MASS, ENERGY, MOMENTUM

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Abstract

The author offers the interpretation that the mass is a relativistic invariant. Next it is evidenced that the formula for the momentum four-vector cannot be applied, given the current interpretations of Physics, neither to particles nor to photons, and that the photons should have mass. Then it is proposed a postulate about the electromagnetic constitution of matter, which gets rid of these problems. Finally, the validity and applicability of the equivalence of mass and energy is evaluated and is confirmed the requirement that the constitution of mass must be electromagnetic.

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1 Introduction

The author makes here an attempt to reveal and correct some of the misinterpretations about the concepts of mass, momentum and energy that are the cause of several problems in Physics.

These concepts are somewhat related with Relativity Theory. After around one hundred years since the publication of

the first paper of Einstein about this theory and several generations of physicists, which have accepted that theory, it is very difficult not to be taken without skepticism in this endeavor.

No change whatsoever is attempted to the concepts not explicitly mentioned in this paper, such as the constancy of the speed of light, which is a fact well supported by several experiments.

The author offers, in section 2, the interpretation that the mass is a relativistic invariant. Next it is evidenced, in sections 3 and 4, that the formula for the momentum four-vector cannot be applied, given the current interpretations of Physics, neither to particles nor to photons, and that the photons should have mass. In section 4 is proposed a postulate about the electromagnetic constitution of matter, which gets rid of these problems. The validity and applicability of the equivalence of mass and energy is evaluated in section 5 where it is also confirmed the requirement that the constitution of mass must be electromagnetic.

2 The Mass is a Relativistic Invariant

In this section the author presents some arguments that suggest that the mass of a body should be considered as a relativistic invariant. In other words, mass should not depend on velocity and, consequently, is the same for any inertial observer, whatever the frame of reference from where it is seen. In such a sense it is comparable with the elementary electric charge.

The recognition that the mass is a relativistic invariant is stated explicitly only in very few good textbooks. Some of the few but important references that support this point of view are Taylor and Wheeler [1], Okun [2] and the same Einstein in a letter to Lincoln Barnett (19 june 1948 - see Okun [2]). Also, the author David Waite [3] expresses the following: "Though **much** more complicated in the long run, the math is consistent and leads to consistent predictions concerning observation and so one might argue that the physics is therefore correct. But, in keeping with Occam's razor this definition and method must be done away. The m in this method is then inappropriately qualified and called the *rest mass*. It is wrong to do this for the following reason. Calling m the *rest mass* infers to the listener that m is not the mass according to other frames for which it is not at rest. We have already noted that m is an invariant as it is the same value as calculated according to any frame. It is not just the value for the rest frame. The relativistic mass method also leads to many erroneous conclusions... In short the terms *relativistic mass* and *rest mass* need to be done away and the real mass m which is actually observed is an invariant. It does not change with speed".

But this concept is not commonplace and, on the contrary, the great majority of Physics' books and papers that introduce relativity theory assume, incorrectly, the existence of a "relativistic mass", which is a function of the perceived velocity of the particle. In particular they assume that, in a comoving frame, the particle has an associated "rest mass", which is a constant.

Several important examples of current Physics' books that maintain this error are: Rindler [4], D'Inverno [5], Mould [6], Born [7]; also, authors from important universities and

journals in the world: Harrison [8], Khrapko [9], Gabrielse [10], Sandin [11], Q. Ter Spill [12]. They define and use the so-called "relativistic mass" as the "rest mass", m_0, multiplied by the Lorentz dilation coefficient, γ:

$$m = \gamma m_0 \tag{1}$$

the Lorentz dilation coefficient is defined as:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2}$$

where "v" is the velocity of the particle as viewed by the observer.

