JUST LIKE THE GRAVITATIONAL FIELD, THE ELECTRIC FIELD TOO, SLOWS DOWN A CLOCK, INTERACTING WITH IT: A WHOLE NEW APPROACH TO THE BOUND MUON DECAY RETARDATION

Tolga Yarman, Okan University, Kadikoy, Istanbul, Turkey &
Savronik, Organize Sanayii Bölgesi, Eskisehir, Turkey
tyarman@gmail.com

Alexander L. Kholmetskii*
Department of Physics, Belarus State University,
4, Nezavisimosti Avenue, 220080 Minsk, Belarus
kholm@bsu.by

ABSTRACT

We show that, just like the gravitational field, the electric field too slows down the internal mechanism of a clock, which interacts with the field. This approach explains substantially, the retardation of the decay of the muon, bound to a nucleus.

Keywords: Muon, Mass Deficiency, Special Theory of Relativity, Electric Charge, Metric Change

1. INTRODUCTION

This work is issued from a much broader angle than the one we will specifically consider herein [1]. Thus, it was the author's idea that, owing to the law of conservation of energy, the overall internal energy of a bound particle should be weakened as much as the binding energy coming into play, no matter what field the particle is bound to, and no matter whether it is bound to an electric field in the atomic world, or a gravitational field in the celestial world. All internal mechanisms, the particle of concern may embody, shall be affected accordingly, provided that the particle's inner articulations in relation to each other, are not degenerated via the binding process.

This is the essence of our approach, and we will show that, it can be successfully applied to predict the bound muon's decay rate.

Let us stress that, the binding coming into play may be any binding, a nuclear binding, or an electric binding, or a gravitational binding, or else.

We know that a gravitational binding does not alter the particle's inner articulations in relation to each other. That is, all parts of the bound particle, is affected in the same way, so that the particle holds up its original identity (unless perhaps the field is exceptionally strong).

Likewise, a charge particle will hold up its original identity when electrically bound, provided that the electric field, affects equally all parts of the charged particle. This, at the first strike, requires that, all parts of the charged particle are charged, and uniformly charged.

In a nuclear binding however, the particle, such as the neutron, may not preserve its original identity, just like in a vegetable soup, ingredients cannot all the way, preserve their initial specific characteristics.

* Corresponding author

For the sake of completeness, we better recall a bit, our previous work (Sections 2, 3, 4, and 5), and only afterwards tackle with the bound muon's decay rate retardation (Sections 6 and 7), yielding a striking conclusion, i.e. the metric change nearby the nucleus vis-à-vis charged particles, such as electrons or muons bound electrically, just like the metric change nearby a celestial body, vis-à-vis any particle bound gravitationally (Section 7).

2. BRIEF SKETCH OF THE PREVIOUS WORK: INSIGHT TO THE BOUND MUON DECAY RATE RETARDATION

Here, for simplicity, though without any loss of generality, we assume that the particle in question, is insignificant as compared to the host object binding it, so that we only have to worry about the changes this particle would undergo [2,3]; in other words, the host object binding the particle in consideration, will remain practically untouched through the binding process.

The binding in question, can be any type of binding provided that it does not degenerate the bound particle, though in this section we will mainly refer to gravitational binding. As we will see, the way we will develop our approach, does not at all restrict us to gravitational interaction; quite on the contrary it allows us to consider any field the particle in hand, interacts with (provided that the field dos not deteriorate the particle's identity).

Let us explain a bit further, why we like to assume that the bound particle is insignificant as compared, to the binding host object.

Suppose an observer on Earth sets free from his elevated right hand, a stone to a free fall, and little after, he catches it with his lowered left hand. The overall energy of the closed system made of the stone in question and Earth (and just the two, i.e. disregarding, the air in between), must stay constant all along the free fall of the stone. The law of linear momentum conservation law, on the other hand, requires that, because Earth is considerably more massive than the stone; in regards to a distant star, it will remain in place. Therefore throughout, it is only the stone, which gains kinetic energy. Once the observer interferes and catches the stone, with his lowered left hand, he retrieves from the closed system (made of the falling stone and Earth), an amount of energy equal to the kinetic energy, the stone would have acquired on the way, and this energy, evidently, is retrieved from the stone alone, once this is stopped [1].

Conversely as the observer highers the stone, he will come to pile up an extra amount of energy equal to the energy he has to furnish to it, to elevate it to the given altitude.

Let us simplify things. Suppose one highers, just one atom of hydrogen. Then, what would it mean that, via highering the hydrogen atom, one piles up in it, an extra amount of energy equal to the energy he would have furnished to it?

Owing to the Relativistic Equivalence of Mass & Energy, the Rest Mass of the Hydrogen Atom Will Get Altered. Wherever This Mass Intervenes, We Will Observe a Related Change.

The answer to the question we just introduced, primarily, is the following.

Owing to the relativistic equivalence of mass & energy, the rest mass of the hydrogen atom will get increased as much.†

Thus, wherever this mass intervenes, we will observe a related change.

We can further analyze the situation in the following way.

Any entity must display an internal dynamics, based on a given "internal mechanism". What do we mean by "internal mechanism"? This is an "intrinsic periodic phenomenon". Already de Broglie has considered such a phenomenon in regards to a given particle at rest, were this, totally transformed into electromagnetic radiation [4].

We can be more specific than that:

A diatomic molecule for instance, vibrates. The motion in question delineates a particular "internal dynamics".

A diatomic molecule can as well rotate. The related motion delineates another internal dynamics.

One can associate a total energy with every specific internal dynamics, coming into play, provided that the "internal motions" in question can be envisaged to be independent from each other.

Thus, we can conceive any entity to embody an "internal dynamics, driven by a given internal mechanism. A given entity may embody many internal mechanisms, working simultaneously. To make things simple, let us assume that, there is only one internal mechanism of concern.

