cos\theta')}{r^2} \quad (26)$$

This formula has the benefits to be able to account for

- The attractive force between two currents aligned to each other(due to the right term)
- The repulsive force between two currents on the same line (as is the case with the "boat" described above

However, in this very extensive paper of Ampère there is no numerical data and therefore one feels compelled to believe that he is mainly exploring the qualitative behaviour of the forces between currents. The way he derives the formula reminds of the way a computer program is computing the coefficients of a polynomial that gives a RMS fitting. Hence, his theory may be opposed by a better one that at least to a good approximation gives rise to the same results but has a higher epistemological quality. Such efforts have been done, as will be seen from the following. On point that sticks into eyes is for example that in his time the electron had not yet been found and there must have been rather vague ideas of what an electric current is made of. Ampère himself directly begins with the currents as the principal cause for behind the forces.

3.1.1. A qualitative judgement of Ampére's law.

When studying literature by Ampère, describing the way he derives 'Ampère's law´, it appears that he just searches for any combination of terms that are able to satisfy certain qualitative experimental evidence. There seems to be lacking numerical experimental data, even though the experiments are described well in a qualitative manner. [30], [31], [32], [36], [37].

The experiments by Ampère are further treating continuous currents, which imply that the formula he is searching for inevitably include more than one physical effect, especially the Lorentz transformation and the effect of retardation.

3.2. Discussion of Ampère's law in connection to other efforts

Considerable efforts have been made to infer the forces between electrical currents by Ampère and there are contemporary scientists, who use the formulae he succeeded in deriving in the 1820's, like Wesley [16] and Keele [39].

3.2.1. Wesley's approach using Ampère's law

Wesley[16] is using solely Ampère's law, without making any reference to Maxwell's equations and he makes an extensive analysis of the forces within a set of Ampère's bridge, that same experiment that the author of this article has been treating [1] before the above relativistic approach was applied.

$$d^2\vec{F} = I_2 I_1 \vec{r}(-2(d\vec{s}_2 \bullet d\vec{s}_1)/r^3 + 3(d\vec{s}_2 \bullet \vec{r})(d\vec{s}_1 \bullet \vec{r})/r^5) \quad (27)$$

$$\cos\omega = 1 \quad (28)$$

3.2.2. Spencer's approach using Ampère's law

Spencer, too, makes reference to Ampère’s law in a paper but is simultaneously giving credit to Maxwell’s equations [43]. She writes the formula for the force between two current carrying conductors as follows [44]:

$$d^2\vec{F}_A = -\frac{I_1 I_2}{4\pi\varepsilon_0 c^2 r^2}\vec{a}_r(2(d\vec{s}_1 \bullet d\vec{s}_2) - 3(d\vec{s}_1 \bullet \vec{a}_r)(d\vec{s}_2 \bullet \vec{a}_r)) \qquad (28)$$

Regrettably, this equation, being referred to as Ampère’s law, is not written in the way Ampère himself wrote it (cf. Eq. (25) and (26) above), instead it resembles the formulation of Wesley above. And it is further not satisfactorily being shown, how the transformation really occurs. Instead a reference is being done to a piece by Maxwell: “The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. …”, in fact a statement lacking any conceptual content. In reality, as shown by this author [8] the electromagnetic fields à la Maxwell are completely unable to predict the parallel force between electric currents appearing in Ampère’s bridge. In connection with his famous “boat” experiment [31], [38], Ampère thoroughly describes how a circuit consisting of two mercury troughs with two copper boats connected by a bridge in the air is able to verify a repulsive force between two rectilinear current elements, in the same line, repel each others. The boats namely begin to travel away from the electric contacts. Ampère also has the benefit to very carefully deduce his formulas, thereby checking with experimental results.

He only makes the claim that there must be parallel components of currents, which interact with each other. He thereby separates both currents in components aligned to the distance vector between themselves and into components that are orthogonal. Only those terms, two by two, contribute, and hence, one finally attains the two famous terms of Ampère’s law [20].

The observant reader may already have seen (above) that this author succeeds in explaining the force between two currents using only the retardation effects with respect to Coulomb’s law. But this was so because that effect remains effective already at low velocities, when the typical v^2/c^2 term of the SRT vanishes. This means that at higher velocities, both effects must be accurately taken into account. But in the case of typical conductor currents, the velocity is of order mm/s and hence, the SRT becomes completely irrelevant.