It must be noted that the same Einstein, in the paper that originated the theory of relativity [15], developed two formulas that express the mass as a function of the speed. He called "longitudinal mass" to: $m = \gamma^3 m_0$, and "transverse mass" to: $m = \gamma^2 m_0$. He concluded: "We remark that these results as to the mass are also valid for ponderable material points" (he means valid not only to his definition of electron). Both of these expressions are incorrect by now. However, is very revealing the intention of some people in hiding, not revealing and even "correcting" this and others Einstein's errors.

The Lorentz dilation coefficient is also defined (in fact it can be derived, but that is not of interest for the problem at hand) as the quotient between a differential of coordinate time divided by a differential of proper time; that is:

$$\gamma = dt/d\tau \tag{3}$$

where τ is called the proper time of the particle and represents the time measured by an observer located in the frame of reference fixed to the particle.

The "coordinate velocity", which is not a tensor, is given by the coordinate differentials divided by the coordinate time:

$$u^a = dx^a/dt = \{c, vx, vy, vz\}. \tag{4}$$

On the other hand, the velocity four-vector, or proper velocity, which is a rank-one tensor, is expressed as:

$$U^a = dx^a/d\tau \tag{5}$$

If, in this expression, we multiply and divide by the differential of coordinate time, and then replace the equivalences given by equations (3) and (4), we obtain the relation between proper and coordinate velocities as shown in the following equation:

$$U^a = \gamma u^a = \gamma\{c, vx, vy, vz\}. \tag{6}$$

Tensor calculus also tells us that the product of a contravariant four-vector by its corresponding covariant produces a scalar, which is an invariant. The product of the velocity four-vector, given by equation (6), multiplied by its covariant, which has negative signs in the spatial components, produces the square of the speed of light:

$$U^a U_a = c^2. \tag{7}$$

Now let us define the momentum four-vector, which is obtained multiplying the mass, m, by the velocity four-vector:

$$\begin{aligned} P^a = mU^a = \gamma m u^a = \gamma m\{c, vx, vy, vz\} \\ = \gamma\{m\ c, m\ vx, m\ vy, m\ vz\} \end{aligned} \tag{8}$$

The last expression is customarily rewritten with the first term expressed as the energy divided by light speed; the remaining terms constitute the coordinate momentum of the particle, $\mathbf{p}$:

$$P^a = \gamma\{E/c, \mathbf{p}\}. \tag{9}$$

This result applies to any frame of reference and not only to the rest frame. We have been assuming a Minkowski space-time, and will continue doing so.

If we examine the four-momentum expressions (8), it is difficult to recognize where the γ factor came from; in particular, does it belong to the mass or to the velocity?. Only after checking the preceding equations one can conclude that the γ factor came from the four-velocity, given in equation (6). Consequently, it is incorrect to associate the γ factor with the mass, as if it were a function of velocity.

From the point of view of mathematics, it is a well-known fact in tensor calculus that an invariant multiplied by a tensor gives rise to another tensor. In our case, mass should be considered as the invariant that multiplies the velocity four-vector. This, clearly, produces the momentum four-vector $P^a = m\ U^a$. On the other hand, if we had constructed this very same expression, but multiplying the "relativistic mass" by the coordinate velocity, u^a, it would not be clear if the result is a tensor. This is a mathematical reason to conclude that mass should be treated as a relativistic invariant. Let us investigate the same problem, but now from the point of view of Physics. Let us assume that the mass is "relativistic" or, more precisely, that equation (1) is correct. From one of the principles of relativity we know that a physical law, such as this one, should apply without modification for an observer in any inertial frame of reference. It is clear that this rule was conceived by assuming that the observer is fixed and that the particle displaces with a velocity "v", with the consequence that the particle appears as if it had increased its momentum and its energy. This is the only range of validity of equation (1), with the mathematical caveat of the previous paragraph,

i.e. that it does not allow a tensorial form.
As a matter of fact, let us assume that the particle is static and that it is the observer who begins to move with velocity "v", relative to the particle. From the point of view of the moving observer it is true that the particle, apparently, has additional energy and momentum. The question is: how could the particle have really acquired some additional mass if we never affected or touched the particle in any way?. Moreover, if we assume the existence of a second observer, moving with respect to the particle at a different velocity, say "$v2$", how can that particle acquire a different mass for the second observer?.
The obvious answer, of course, is that mass is an invariant and that both energy and momentum are variables, which depend on the relative velocity of the observer via the γ factor.