Clock Labor, Clock Mass, Clock Space, Clock Unit Period of Time

Any such mechanism will consist in a "clock labor", taking place in a given "clock space", and achieved by a "clock mass", displaying a "unit period of time". The resulting "internal dynamics" is founded on a "total energy", framing the "clock's mass motion".

Based on the Bohr Atom Model, the internal mechanism turns out to be the rotational motion of the electron around the proton; the clock mass is the reduced mass of the electron and the proton; he unit period of time is the period of time, a mass equal to the reduced mass in consideration, takes to rotate around the center of mass of the electron and the proton; since the reduced mass is practically the mass of the electron, then the unit period of time virtually becomes, the period of time the electron takes to rotate around the proton.

In more modern terms, the internal dynamics we refer to, can be characterized with the (no more than probabilistically predictable, yet still) measurable momentum of the electron, to be considered along with the usual quantum mechanical total energy.

† Note that within the frame of the General Theory of Relativity, a mass imbedded in a gravitational field dilates, and a mass carried away from a gravitational field, contracts. But then, the relativistic equivalence between mass & energy is broken. The present approach does not give rise to such annoyances.

Based on a non-relativistic approach, the total energy $E_{\infty n}$ of the hydrogen atom, at the n^{th} principal level, in empty space, is (in CGS unit system), as usual, given by

$$E_{\infty n} = -\frac{2\pi^2 e^4 \mu_{0\infty}}{n^2 h^2} \;; \tag{1}$$

(total energy of the hydrogen atom)

here e is the charge intensity of the electron or that of the proton, $\mu_{0\infty}$ is the reduced mass of the electron and the proton in empty space, and h is the Planck Constant; recall that $\mu_{0\infty}$ is practically equal to the electron's rest mass in empty space.

Decrease of the Mass, and Red Shift of Gravitationally Bound Hydrogen's Light

A transition in the hydrogen atom, between an upper level n, to a lower level m, in empty space, yields an electromagnetic radiation of frequency $\nu_{\infty n \to m}$, so that

$$h\nu_{\infty n \to m} = \frac{2\pi^2 e^4 \mu_{0\infty}}{h^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) . \tag{2}$$

(electromagnetic energy released from the hydrogen atom, through a n to m *transition)*

When bound, at an elevation R, to a uniformly structured celestial body of mass $\mathcal{M}$, the reduced mass $\mu_{0\infty}$ of the hydrogen atom (just like any mass taking place within the frame of this atom), is decreased as much as the gravitational binding energy $E_B(R)$ coming into play [5,6,7,8,9] to become $\mu_0(R)$:‡

$$E_B(R) = [\mu_{0\infty} - \mu_0(R)]c^2 \;; \tag{3}$$

(gravitational binding energy, in terms of the mass variation the bound particle displays)

here c, is the velocity of light, in empty space.

‡ The proton's mass $m_{P\infty}$ is decreased as much as the binding energy; so is the electron's mass $m_{e\infty}$. Thus the reduced mass $\mu_{0\infty} = m_{P\infty} m_{e\infty} / (m_{P\infty} + m_{e\infty})$ is also decreased as much.

The gravitational binding energy $E_B(R)$, to a first approximation, can be expressed as

$$E_B(R) = G\frac{\mathcal{M}\mu_{0\infty}}{R} \; ; \tag{4}$$

(gravitational binding energy, derived from Newton's law of attraction)

G is the universal gravitational constant, $\mathcal{M}$ the mass of the hosting celestial object, and R is the elevation at which the atom is bound.

Here we have tacitly adopted the Newton's law of gravitational attraction. We will though soon elucidate the fact that, the structure of this law is imposed by the special theory of relativity.

Eq. (3) and (4) furnish $\mu_0(R)$:

$$\mu_0(R) = \mu_{0\infty}[1-\alpha(R)] \; , \tag{5}$$

(mass of the bound object)

where $\alpha(R)$ is given by

$$\alpha(R) = \frac{G\mathcal{M}}{Rc^2} \; . \tag{6}$$

Note that, to be rigorous one should consider the continuous change on the mass $\mu_0(r)$, through the binding process [1]. One should then, reconsider Eqs. (3) and (4):

$$dE_B(r) = -c^2 d\mu_0(r) \; , \tag{7}$$

$$dE_B(r) = -G\frac{M\mu_0(r)}{r}dr \; ; \tag{8}$$

$d\mu_0(r)$, here, is the infinitely small increase the rest mass $\mu_0(r)$ undergoes, if it is moved quasistatically through dr.

Thus at the elevation R, one arrives at

$$\mu_0(R) = \mu_{0\infty}e^{-\alpha(R)} \; . \tag{9}$$

(rigorous expression for the gravitationally bound mass)

In our approach, both the electron charge and the Planck Constant, are universal constants, and they remain untouched in either a gravitational field or an electric field, or seemingly, any other field. Note that they are as well Lorentz invariant quantities. Recall on the other hand that, the universal gravitational constant G is not Lorentz invariant. (So it is not as "universal", as one may think it is.)