3.2.3. Wesley’s derivation of the force between currents

As mentioned above, Wesley made a very deep-going effort to derive an expression for the force between the two parts of Ampère’s bridge [16], thereby evaluating Ampèrs’s law for several cases measured by Pappas and Moyssides [15]. He succeeds rather well in doing so, having some success in predicting the measured force. However, as can be seen from the figure he has made [16], the slope is not exactly correct. Jonson, instead [1] succeeds better in this respect, indicating that the basic space variables appear with right coefficients. He attains a curve with almost exactly the correct slope, but the level is 0.42 times the measurement results [8]. The missing constant factor would easily be described as an effect of the scaling, i.e. fitting of constants when determining the strength of the measured current in a typical measurement instrument. Since – according to Jonson – the commonly used formulas today are inherited with fundamental faults, as becomes clear above – a new scaling for measurement instruments becomes inevitable if stating another fundamental law to be ruling. Jonson also gives a closer description of the way currents are fallaciously being measured by today’s instruments, due to a fundamental flaw in understanding the very measuring. [41], [42]. Nonetheless, the basic proportionality to the currents involved in the expression for the force is the same in both the formulas used today [7] as in the Coulomb model [1]. In Ampère meters (and in Volt meters as well) there is an angular momentum that forces the pointer or needle to move. Both Wesley and Jonson presume usual instruments being used when measuring the currents of course, and hence, there will appear a fitting problem for both of them. One gets a better conformity between the values the other between the slopes. The result of the discussion above clearly favours the conformity between slopes as decisive for giving credit to a theory.

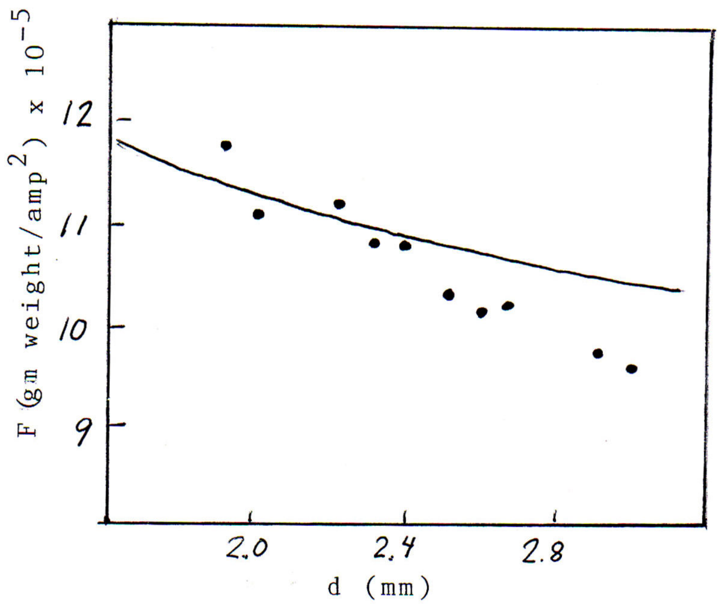

Figure 4. Wesley describes how his theory (curved line) fits with experiments on Ampère's bridge [16], [17]

3.3. The effort by Keele to derive Ampère's law using Coulomb's law and the Special Relativity Theory (SRT).

Keele in a paper [39] presents a theory, according to which it is possible to derive Ampère's law
from the Lorentz force, given below. He claims that the expression may be derived using the rudimentary definition, the 'Lorentz force law':

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B}) \qquad (29)$$

thus arriving at:

$$\vec{e}_s = \frac{kq_s \dfrac{\vec{r}}{\gamma^2 r^3}}{(1 - \dfrac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}} \qquad (30)$$

for the electric force term in Eq. (15) above..
This expression is by its content equivalent with the expressions derived by Resnick [14].

Keele also shows that Coulomb's law can be attained as a special case of Eq.(30), i.e. the Lorentz force (comment made by this author), provided that v=0 and γ=1.

