Mathematically Defined Speed of Light

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Abstract: The methodologies used to determine the numerical value of the speed-of-light are limited to the precision of the instruments available and to the defined limits of the units of measurement, which are the meter and the second. A mathematical method can be used to define the numeric value for the speed-of-light using a physical science and a mathematical constant which will be independent of the meter and the second, but readily related to those units. The mathematical method will be defined relative to a physical science and a mathematical constant utilizing a trigonometric function that will exploit electromagnetic relationships. The result will be a numeric value for the speed-of-light that has nearly unlimited precision.

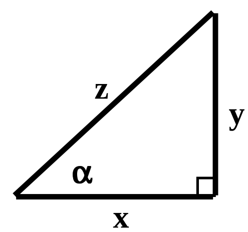

Figure 1.

Trigonometry is derived from the geometric and angular relationships of a right triangle, Figure 1. Many of the characteristics of electromagnetic (EM) waves are described using trigonometric functions, typically the sine and the cosine, but the cosecant function has not been used for this purpose. The dimensional and trigonometric relationships of a right triangle can be exploited in a unique manner by giving the lines a dimensional duality that is commonly used in describing EM waves, their wavelength (**λ**) and frequency (**f**). **When the triangle lines are expressed as wavelengths and as frequencies, they will retain relationships that are both trigonometric and electromagnetic.** When the triangle is in the cosecant form, where the value of **y** equals one, the vertical leg (**y**), the hypotenuse (**z**) and the angle (**α**) are the only elements needed to define the trigonometric-electromagnetic relationships.

EM waves have their own duality because a wavelength (**λ**) can be represented by an actual length or an angular length, 2π radians, and frequency (**f**) can be defined as a numeric frequency or as an angular frequency (**ω**). Pure EM relationships are defined by the wavelength-frequency formula.

$\lambda = c/f$ and $f = c/\lambda$ and $c = f*\lambda$; where (**c**) represents the speed-of-light (SOL)

The numeric value of the SOL is defined by the System International (SI) and is expressed in meters per second, but the meter and the second are not in themselves either a physical science or mathematical constant. **The SOL can be defined in relationship to a known physical science constant and a mathematical constant, and it will result in a numeric value that is independent of the meter and the second, but readily related to those units.**

The wavelength of the precession emission of neutral hydrogen is a physical constant and is typically referred to as the 21 cm line, and will be referred to as λ_H. The second will be replaced by a time unit (**τ**) with a duration that will vary as a function of the angle, effectively a time angle. Dividing 2π radians by the time unit has technically the same meaning as frequency, but it is customarily called angular frequency (ω_f) and expressed in radians/sec. In this application it will be expressed in rad/**τ**. To accommodate the large frequency value related to the λ_H wavelength the angular frequency will be given a 10^6 multiplier, ie; $\omega_f = 2\pi(10^6)$. Frequency will be expressed in cycles per time unit, **cpτ**, rather than hertz (Hz).

To create the trigonometric-electromagnetic relationships the values **y** and **z** each are multiplied by λ_H and ω_f, and the resultant values are converted by the wavelength-frequency formula. The resulting equation sets are shown as (1), (2), (3) and (4), and must be solved simultaneously. Alternatively, when the angle is known or needs to be calculated, substitute equations (1a) and (2a) for (1) and (2) respectively.

$\omega_f * z = f_1$	$\lambda_1 = c / f_1$	(1)		$\omega_f * \csc(\alpha) = f_1$	$\lambda_1 = c / f_1$	(1a)
$\lambda_H * z = \lambda_3$	$f_3 = c / \lambda_3$	(2)		$\lambda_H * \csc(\alpha) = \lambda_3$	$f_3 = c / \lambda_3$	(2a)
$\omega_f * y = f_2$	$\lambda_2 = c / f_2$	(3)				
$\lambda_H * y = \lambda_4$	$f_4 = c / \lambda_4$	(4)				

When the equation sets are calculated, their solutions will result in a numeric mirror symmetry between equations (1) and (4) and between (2) and (3), but each with a 100 factor difference. It will become apparent upon examining the calculation process that the SI SOL numeric value will be valid for just one angle because the unit of time (τ) is a function of the angle.

The simplest process is at 45 degrees where the value of the hypotenuse is the square root of 2, and the unit of time equals "one". The common unknown is the numeric value for the SOL at that time angle. In the equation below the value of λ_H is equated to one, and any length result based upon that will be denoted as **L**. Iterative algebra was used to calculate the results.

$f_1 = 8.8857\ (10^6)$ cpτ	$\lambda_1 = 100.00$ L	(5)
$\lambda_3 = 1.4142$ L	$f_3 = 628.31\ (10^6)$ cpτ	(6)
$f_2 = 6.2831\ (10^6)$ cpt	$\lambda_2 = 141.42$ L	(7)
$\lambda_4 = 1.0000$ L	$f_4 = 888.57\ (10^6)$ cpτ	(8)

Based upon the trigonometric-electromagnetic relationships expressed with the cosecant value at 45 degrees, and using a unity value for λ_H, the numeric value for the SOL will be limited in precision only by the computational capability to multiply 2π by the sqrt of 2.

The following results illustrate the equation results when the value of λ_H is in centimeters, 21.106 cm., at the time angle that coincides with the second (26.2540 deg) and at 45 degrees..

26.2540 degrees			**45 degrees**		
$f_1 = 14.204\ (10^6)$ cpτ	$\lambda_1 = 2110.6$ cm	(9)	$f_1 = 8.8857\ (10^6)$ cpτ	$\lambda_1 = 2110.6$ cm	(13)
$\lambda_3 = 47.714$ cm	$f_3 = 628.31\ (10^6)$ cpτ	(10)	$\lambda_3 = 29.848$ cm	$f_3 = 628.31\ (10^6)$ cpτ	(14)
$f_2 = 6.2831\ (10^6)$ cpτ	$\lambda_2 = 4771.4$ cm	(11)	$f_2 = 6.2831\ (10^6)$ cpτ	$\lambda_2 = 2984.8$ cm	(15)
$\lambda_4 = 21.106$ cm	$f_4 = 1420.4\ (10^6)$ cpτ	(12)	$\lambda_4 = 21.106$ cm	$f_4 = 888.57\ (10^6)$ cpτ	(16)

The precision of the above calculations cannot exceed the known precision of the value of λ_H and the SI SOL value, nor can the angle (26.2540...+ deg) associated with the SI SOL be calculated to exceed that precision. The duration of τ is equal to the duration of the second in the 26.2540 degree case.

It must be noted that within the EM spectrum, the numeric frequencies represented by f_1, $14.204(10^6)$ cpτ, and f_4, $1420.4(10^6)$ cpτ, are at the same spectrum position when the numeric values are $8.8857(10^6)$ cpτ and $888.57(10^6)$ cpτ respectively. This applies to any specific numeric frequency value that has been angularly translated by the relationships, their spectrum position is the same even though the numeric value is different.

Using the time unit τ duration defined by the 45 degree angle as the reference, the time duration of the second can be expressed as the ratio of the hypotenuse values between 26.2540 and 45 degrees, or about 1.5984... longer than the reference.

By establishing the length first, the unit of time and the speed of light are mathematically defined relative to that constant, essentially establishing a set of scientific base units.

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