Thus, in a gravitational field, a change in $E_{\infty n}$ of Eq.(1), and accordingly a change in $\nu_{\infty(n\rightarrow m)}$ of Eq.(2), must be based on a corresponding change, the reduced mass of the hydrogen atom undergoes:

$$h\left|\Delta\nu_{\infty(n\rightarrow m)}\right| = \frac{2\pi^2 e^4 \left|\Delta\mu_0\right|}{h^2}\left(\frac{1}{m^2}-\frac{1}{n^2}\right) = h\nu_{\infty(n\rightarrow m)}\frac{\left|\Delta\mu_0\right|}{\mu_{0\infty}} \quad ; \tag{10}$$

(change in the energy of the hydrogen atom's light, due to the change of mass)

here the change $\Delta\mu_0$ in the reduced mass $\mu_{0\infty}$ of the hydrogen atom, in empty space, were this embedded in the gravitational field in consideration, is given by [cf. Eq. (9)]

$$\Delta\mu_0 = \mu_0(R) - \mu_{0\infty} = -\mu_{0\infty}(1-e^{-\alpha}) \ . \tag{11}$$

(change in the hydrogen atom's reduced mass due to gravitational binding)

The negative sign that appears over here, points to the fact that, the hydrogen mass in the gravitational field, decreases.

Eq.(3), along with Eq.(10), leads to

$$\left|\Delta\nu_{\infty(n\rightarrow m)}\right| = \nu_{\infty(n\rightarrow m)}\frac{E_B(R)}{\mu_{0\infty}c^2} \quad . \tag{12}$$

(red shift in the energy for the hydrogen atom's light, due to gravitational binding)

This is nothing else, but a red shift in the energy of the hydrogen light, due to gravitational binding.

Note that, along with Eq.(11), the binding energy $E_B(R)$ of the particle bound at the elevation R, becomes

$$E_B(R) = \left[\mu_{0\infty} - \mu_0(R)\right]c^2 = \mu_{0\infty}c^2(1-e^{-\alpha}) , \tag{13}$$

(rigorous expression of the binding energy)

which, for a small α, and via Eq.(6), yields

$$E_B(R) \cong G\frac{\mathcal{M}\mu_0}{R} \ , \qquad (14)$$

which well turns out to be Eq.(4).

The frequency $\nu_{n\to m}(R)$, the hydrogen atom would produce at the altitude R, through a n to m transition, becomes

$$\nu_{n\to m}(R) = \nu_{\infty(n\to m)} - \nu_{\infty(n\to m)}\frac{E_B(R)}{\mu_{o\infty}c^2} = \nu_{\infty(n\to m)}\left[1 - \frac{E_B(R)}{\mu_{o\infty}c^2}\right] \ . \qquad (15)$$

(red shifted frequency of the bound hydrogen atom's light)

It is weakened as much as $1 - E_B(R)/(\mu_{o\infty}c^2)$.

Decrease of the Mass and Stretching of the Unit Period of Time, as well as the Size of the Clock Space of a Gravitationally Bound Clock

As stated, de Broglie in his doctorate thesis, considered the electromagnetic energy amounting to the entire mass of the particle, i.e. the overall relativistic energy of it (at rest), and this, even long before the annihilation of the electron with a positron was discovered.

Thus, he would write

$$h\nu_{0\infty} = m_{0\infty}c^2 \ , \qquad (16)$$

(the frequency de Broglie has associated with the presumed intrinsic periodic phenomenon an object of a given mass would delineate, or the same the frequency to be associated with the overall relativistic energy of the object in hand)

for a particle of mass $m_{0\infty}$ in empty space; $\nu_{0\infty}$ is the frequency of the electromagnetic radiation, were the mass $m_{0\infty}$ somehow annihilated.

If the particle is embedded in a gravitational field created by the host celestial body of mass $\mathcal{M}$, at the altitude R, then according to our approach, the electromagnetic energy $h\nu_{0\infty}$ will become $h\nu_0(R)$, which can be, via Eq.(9) written as

$$h\nu_0(R) = m_{0\infty}c^2e^{-\alpha(R)} = h\nu_{0\infty}e^{-\alpha(R)} \ . \qquad (17)$$

(the overall relativistic energy of the given particle when this is embedded, at rest, in the gravitational field in consideration)

This equation, via Eq.(13), but written for the mass $m_{0\infty}$, can be written as

$$h\nu_0(R) = h\nu_{0\infty}\left[1 - \frac{E_B(R)}{\mu_{o\infty}c^2}\right] \ , \qquad (18)$$

*(the overall relativistic energy of the entity in hand,
weakening as much as the binding energy coming into play)*

where $E_B(R)$, now becomes the binding energy of the particle of concern, at R.

Eq.(18) tells us that, when bound, the "overall relativistic energy" of the entity in hand, is weakened as much $1 - E_B(R)/(\mu_{o\infty}c^2)$.

For an electromagnetic radiation, one by definition, has

$$c = \frac{\text{wavelength}}{\text{period of time}} = \text{wavelength x frequency}. \qquad (19)$$

In other words, the frequency and the corresponding period of time associated with the given electromagnetic radiation are inversely proportional to each other.

Thus, let $T_{0\infty}$ be the period of time associated with the frequency $\nu_{0\infty}$. When the particle is embedded in the gravitational field in consideration, its internal energy weakens, $T_{0\infty}$ stretches just as much, to become $T_0(R)$, i.e.

$$T_0(R) = \frac{T_{0\infty}}{1 - \frac{E_B(R)}{m_{o\infty}c^2}} \ . \qquad (20)$$

*(the period of time associated with the overall relativistic energy of the
entity in hand, stretching as much as the binding energy coming into play)*

The same should be expected to occur in relation to any package of energy driving a given internal dynamics, within the particle in hand.

Let us for instance consider the ground rotational period of time $T_{e\infty}$ of the electron of mass $m_{e\infty}$ around the proton in empty space, within the frame of Bohr Atom Model.

$T_{e\infty}$ turns out to be[§]

[§] The Bohr ground rotational velocity is given by

$$v_n = \frac{2\pi e^2}{h} = \frac{2\pi r_{e\infty}}{T_{e\infty}} \ ,$$

where $r_{e\infty}$ is the Bohr ground orbit radius in empty space; it is expressed as

$$r_{e\infty} = \frac{h^2}{4\pi^2 e^2 m_{e\infty}} \ .$$

This then, well leads to Eq.(21).

$$T_{e\infty} = \frac{h^3}{4\pi^2 (e^2)^2 m_{e\infty}} \quad . \tag{21}$$

(electron's rotational period of time around the proton, in empty space)

Here for simplicity we have assumed that the proton is infinitely more massive than the electron.