3 The Photons have Mass

In what follows let us try to compute the four-momentum of a photon by using equation (8).

First, we have to assess whether equations (6) and (8) are the correct structures for the velocity and momentum four-vectors of a photon.

Since light moves at light speed, the velocity v should be changed by c for the photons. But, in such case, the Lorentz factor γ becomes infinite and, after replacing this value in equations (6) and (8), all the components of the velocity and four-momentum grow to infinite also, which implies that both energy and momentum of the photon become infinite. Consequently, with this combination of values, no reasonable results

can be obtained. To avoid the problem of infinite energy and momentum the physicists find necessary to assume that the mass is zero, with which they can obtain indeterminacy.

It is necessary to find a valid alternative to represent the photon's four-momentum. One possibility is that the γ factor has always the value of one (1) for the photons, that the temporal component (first component in our terminology) of the momentum four-vector is zero and that the (spatial) speed v of light is equal to c. If we, for simplicity, assume displacement only in the x direction, then the four-momentum becomes:

$$P^a = \{0, m\ c, 0, 0\}. \tag{10}$$

This four-vector is automatically Lorentz invariant since both the mass and the speed of light are invariant. It is reasonable because it associates with the photon the momentum "$m\ c$". However, it still needs to be confirmed with further studies and applications which integrate with the corresponding formulas for the mass particles.

It should be clear that one needs a mass m different from zero, should the photons have some energy and associated momentum, which are experimentally confirmed. If the mass were zero then the energy and the momentum of the photon would also be zero. Therefore, the correct combination of variables seems to be that photons have mass, don't have velocity in the temporal coordinate but do have velocity in the spatial coordinates. Otherwise, the equations physicists write are useless. In particular, in this section we have proved that equation (8) for the momentum four-vector cannot be applied to photons, when the classical values are replaced for velocity and mass; and, in the next section, it will be shown that such equation cannot be applied to particles either.
Current Physics presents in this case a unified front; it consid-

ers, without exceptions, that the photons do not have mass and that light moves at light speed both in the temporal and space coordinates. There are no references supporting or proposing some alternative point of view about the structure of the velocity or momentum four-vectors of a photon. But, with respect to the mass of light, Einstein wrote in his book "The evolution of Physics" [13]: "a beam of light carries energy and energy has mass"; consequently, by an immediate logical inference, we conclude that *light carries mass.*

4 The Particles move at Light Speed

In [14] Einstein wrote: "The inertial mass of a body is not a constant, but varies according to the change in the energy of the body". This phrase reflects his formula, $E = mc^2$, but with a caveat: as was explained in section 2, the mass is an invariant; accordingly, it is an imprecision to say that the mass of a body "varies according to the change in the energy of the body". It is true that if to the mass of the body is added the mass of some photons then the sum is greater than its parts, but this does not mean that the mass of the original body has changed. In 1905 Einstein didn't know about tensors so the "mathematical reason" proposed earlier, in section 2, was out of his reach, but I consider that the physical argument (two observers with two different velocities looking at the same mass) is not very difficult to develop and follow, and should not have been missed by him and his thousands or millions of followers during one hundred years. Moreover, for a massive particle, energy does not go to zero when the spatial velocity approaches to zero. Physicists are accustomed to the result that particles possess energy even when they are "at rest".
In fact, when the spatial velocity v of the particle goes to

zero in equations (8) or (9) then the four-momentum of the particle becomes:

$$P^a = \{m\ c, 0, 0, 0\}. \tag{11}$$

If we were to ask now what is the momentum of the particle at rest, the answer of current Physics is, almost certainly, that the momentum of the particle is zero. Therefore, current Physics "forgets" that the momentum tensor has a *momentum* component, which is different from zero, for the particle at rest. Such component is interpreted as energy divided by light speed, with the secret intention of avoiding the evident need of the velocity of light in the expression for momentum as well as in the expression for the energy of the particle. Actually, both expressions require a velocity equal to that of the light. As physicists currently cannot imagine the existence of a velocity associated with a particle at rest, they conclude, illogically, that the magnitude of the momentum four-vector is zero.