(Below – please see section 3.3.4 - it is convincingly being shown that there is no such system, where the second 'magnetic' term of Eq.(29) disappears)

Applying a binomial series expansion, thereby eliminating the higher order terms of $\frac{v^2}{c^2}$, and subtracting out the stationary Coulomb force, gives, according to Keele,

$$\vec{f} = \frac{kq_t q_s}{r^3}\frac{v^2}{c^2}\vec{r}(0.5 - 1.5\cos^2\theta) \qquad (31)$$

Transformation into continuously distributed charges, constituting thereby electric currents, he gets on differential form:

$$d^2\vec{f} = -\vec{r}\,\frac{kI_t I_s ds_t ds_s}{c^2 r^2}(2\sin\theta_t \sin\theta_s \cos\eta - \cos\theta_t \cos\theta_s) \qquad (32)$$

This expression may be rewritten using vector notation as follows:

$$\vec{f}_{12} = -\vec{r}_{12}\,\frac{kI_1 I_2 ds_1 ds_2}{c^2 {r_{12}}^2}(2d\vec{s}_1 \bullet d\vec{s}_2 - 3(d\vec{s}_1 \bullet \vec{r}_{12})(d\vec{s}_2 \bullet \vec{r}_{12})) \qquad (33)$$

This expression is the same that Wesley arrives at above, in section 3.2.1. For the reader's convenience, the original expressions of the respective authors have been used, thereby causing slight confusion due to a differing terminology. Please take a look at the variable list below if needed!

3.3.1. A discussion of Keele's approach.

The criticism that one can direct against the approach by Keele, if one is intending to create a fundamentally new understanding of electrodynamics, is that he sticks to the Lorentz force, as if it were to be a "simplest presumption" within electrodynamics. According to the principles of the philosophy of sciences, one ought to search for a simplest thinkable theory for action, and apparently Coulomb's law does constitute such a one, the Lorentz force not, since it involves three vector functions, providing the basis for the action, namely: the electric field, the velocity vector and, finally, the magnetic field., contrary to Coulomb's law, which reduces that amount by one, using only the first two vector functions (the velocity when taking into account the effects of the SRT)

3.3.2 A computational error in another derivation by Keele.

Regrettably, a careful check of the steps performed by Keele in his derivation in a memberchatpaper 2006 [40], Appendix 2, implies that he should not have arrived at his Eq.(31) if applying the Lorentz transformation to Coulomb's law, as he says he is doing. The fault is due the missing γ within the distance vector $\vec{r}\,'$ that should replace $\vec{r}$ in the numerator term of the electric force due to Coulomb's law:

$$\vec{e}_s = kq_s\frac{\vec{r}}{r^3} \qquad (34)$$

Assuming the famous standard configuration, and no time present, due to the steady state current namely gives

$$\vec{r}\,' = (x\gamma, y, z) \qquad (35)$$

and accordingly $(r')^3 = (\gamma^2 x^2 + y^2 + z^2)^{\frac{3}{2}}$ (36)

The fault was already indicated above, in connection with Eq. (30).

Using $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ (37)

And working straightforwardly, thereby applying binomial series expansions, accordingly and subtracting out the stationary Coulomb term gives

$$\vec{f} = kq_t q_s \frac{(\gamma x, y, z)}{r^3}\frac{v^2}{c^2}(-1.5\cos^2\theta) \qquad (38)$$

Apparently, there is a huge difference between Eq. (31) of Keele and Eq. (38) which is the result, which Keele should have arrived at if performing the computations correctly.

3.4. Implications by the flaws introduced by the Lorentz force.

It may at once be recognized that the correct computation of Coulomb's law implies that

Ampère's law has to be regarded as a strictly empirical law, though in a very restricted, qualitative manner (please see section 3.1. above). The mere fact that Ampère's law to the lowest orders of v/c coincides with the Lorentz transformed electric field term does not prove any physical entity. And, leaving the lowest orders of v/c, they indeed diverge.

3.4.1. Principal flaws embedded in the Lorentz force

Firstly, the Lorentz force is used as a ***definition*** of the total electromagnetic force between charges. It consists of two disparate terms, depending on altogether three vector fields, the electric, the magnetic and the velocity field. For the special case of stationary charges it coincides with Coulomb's law, but it is not Coulomb's law itself. Secondly, a traditional derivation of the transformation of forces between two coordinate systems (here inertial frames) [11] leads to the requirement that

$$F_x = F_x' , \qquad F_y = \frac{F_x'}{\gamma} \quad \text{and} \quad \frac{F_Z'}{\gamma} \qquad (39)$$

respectively in order to grant the conservation rules of mechanics when the SRT is applied upon the equations. The proof is omitted here but can be followed in detail in the book on special relativity by Resnick [18].

However, there is also a built-in flaw in the very idea to assume that, what has to be proved. That means that making the Lorentz transformation of Coulomb's law should lead to eq.(39) ***if the assumption is correct***. One may not beforehand prescribe the result! But current science does so and

according to that model, these relations, in turn, are accordingly being imposed upon the Lorentz force law.