When the hydrogen atom is embedded in the gravitational field in consideration, $m_{e\infty}$ will get decreased in accordance with Eq.(9); thus $T_{e\infty}$ stretches as much, to become $T_e(R)$, at the altitude R, so that

$$T_e(R) = \frac{h^3}{4\pi^2 (e^2)^2 m_{e\infty} e^{-\alpha(R)}} = T_{e\infty} e^{\alpha(R)} = \frac{T_{e\infty}}{1 - \frac{E_B(R)}{m_{e\infty} c^2}} \quad . \tag{22}$$

(electron's rotational period of time around the proton, stretching as much the binding energy coming into play)

This means that the related internal dynamics loosens as much as the internal energy loss, amounting to the binding energy (c.q.f.d.).

Any information coming from the atom, such as an electromagnetic radiation, based on the energy difference between two states, must as well weaken as much [cf. Eq.(15)]; thus, once again, the gravitational red shift.

Conversely, as we elevate the hydrogen atom, in a gravitational field, owing to the relativistic equivalence between mass & energy, we come to increase its mass. This in return strengthens just as much, the total energy of the atom, which concurrently shortens as much the period of time, one can associate with the electromagnetic radiation, one would obtain, if the entire mass of the hydrogen atom were transformed into electromagnetic energy.

Note that by the same token, the size of the clock space of the object embedded in the gravitational field, stretches. This can be right away seen from Eq.(19). The fact that the mass decrease [via Eq.(16)] yields, a decrease of the frequency corresponding to the electromagnetic energy, were the object annihilated, means that (since c remains constant), the corresponding wavelength must stretch just as much.[**]

[**] Here again note that, what we draw in regards to masses and lengths, is the opposite of what the General Theory of Relativity establishes; in this latter theory indeed, when embedded in a gravitational, masses increase, and lengths contract.

Thus, let $\lambda_{0\infty}$ be the period associated with the frequency $\nu_{0\infty}$. When the particle of mass $m_{0\infty}$ is embedded in the gravitational field in consideration, $\lambda_{0\infty}$ stretches just as much, to become $\lambda_0(R)$, i.e.

$$\lambda_0(R) = \frac{\lambda_{0\infty}}{1 - \frac{E_B(R)}{m_{o\infty}c^2}} \quad . \tag{23}$$

(clock space size stretching as much as the binding energy coming into play)

Accordingly, any space size, in which a given internal dynamics takes place, must as well stretch, as much.

We Have Elaborated on the Idea We Developed, to Predict All of the Measurable End Results of the General Theory of Relativity, Without Though Having to Assume the "Principle of Equivalence". As a Result, Our Approach Opens a Whole New Horizon.

We have elaborated on this interesting idea to predict, all of the measurable end results of the General Theory of Relativity (GTR), without though any further assumption than the energy conservation law. It is evidently striking to obtain the same results as those of the GTR, through a completely different set up than that of this latter theory, up a second order Taylor Expansion [10, 11].

We would further like to emphasize that, it really must take quite a captivation, not to have considered for many decades, that a gravitationally bound object, would indeed exhibit straight, conventional changes, at the atomic level. The fact that Dirac, the monumental father of the relativistic quantum mechanics (who was deeply interested in the GTR, and who has published a book about it, just before he died), seems not to have given any thought, to quantum mechanical changes that a gravitationally bound object may exhibit, furnishes a firm sign about the strength of the captivation in question [12].

Anyway, as we have demonstrated, given that along with our approach, one does not have to assume the "principle of equivalence" of the GTR, in order to arrive at the end results of this theory, he comes to discover a whole new horizon.

Generally speaking, the bound particle in consideration may just not be a mass (such as a stone) gravitationally bound to a celestial body; but it may also be a charged particle electrically bound to a charged body. The particle of concern, may even be a nuclear entity such as neutron, bound to a nuclear field, provided that, the binding process does not destroy the inner articulational characteristics of the original entity in hand.

At the first strike, our claim (that the internal energy of the bound particle must be decreased as much as the binding energy coming into play), may seem trivial, since it is nothing else but the energy conservation law. Yet, not only that our approach was overlooked for gravitationally bound objects, but also, even the "internal energy" of a charged particle, such as that of an electron, was not given a particular consideration, for say, the electron was always considered as a point-like particle; thereby an eventual change of its internal energy, was not considered at all.

The Internal Dynamics of an Electron Bound to a Proton, Must Weaken As Much As the Binding Energy Coming Into Play.

What we basically do here, is to consider the internal dynamics of the entity in hand. According to our approach, the internal dynamics of the given entity does not only weaken when gravitationally bound to a celestial body, but it also weakens, say in the case of a charged particle, such as an electron or a muon, electrically bound to a charged object, such as a nucleus [[13]].

Then, the internal dynamics of an electron bound to a proton, must weaken as much as the binding energy coming into play.

Well, what is the internal dynamics of an electron? We do not know. So far, no one knows. But one way or the other, the electron must have an internal dynamics. The electron cannot be reduced to just a point. It has a mass, and a charge. These cannot be reduced to an imaginary point.

We may not know what the internal dynamics of an electron consists in. Nonetheless, we can well consider a muon, instead. This particle is unstable. It sure has a certain internal dynamics. Just like any other internal dynamics, the muon's internal dynamics too, constitutes a clock. It can be sensed via the muon's decay rate.