Some years ago I proposed the postulate that the elementary particles are constituted by photons. The current argument only confirms, and should be enough to accept, such postulate, which was advanced by the present author, for example, in a former paper entitled "The electromagnetic constitution of matter". I sent that paper to the magazine *Galilean Electrodynamics* but it was never published, among other reasons, because one of the peer reviewers found "the absence of a fruitful discussion about a possible origin of the curvature of the trajectory of the photons". Also, the Editor of that magazine, Dr. Cynthia Whitney, found that "One problem is to justify a ring structure for electromagnetic radiation, since we usually think of electromagnetic radiation traveling in straight lines." [16]

Granted that current Physics has not devised yet a theory

about the electromagnetic explanation for the proposed curvature of the photons, but the alternative is to stay with unsatisfactory answers, or no answers at all, to the questions posed above. Not only the electromagnetic waves are curved slightly by gravity but it is found that they can be forced to travel through a medium such as fiber optic in very closed loops. Moreover, its proper constitution has the potential to explain the mechanisms that maintain the circular configuration. Another reason, that currently nobody justifies, is given by the spiral figures which appear in the bubble chambers when particles "unwind" in disintegration processes.

Anyway, the speculation here is that the (admittedly imperfectly known) electromagnetically constituted particles could explain better several Physics' formulae and phenomena, such as the velocity of light within a particle, needed in the equation of momentum; the spin of the particles, and the origin of the particle-wave duality, first discovered by the insight of Louis de Broglie.

5 Equivalence of Mass and Energy

The classical Einstein's formula, $E = mc^2$, establishes the equivalence of mass and energy. I am going to demonstrate in this section that this is another example where the physicists do not apply the equations of Physics when those equations contradict the current dogma.

As is well known, the mentioned formulas for four-momentum and energy are not applied to photons because they are believed to have zero mass. If such equations were applied, with the value zero for the mass, then the energy and momentum of the photon would be zero, which amounts to contradicting a whole set of obvious and evident notions, such as the radia-

tion energy received from the sun. Physicist have some ways to overcome whatever annoying formula appears by the well known expedient of "forgetting" the conflicting equations or using another equations which could provide the same result, instead of trying to change the false premise and construct a coherent theory. In this case, the alternative formula computes the energy of the photon as the product of Planck's constant by the frequency of the photon. This conceals the mass within the Planck's constant. In order to see the mass explicitly let us remember that the Planck's constant is equal to the product of the mass of the electron (or proton), the Compton's wave length of the electron (or proton) and the speed of light.
With this method the problem is displaced but not avoided. This preserves several problems, which compromise the logic, integrity and validity of Physics. First, with this practice, the physicists are explicitly denying the possibility of an authentic and general equivalence and convertibility between mass and energy. To be precise, for a certain amount of energy, that of a photon, there would be no corresponding mass, because photons are assumed to have no mass.

Second, if energy does not necessarily have an equivalent mass, then at least one of the directions of the equality $E = mc^2$ is lost or needs a new artifact, such as to presuppose a special kind of energy that has no corresponding mass. Nevertheless, the other direction of the equality is also compromised. In fact, if it were possible to transform the mass into energy, this energy would not have the original mass from where we departed, because to the energy of the photons would correspond zero mass and not the original one of the particle.
Consequently this equation would have no real validity.