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B}) \qquad (29)$$

Going from one frame K, where the conductors are held at rest, to K', following the movement of the electrons, which constitute the current, leads to the presumption that whereas K has a magnetic field due to this current, K' doesn't. Hence, whole the electromagnetic force must in this case be attributed to the electric force term of the Lorentz force. But in doing so, a flaw is introduces, since it is simultaneously forgotten that the positive conductor ions will be regarded as moving in K', thus constituting the current of that system, thereby giving rise to a 'magnetic force' term, though with the opposite charge as well as direction of the velocity.

And if looking at free electrons in space, due to the general assumption of total charge neutrality, there will necessarily be positive charges somewhere, if not moving in the system where the electrons are regarded to be moving, they will nonetheless be moving in the system, where the electrons are at rest. And, if there is no force working upon an electron this is equivalent to regarding the ‘mass centre’ of the positive charges as being situated at the electron.

Another matter necessary to take into account is that if an electron is moving in one direction, there must be another negative charge moving back at some other location in order to grant that no net work is being done within the closed system – a fundamental law of mechanics - consisting of moving electrons (thereby neglecting the positive charges, if they are assumed to be totally at rest, thereby doing no work). This discussion is performed in another paper [7]. A reasonable conclusion is that also electrons in free space may be regarded as participants in a closed electric circuit.
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3.5. The need for Jonson’s corrections

Though mainly a correct approach, besides the mistakes treated above, the derivation by Keele does not take into account the consequences of retarded action. Hence, one more step is still needed in order to attain expressions that are able to predict the tru force between currents. The analysis is already done in section 2.2 above. Simultaneously, he is also successful in showing that for the case of currents carried by conductors, the effects of the SRT can be neglected in comparison with the effects of retardation.

4. Discussion of the different results

Experimental evidence supports Ampère's law. It is also possible to show that Ampère's law may be inferred by using Coulomb's law and take into account the effects the length contraction, expressed through the Lorentz contraction, introduces. Thereby the origin to the two terms in Ampère's law can be understood. Ampère deduced the two terms of his law, being still unable to give a reason for their appearance. Further, in the case of low velocities compared with the velocity of light, the effects of retarded action predominates, as in the case of conductor currents. Efforts to use Ampère's law in order to explain experimental results with Ampère's bridge give ambiguous results; numerically a good approximation but with the wrong slope of the curve; the straightforward use of retardation gives numerically worse resemblance but the slope is the correct one. A discussion of measurement sciences indicate that there is a scaling problem, depending on which fundamental law one assumes when measuring currents. Hence, the discrepancy between theory and measurements when using the retardation approach is being explained.

5. Conclusions

The conclusion to be drawn is that Coulomb's law is the utmost cause behind electromagnetic action, independently of whether the charges are moving or at rest. The apparent problem in analysing the electromagnetic forces due to moving charges, or in the case that has been studied above, continuous electric currents, arise due to two effects related to relative velocity: the length contraction of the Special Relativity Theory and retarded action do to the path the field has to cover. A strict analysis in order to establish which one predominates is necessary to be performed. Ampère's law has to be understood as a good approximation to the force, whenever the length contraction has been applied, though not retarded action.

6. Variable list and some explanatory points

6.1. Comment.

The variables are mainly given in the order they appear in the paper, but the same kind of variables will appear together. When references to certain authors require other choices of names, this will be indicated. Please observe that the original terminology of respective author has been used.

6.2. The list.

$\vec{F}$	total electric force between charges
dx_1	incremental part of the first conductor
dx_2	incremental part of the second conductor
ρ_1	charge density of the moving electrons of the first conductor
ρ_2	charge density of the moving electrons of the second conductor
$\vec{u}_R$	unit vector along the distance vector from the first to the second conductor
$\vec{R}$	distance vector from the first to the second conductor
R	length of the distance length from the first to the second conductor
ρ_1'	virtual, i.e. observed charged density of the moving electrons of the first conductor
ρ_2'	virtual, i.e. observed charged density of moving electrons of the second conductor
$\vec{v}_1$	velocity of the electrons of the first conductor
$\vec{v}_{2_1}$	velocity of the electrons of the second conductor

ρ_1'' net virtual charge density of to the first conductor, including positive charges and electrons

ρ_{2_1}'' net virtual charge density of to the second conductor, including positive charges and electrons

I_1 current of the first conductor

I_2 current of the second conductor

θ angle between the direction of the first current and the distance vector to the second current

ψ angle between the direction of the second current and the distance vector to the first current

ω the angle betweens the planes the currents lie within respectively and which also contains the distance vector