Thus, what we claim is that, when gravitationally or electrically bound, due to the energy conservation law, the internal dynamics of a muon must weaken, as much as the binding energy coming into play (assuming that the field of concern acts equally on all parts of the muon). Such a weakening must concurrently cause the retardation of muon's decay.

According to our approach, the muon's decay rate must slow down in an electric field, just like it is expected to slow down in a gravitational field.

That is the heart of the present approach.

Alpha Decay of a Gravitationally Bound Nucleus

Note that along our approach, the gravitational slowing down of a radioactive process, can be easily checked, for instance, on the basis of alpha disintegration.

The alpha disintegration half life $T_{\alpha\infty}$ in empty space, can be as usual expressed, as [14]

$$T_{\alpha\infty} = \frac{2\ell n2\{m_{\alpha\infty} R_{\infty}^{2}\} e^{\gamma}}{h} \quad ; \tag{24}$$

(the alpha disintegration half life in empty space)

here $m_{\alpha\infty}$ is the mass of alpha particle in empty space, e^{γ} the barrier transmission coefficient, and R_{∞} the radius of the nucleus alpha disintegrating (still in empty space).

The mass of the alpha radioactive nucleus is decreased in a gravitational field, along with Eq.(9). Its size stretches as much, along with Eq.(23). One can show that, the barrier transmission coefficient is not altered.††

The alpha disintegration half life $T_\alpha(R)$ at the altitude R, in the gravitational field, then becomes [cf. Eq.(20)]

$$T_\alpha(R) = \frac{T_{\alpha\infty}}{1 - \frac{E_B(R)}{m_{o\infty}c^2}} \quad . \tag{25}$$

(the alpha disintegration half life stretching as much the binding energy coming into play)

This gives us a clue about how the muon's decay rate would retard in a gravitational field. Anyway we should expect that muon's decay rate would retard, in a gravitational field, along with the general deduction we achieved along with Eq.(20). Though, below, we will have to specifically elaborate on the muon's disintegration description.

How is the Half Life of a Bound Muon Would Get Perturbed in a Gravitational Field?

The muon's half life $T_{\mu\infty}$ in empty space, to a first approximation, can be expressed through a quite complicated set up and derivation, based on the theory of β-decay, as (in a unit system, where $\hbar$ and c are taken to be unity) [15]

$$T_{\mu\infty} = \ln 2 \frac{192\pi^3}{m_{\mu\infty}^5 g_\infty^2} \ ; \tag{26}$$

(the muon disintegration half life in empty space)

here $m_{\mu\infty}$ is the muon mass in empty space, and g_∞ is the Fermi constant (still in empty space), i.e.

$$g_\infty = 1.431 \times 10^{-49} \text{ erg x cm}^3 \ . \tag{27}$$

(the Fermi constant in empty space)

†† To a good approximation we have [6],

$$\gamma = \pi\left(\frac{2Zzc}{137V}\right) - \frac{4}{137}\left(2z\frac{M_\infty}{m_{\alpha\infty}}\frac{R_\infty}{r_{0\infty}}\right) .$$

Z is the proton number of the daughter nucleus (resulting from the decay of the parent, achieved via throwing the alpha particle) , z is 2, V is the alpha particle's velocity with respect to the daughter nucleus, M_∞ is the mass of the daughter, R_∞ is the radius of the daughter, and $r_{0\infty}$ is the classical electron radius, i.e. $e^2/(m_{e\infty}c^2)$, where $m_{e\infty}$ is the electron's mass in empty space. Since M_∞ and $m_{\alpha\infty}$ on the one hand, and R_∞ and $r_{0\infty}$ on the other hand, are altered in exactly the same manner in a gravitational field, the barrier transmission coefficient, remains untouched (c.q.f.d.). (Recall that because both lengths and periods of time stretch, the velocities remain the same.)

It is evident that, the form of $T_{\mu\infty}$, does not look like the form of Eq.(24) [where the half life is proportional to (mass) x (space size)^2]. The reason is that the Fermi constant g_∞, introduced, to replace the electron or the proton charge e, to describe the weak interaction, does not bear the same dimension as that of e. [The Fermi constant as seen from Eq.(27), bears the dimension of force x L x L^3. But force bears the dimension of (electric charge)^2 x L^{-2}. Thus, the Fermi constant has the dimension of (electric charge)^2 x L^2.]

This makes that, g_∞ is not Lorentz invariant (whereas e is). In other words g_∞, should be altered in a gravitational field (whereas, according to our approach, e or h, are not).

And how will g_∞ be affected in a gravitational field? We can deduce the answer, from the discussion we presented above. We figured out that, mass is decreased as much as the binding energy [cf. Eqs. (9), (16) and (18)]; the periods of time stretch just as much [cf. Eq.(20)]; the lengths also, stretch just as much [cf. Eq.(23)]. Let us define γ as

$$\gamma = 1 - \frac{E_B(R)}{m_{\mu\infty}c^2} \ . \tag{28}$$

(definition)

Thus g_∞, in a gravitational field (where mass is multiplied by γ), becomes g, which can be expressed as

$$g = g_\infty \gamma^{-2} \ . \tag{29}$$

(the Fermi constant of the gravitationally bound muon)

This means that, the half life given by Eq.(26), in a gravitational field, should be transformed, to yield T_μ:

$$T_\mu(R) = \mathit{ln2}\frac{192\pi^3}{m_{\mu\infty}^5\gamma^5 g_\infty^2\gamma^{-4}} = T_{\mu\infty}\gamma^{-1} = \frac{T_{\mu\infty}}{1 - \dfrac{E_B(R)}{m_{\mu\infty}c^2}} \ . \tag{30}$$

(the muon disintegration half life stretching as much the binding energy coming into play)

Thus, we expect the bound muon’s half life indeed, to stretch as much γ^{-1} [cf. Eq.(20)].