As was suggested in a previous section, my standpoint is

that photons do have mass. This permits a coherent explanation for the existence of momentum and energy in photons and justifies, for example, the Compton effect. This postulate, also, restores the validity of the formula for the equivalence of mass and energy and justifies the experimental results of Carl David Anderson, who proved, in 1932, that when a gamma-ray, with an amount of energy of about 1.02 mega-electron volts (MeV), is absorbed in some point of space, then two particles emerge out of this point: a positron (discovered by him) and an electron, each of about 0.511 MeV. Conversely, when an electron meets a positron, they may disappear producing an equivalent radiation of the same magnitude as the original. Some physicists misinterpreted the Anderson's experiments (and also other experiments such as at the SLAC – Stanford Linear Accelerator Center) claiming that radiation "creates" electrons and positrons out of empty space or from the "virtual particles that inhabit the vacuum" [17]. This explanation coincides with the magicians' explanation that the cards and rabbits appear and disappear from empty air and not from the sleeve or the hat. In Physics the obvious explanation would be that energy converts into mass, and vice versa.

The correct interpretation of this classical Einstein's formula allows us to recognize not only that mass and energy can be *converted* one into the other, but, what is more important, that mass and energy are only two manifestations of the same substance.

6 Conclusions

In this work several concepts such as mass and momentum, for particles and photons, have been identified that have

seemingly been managed incorrectly. The subtle differences that have been mentioned produce a radical change in Physics.

If the postulate proposed by the author about the electromagnetic constitution of matter is correct, then the formulas of Physics should be applied without modifications both to particles and to photons. The complete application of this proposal should lead to the unification of the formulations for Mechanics and Electromagnetism.

References

[1] E. F. Taylor and J. A. Wheeler. *Spacetime physics, introduction to special relativity.* 2nd edition. Freeman and Company Eds. New York. 1992.

[2] Lev B. Okun. *The concept of mass.* Physics Today. june 1989.

[3] David Waite. *The Special Relativity - Chapter 3.* In URL: http://home.aol.com/zcphysicsms/sr.htm

[4] Wolfgang Rindler. *Relativity: Special, General and Cosmological.* Oxford Univ. Press. 2001.

[5] Ray D'Inverno. *Introducing Einstein's Relativity.* Oxford Univ. Press. 1992.

[6] Richard A. Mould. *Basic Relativity.* Springer Verlag. 1994.

[7] Max Born. *Einstein's Theory of Relativity.* Dover Publ. 1962.

[8] David M. Harrison. *The Special Theory of Relativity.* Department of Physics, University of Toronto. April 1999. Web page: *SpecRel.html.* In URL: http://www.upscale.utoronto.ca/GeneralInterest/Harrison/SpecRe

[9] R. I. Khrapko. *What is mass?.* Physics - Uspekhi. Vol.43, No.12. 2000.

[10] Gerald Gabrielse. Harvard University. *Relativistic mass increase at slow speeds.* American Journal of Physics. Vol.63, No.6, June 1995.

[11] T. R Sandin. *In defense of relativistic mass.* American Journal of Physics. Vol.59. 1991.

[12] Q. Ter Spill. *Mass and Energy.* 's Gravesande Institute for Physics Education. Netherland. In URL: http://www.teleles.nl/pdf/total_artikel.pdf

[13] Albert Einstein and Leopold Infeld. *The Evolution of Physics.* Simon and Schuster. 1966.

[14] Albert Einstein. *Relativity - The Special and the General Theory. A Popular Exposition*, 1916. In web page: http://ourworld.compuserve.com/homepages/eric_baird/

[15] Albert Einstein. *On the electrodynamics of moving bodies.* June 30, 1905. In web page: http://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf

[16] Cynthia Whitney. Editor Galilean Electrodynamics. *Personal communication.* Unpublished, 2000.

[17] David Ehrenstein. *Conjuring matter from light.* Science Magazine, 277(5330):1202+, August 1997.