$\rho_1'_e(SRT)$ observed charged density due to the moving electrons of the first conductor, taking into account the effects of the Lorentz transformation

$\rho_1'_p(SRT)$ observed charged density due to immobile positive charges of the first conductor, taking into account the effects of the Lorentz transformation

$\rho_2'_e(SRT)$ observed charged density due to moving electrons of the second conductor, taking into account the effects of the Lorentz transformation

$\rho_2'_p(SRT)$ observed charged density due to immobile positive charges of the second conductor, taking into account the effects of the Lorentz transformation

6.2.1. Variables of Resnick

ρ charge density (generally), taking into account SRT (Lorentz transformation)

ρ_0 charge density (generally), regarded in the reference frame, where they are at rest

$\vec{u}$ velocity vector of the electrons of a current

u absolute of the velocity vector of the electrons of a current

γ Lorentz factor (generally)

$\gamma(u)$ Lorentz factor due to a specific velocity

6.2.2. Variables of Weidner and Sells

x, y, z, t Cartesian space-time variables of a reference frame

dx, dy, dz, dt Cartesian incremental space-time variables of a reference frame

6.2.3. Variables of Ampère

θ angle between the direction of the first current and the distance vector to the second current

θ' angle between the direction of the second current and the distance vector to the first current

ω the angle betweens the planes the currents lie within respectively and which also contains the distance vector

r length of the distance vector from the first to the second conductor

6.2.4. Variables of Wesley

$\vec{r}$ distance vector from the first to the second conductor

r length of the distance vector from the first to the second conductor

$d\vec{s}_1$ incremental part (vector) of the first conductor

$d\vec{s}_2$	incremental part (vector) of the second conductor
	6.2.5. Variables of Spencer
$d\vec{F}_A$	the total incremental force between two current carrying conductors
$\vec{a}_r$	distance vector from the first to the second conductor
	6.2.5. Variables of Keele
$\vec{e}_s$	electric field intensity at a stationary point, emanating from a moving charge q_s at a 3-vector distance of **r**
k	short for $1/4\pi\varepsilon_0$
q_s	point charge of the sending point
q_t	point charge of the target
$\vec{f}$	resulting electromagnetic force between two charges after having subtracted out the stationary Coulomb force
$d^2\vec{f}$	resulting electromagnetic force between two continuously distributed charges, thereby constituting a current, after having subtracted out the stationary Coulomb force
θ_s	angle between the direction of the first current and the distance vector to the second current
θ_t	angle between the direction of the second current and the distance vector to the first current
η	the angle betweens the planes the currents lie within respectively and which also contains the distance vector
$\vec{f}_{12}$	the same as $d^2\vec{f}$
$\vec{r}_{12}$	distance vector from the first to the second current
r_{12}	length of the distance vector from the first to the second current
$d\vec{s}_1$	incremental part (vector) of the first conductor
$d\vec{s}_2$	incremental part (vector) of the second conductor

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8. Appendix 1. Jonson, J. O.,
"THERE'S NO NEED FOR AN ETHER -Einstein was right – again! But also wrong!", Memberschatpaper within NPA July 24, 2008

1. Introduction
It has repeatedly been claimed that the Sagnac effect implies that there must be something wrong with the Special Relativity Theory (SRT).
Paul Marmet in a paper (1) describes how correction terms being used in the GPS applications imply that the customary way to compute the tracks of light in coordinate systems moving with respect to each others must be revised. Normally it is assumed that both ways for a light beam requires the same time, as prescribed by the Einstein light clock synchronization techniques (2), which more correctly ought to be described as an 'intellectual experiment' rather than a real and feasible one.
Marmet describes that this model fails for the case of the corrections being made within the GPS system, thereby prescribing an addition for the case radiation is sent westwards, and subtraction for the case radiation is sent eastwards (3).
The null result of Michelson-Morleys experiment is explained through the cancellation of the two correction terms needed when first sending a light beam westwards, thereafter eastwards^2 (4) . He presents the correction to be proportional to $\frac{v}{c^2}$, thereby referring to Kelly (5).
A university search for this reference paper gave no result – it was not available on the advanced search systems.
Eager to understand the phenomenon myself, I propose the following explanation.