It is clear that in Eq.(30), there is nothing special with respect to a gravitational field. Wherever the muon may be bound, provided that the field in consideration acts on the muon uniformly, and does not deteriorate it, Eq.(30) will be valid, and this is the essence of our approach.

3. RECAPITULATION: EXPECTED OCCURRENCES ABOUT A PARTICLE WHOLLY BOUND TO A FIELD

Let us state briefly, what we have so far established, no matter whether we may have, time to time, lacked generality. We will anyway soon introduce a general quantum mechanical theorem, encompassing all of our foregoing derivations.

Theorem 1: The energy conservation law requires that, a particle at rest, when embedded in a field, it interacts with, must discharge an amount of energy equal to the binding energy coming into play. Likewise, as the bound particle is carried out of the field in consideration, it will pile up, an amount of energy equal to its binding energy, which is in fact, the energy one has to furnish to the particle, in order to remove it out of the field. Here, for simplicity, though without any loss of generality, we assumed that the bound particle is insignificant as compared to the host object binding it.

Theorem 2: The energy conservation law, in the broader sense, drawn by the relativistic equivalence between mass & energy, requires that the "rest mass of a particle", when embedded in a field, the particle interacts with, in its entirety, decreases as much as the binding energy coming into play.

Theorem 3: Any internal dynamics the particle may embody, along with a given mass $m_{0\infty}$, which we call "clock mass", interacting with the field of concern, in its entirety, must accordingly, slow down. Thus, the frequency associated with a given internal phenomenon is red shifted as much as $1 - E_B / (m_{o\infty} c^2)$, where E_B is the binding energy coming into play. This result is the same as the red shift predicted by the GTR, were the particle embedded in a gravitational field, though it is obtained through a totally different set up than that of this latter theory. The corresponding period of time is, accordingly, stretched as much as $1 / [1 - E_B / (m_{o\infty} c^2)]$. This result is the same as that related to the clock retardation, predicted by the GTR, were the particle, still embedded in a gravitational field. The present approach furnishes the end results of the GTR, though through a totally different set up.

Theorem 4: Concurrently to the decrease of mass, and the stretching of unit period of time, the clock space size, the clock motion takes place in, stretches as much as $1 / [1 - E_B / (m_{o\infty} c^2)]$.

The above theorems are derived based on plain insights and simple checks. We will improve our approach by providing a mathematically sound and general quantum mechanical theorem, embodying all of the foregoing theorems, at once. But before this, it is worth to review the way we conceive the notion of "field".

4. DISCUSSION ABOUT THE CONCEPT OF FIELD: THE CLASSICAL COULOMB'S LAW, OR NEWTON'S LAW REIGNS IN BETWEEN, EXCLUSIVELY, STATIC CHARGES AND STATIC MASSES, RESPECTIVELY, WHILE THEIR $1/r_0^2$ DEPENDENCY IS A REQUIREMENT IMPOSED BY THE SPECIAL THEORY OF RELATIVITY

Sure, according to our approach the concept of field, has to be revised, and we are to clarify our stand point. The concept of force is the fundamental concept, to be experimentally relied on; the concept of field, though useful, is only, an extended concept. It cannot be measured; only force can be measured. Two interacting masses exert upon each other a gravitational force, just like two interacting charges exert upon each other an electric force.

We should stress that, the "total relativistic energy" delineated by two masses or two electric charges, according to our approach, is not anyway materialized by the surrounding space, but only by the "internal dynamics" of the charges of concern.

What is essential is the "conventional Coulomb's Force reigning in between two static charges, only", or the "conventional Newton's Force reigning in between two static masses, only", or any similar force, say the weak force, but expressed in similar terms.

Let us elaborate on this; let us first consider the Coulomb's Force.

The frame of Coulomb's Force is essential in the following way: The electric charges are Lorentz invariant; owing to this fact, the 1/distance^2 dependency of the Coulomb's Force between two static charges, can be shown to be imposed by the special theory of relativity, if this dependency is assumed to be in the form 1/distance^n. A derivation of this fundamental result is provided in Appendix A.

Thus Coulomb's Force, reigning between only two static electric charges, as adopted, is thoroughly compatible with the special theory of relativity [16]. Recall however that, here we consider static charges, exclusively.

Our reasoning regarding Coulomb's Force holds for Newton's Force, with the difference that, in the latter case the masses are obviously not Lorentz invariant, but the product [(universal gravitational constant) x (mass one) x (mass two)] appearing on the denominator of the Newton's Force expression, is well Lorentz invariant. Here again, the 1/distance^2 dependency of the Newton's Force between two static masses, is imposed by the special theory of relativity. Thus, Newton's Force, as it is, but reigning between only two static masses, is also thoroughly compatible with the special theory of relativity [17].

It seems that any other force law, must be built on similar characteristics. The Yukawa mesonic force law, constitutes well a proof of this claim.

To simplify our reasoning, let us continue on the basis of Coulomb's Law.

What is believed so far, is that Coulomb's Force holds, if the source charge is static, regardless whether the test charge is at rest or in motion. However, we discover that, this is not so; if the test charge is in motion, then Coulomb's force is decreased by the factor $\sqrt{1-v_0^2/c_0^2}$ [9].

This occurrence drives us to consider the electron (contrary to what has been so far done) not in the accustomed simplistic way; we sympathize by the fact that, the electron is generally considered as a "point-like particle". It must be obvious though, as tiny as it may be, the electron cannot be reduced to a point, given that a "point" cannot be a "material being". Thus, it is pointless to consider the electron as a point-like particle. The electron must embody an "internal dynamics", just like any other particle. Perhaps its "mass" is simply the "internal energy" of the "electric property", which we call "electric charge". This internal energy, is thus to be associated with (how ever it may be), the internal dynamics delineated by the electric charge.

When the electron is bound, say, to a proton, its internal dynamics is then (as a requirement of the energy conservation law), slown down, as much as the binding energy coming into play, assuming for simplicity that the proton (being much more massive than the electron), is not affected by the process of binding.

Our claim regarding the weakening of the internal dynamics of the bound electron can be rechecked, right away, through a backwards process (just the way we proceeded with the elevated hydrogen atom vis-à-vis a gravitational field). Suppose then we propose to bring back to infinity, the bound electron. Accordingly, we have to furnish to it, an amount of energy equal to its binding energy (still supposing that, moving away the electron, would not disturb, the supposedly infinitely more massive proton). The two particles, forming a "closed system"; furnishing energy to the electron, owing to the energy conservation law, will increase the internal energy, thus the rest mass of the latter. In other words, when entirely detached from the interaction domain, with the proton, the electron's rest mass would then get increased as much as the energy we would have furnished to it, i.e. by an amount equal to its original binding energy.

Hence, the free electron is not anymore the previous bound electron, or vice versa, the bound electron is not anymore the same as the free electron. It is indeed hard to accept that it would be, given that one cannot make an omelette, and keep the eggs as they are, prior to cooking!

The bound muon decay rate retardation, that we will consider herein seems to be an experimental proof of our assertion.

One still would question, "How the interaction between the proton and the electron occurs, if their respective energy is not spread in the surrounding space"; we have worked that out elsewhere [9]. In fact the same occurs between two celestial bodies, in exactly the same manner [10].

5. GENERAL QUANTUM MECHANICAL THEOREM: THE QUANTITY {(TOTAL ENERGY) X (CLOCK MASS) X (CLOCK SPACE SIZE)^2}, COMPOSED WITH RESPECT TO A WAVE-LIKE OBJECT, TURNS OUT TO BE A UNIVERSAL INVARIANT STRAPPED TO THE SQUARE OF THE PLANCK CONSTANT

Let us now give a rigorous prove of the above theorems we have drawn above, based on rather simple considerations.

In order to do that, we demonstrate a general quantum mechanical theorem, in Appendix B.

Thus, for a "real" atomistic or molecular wave-like object, i.e. a wave-like object existing in nature, we have shown elsewhere [18] the following theorem, first, on the basis of the Schrodinger Equation, as complex as this may be, then on the basis of the Dirac Equation, whichever may be appropriate, in relation to the object in hand. A "real" atomistic and molecular wave-like object, involves a potential energy, whose appearance is imposed by the special theory of relativity, just like a "Coulomb Potential energy" or, a "Newton Potential energy", or anything as such. Thence, even a relativistic Dirac description, embodying potential energy terms made of potential energies other than the mentioned potential energies (compatible with the special theory of relativity), may not represent a "real" description, for such an object.

Theorem 5: Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an internal dynamics which consists in a "clock motion", achieved in a "clock space", along with a "unit period of time". The description excludes "artificial potential energies" (which may otherwise lead to incompatibilities with the special theory of relativity). It is supposed to be based on $\mathcal{K}$ particles, altogether. If then masses m_{k0}, k = 1, …, $\mathcal{K}$, involved by this description, are overall multiplied by the arbitrary number γ , the following two general results are conjointly obtained:

a) The total energy E_0 associated with the given clock's motion of the object is increased as much, or the same, the unit period of time T_0, of the motion associated with this energy, is decreased as much.

b) The characteristic length, or the size $\mathcal{R}_0$ to be associated with the given clock's motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0}, k = 1, \ldots, \mathcal{K}) \rightarrow (\gamma\, m_{k0}, k = 1, \ldots, \mathcal{K})] \;\Rightarrow\; [(E_0 \rightarrow \gamma\, E_0),\ [T_0 \rightarrow \frac{T_0}{\gamma}],\ (\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\gamma})] \;. \quad (31)$$

What this theorem fundamentally says, is that, if an object ever experiences, for instance an overall mass decrease, then its total energy weakens as much, yielding the stretching of the period of its internal motion framed by the total energy in question (which should be considered quite understandable).

Next, we define a quantity M_0, which we call the "clock mass"; it is a compound mass whose motion constitutes the internal dynamics of the object; it is manufactured based on different masses embodied by the object in hand; thus multiplying these masses by γ, alters M_0 just as much. The clock mass, for instance is, as mentioned, the reduced mass of the proton and the electron, in the case of the hydrogen atom.

Eq.(31) immediately yields the invariance of the quantity $E_0 M_0 \mathcal{R}_0^2$. We call this invariance the "quantum mechanical invariance" of $E_0 M_0 \mathcal{R}_0^2$. It is remarkable, since $E_0 M_0 \mathcal{R}_0^2$, turns out to be Lorentz invariant as well (were the object brought into a uniform translational motion).

It is sure striking that the quantum mechanical invariance of $E_0 M_0 \mathcal{R}_0^2$, and the relativistic invariance of it, are identical.

This makes that, as uncomplicated as it may seem, the gravitational field, or any other field interacting with the object in hand, affects it, just like the uniform translational motion affects it.

Indeed, our original claim was that, the rest mass of a given object embedded in a field, interacting with it, is decreased as much as the binding energy coming into play [cf. Eqs. (9), (13), and (18)], to be then input to the object's quantum mechanical description at rest; this happens to be, as we have demonstrated, the basis of the quantum mechanical invariance of $E_0 M_0 \mathcal{R}_0^2$, which in return, is nothing else, but a relativistic invariance yeld by the special theory of relativity.