2.Explanation.
Assuming first the speed of light is c in all inertial systems, it must be recalled that a coordinate system based at the surface of the Earth does not satisfy the condition to be an 'inertial system', since it is orbiting around its own axis.
One could nonetheless favourably define an inertial system, let's call it **K'**, which has touches the surface of the Earth at one point and at that point also follows the eastwards movement of the Earth with the velocity **v** relative to an inertial system **K,** with origin in the center of the Earth. That's however no more need to discuss the system K. The interesting feature of the movement of the surface of the Earth, presumably along a latitude line at an arbitrary distance from the equator, is that its x' component changes **continuously** with respect to K'.
Using the Special Relativity Theory (SRT), one could speak of continuously changing inertial systems **K''.** The interesting thing that –**still using the SRT** -can explain the need for a positive time correction when a body (or radio waves) are sent westwards (14 ns between N.Y. and S.F. according to Marmet (6), and a corresponding negative correction, when a body

is moved eastwards, is the so-called time dilation between inertial systems with a reciprocal relative velocity.
Using the so-called standard configuration, an expression for the time dilation can be attained (7)

$$\frac{dt'}{dt} = \gamma(v)(1 - \frac{v}{c^2}\frac{dx}{dt}) \qquad (1)$$

With our choice of indices, the expression would read

$$\frac{dt''}{dt} = \gamma(v)(1 - \frac{v}{c^2}\frac{dx'}{dt}) \qquad (2)$$

The term $\frac{dx'}{dt}$ indicates the x' component in K' of the velocity of the surface of the Earth, being zero at the 'touching point' between K' and the surface of the Earth, being minimal at the east and west 'end points' of the Earth during its revolution.
Integrating the expression gives principally

$$t'' = \gamma(v)(t' - \frac{v}{c^2}x') \qquad (3)$$

As can be seen, the time dilation follows the form given by Kelly that has also been corroborated through experimental evidence.

3. Why Einstein is wrong
It is often being made reference to the so-called Einstein synchronization of clocks, when proving the famous 'null result' of the Michelson-Morely experiment.
It is usually proved that there can be no 'aether' affecting the velocity of light, and hence the conclusion has been made that it must take the same time for a light beam to travel both directions.
However, with the above results, it must be conceived that the null result appears due to the cancellation of the correction terms due to their different sign due to direction. The last term in Eq.(3) above namely cancels, when adding the two contributions, one due to the westward movement the other due to the eastward.

4. Why Einstein is right
The Einstein synchronization method for moving clocks is of course correct as long as two inertial systems are regarded, with no change of the directions of any system with respect to time.

5. Conclusions.
As can be seen from above the SRT is able to account for the behaviour described by the Sagnac effect. Maybe the above derivation is the same one as that by Kelly, but regrettably the Kelly paper has not been found and Dr. Marmet is not more among us.
It must also be mentioned that Marmet in the frequently referred paper repeatedly makes his refernces to the SRT, though without presenting any proofs of his own for the claims (1).

6. Figure

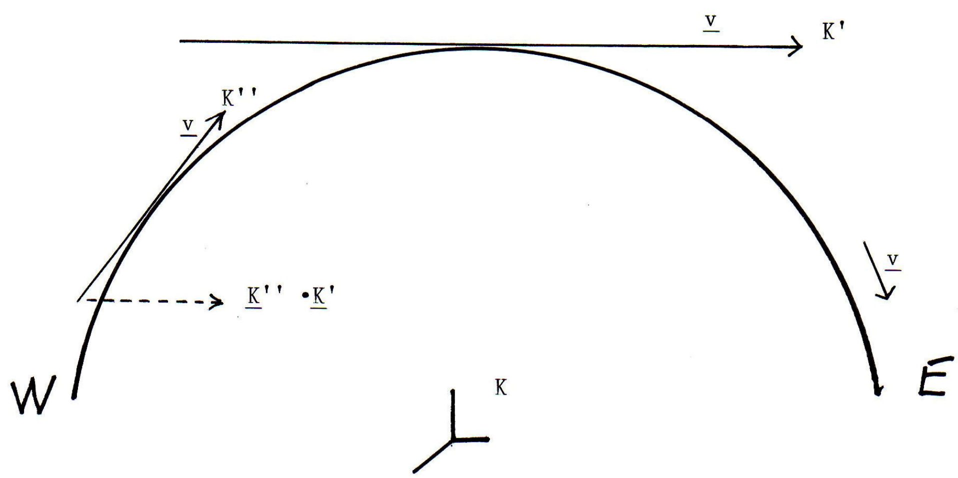

Figure. Above it is possible to study how the x' component of the velocity of a coordinate frame, following the revolution of the Earth, K'', gets lower at both sides of the highest point.

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