Note that the quantum mechanical invariance of $E_0 M_0 \mathcal{R}_0^2$, is obtained no matter what the description in hand is relativistic or non-relativistic. The only requirement, as mentioned, is that the potential energy terms input to the description in hand, are compatible with the special theory of relativity. Coulomb Potential is; so is Yukawa potential. These are the mere two forms, yeld by the Klein Gordon equation built on the fundamental relativistic relationship between momentum and energy (see Appendix B). Thus, any other potential representing any other possible real field, must be made similarly.

We further show that, the quantity $E_0 M_0 \mathcal{R}_0^2$ is necessarily strapped to the square of the Planck Constant, h^2 (being proportional to it, through generally a complex, anyway dimensionless, and relativistically invariant quantity, which is somewhat a characteristic of the bond configuration of the elements making up the wave-like object in hand).

We call the relationship

$$E_0 M_0 \mathcal{R}_0^2 \sim h^2, \tag{32}$$

(quantum mechanical invariance yeld by the change of mass input to the quantum mechanical description in consideration, strapped to the square of the Planck Constant

the UMA (Universal Matter Architecture) Cast.

It discloses already many structural properties, otherwise left obscure since several decades [19,20,21].

Note that primarily what we state along with Eq.(32), is not the result of a "dimension analysis"; indeed $E_0 M_0 \mathcal{R}_0^2$ would not be invariant in regards to a mass change, if the wave-like object description in consideration were not made of potential energy terms compatible with the special theory of relativity, though of course, dimension-wise there would still be no problem.

It is that the special theory of relativity, stringently imposes an interrelation in between E_0, M_0 and $\mathcal{R}_0$ (and this, already at rest), which is precisely the proportionality of $E_0 M_0 \mathcal{R}_0^2$, to a Lorentz invariant universal constant, i.e. h^2. The spatial dependency of the forces bringing the elements of the object in hand, already constitutes a major ingredient of it.

In other terms, in order to insure the end results of the special theory of relativity, when brought to a uniform, translational motion, or the end results of the GTR when embedded in a gravitational field, or any other field capable to interact with the object in hand, the object, already at rest, must be structured in just a given way.

Or, the other way around, because the object, already at rest is structured in just a given way, it well yields the end results of the special theory of relativity, when brought to a uniform, translational motion, or the end results of the GTR, when embedded in a gravitational field, or possibly any other field capable to interact with the object.

In short, the mass increase we introduced, to state Theorem 5, may very well not be arbitrary, and this is indeed what one experiences for instance, when a clock is removed out of a gravitational field; its rest mass, following our claim, as required by the mass & energy equivalence of the special theory of relativity [22], should be increased as much as the binding energy the object displays vis-à-vis the host celestial body of concern (just like the mass of the hydrogen atom is increased, as the electron is removed away, from its orbit around the proton). The unit time displayed by the internal dynamics of the object in hand, according to our Theorem 5, should then be altered as much. This is exactly what happens in the scope of the GTR.

According to our approach, the same phenomenon would occur, in exactly the same way, for ionized wave-like clocks in an electric field, or for wave-like clocks bearing an electric dipole, still in an electric field, or for wave-like clocks bearing a magnetic dipole in a magnetic field [23].

Similarly, if a muon is bound to a proton, its half life should quantum mechanically stretch, as much as its binding energy. This happens, to our knowledge, something totally overlooked. And it is worth to state Eq.(32), as our next theorem.

Theorem 6: The quantity [total energy x clock mass x (clock space size)^2], composed with respect to a wave-like object, turns out to be a universal invariant strapped to the Square of the Planck Constant, and this, vis-a-vis any field the object interacts with.

6. CALCULATION OF THE BOUND MUON DISINTEGRATION HALF LIFE

The muon μ^-, is a negatively charged particle. It has a total relativistic energy of 105.6 Mev. It is unstable. It may undergo different disintegration patterns, but all of them, except one, with considerably law probabilities. Its main disintegration channel thus, is the following:

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e \ ; \tag{33}$$

e^- is the electron, $\bar{\nu}_e$ is the antineutrino accompanying the electron, and ν_μ is the neutrino left out of the loss of the previous two particles.

The half life $T_{\mu\infty}$ of this disintegration, in empty space, is

$$T_{\mu\infty} \cong 1.4x10^{-6} \sec \ . \tag{34}$$

Keeping temporarily aside the relativistic effect due to (had we assumed so) the motion of the bound muon around the nucleus, and assuming that the muon preserves its original identity when bound (besides, the energy of its internal dynamics weakens); the disintegration half life of the bound muon, based on Theorem 5, can be written as [cf. Eq. (30)]

$$T_{\mu} = \frac{T_{\mu\infty}}{1 - \frac{E_B}{m_{\infty} c^2}} \quad ; \qquad (35)$$

(half life of the bound muon based on Theorem 5)

T_{μ} is the decay half life of the bound muon; E_B is the binding energy of the muon to the nucleus of concern. We suppose that the muon is bound to the 1s state, of the muonic hydrogen-like atom of concern.

Here m_{∞}, should be the mass $m_{\infty\mu}$ of the free muon, supposing that, the negative electric charge of the muon is distributed uniformly to its entire mass, and that the muon internal dynamics is altered accordingly, when bound to a nucleus.

However, this may not be true. Indeed, what is bound to the positively charged nucleus, should most likely be the "muon's electron", and not the "muon" as a whole. This "muonic electron" should then pull, the neutrino and the antineutrino, together with itself, to the binding state.

This reasoning may be objected, claiming that one does not have any right to talk about the "muonic electron". Yes but i) disintegration calculations are based on the likely decay constituents [8], ii) most important, the energy of the decay electron of the bound muon, in comparison with the energy of the decay electron of the free muon, as we will soon detail, happens to be shifted just as much as the bound muon's binding energy.

Hence, we anticipate that, m_{∞} should be considered as the highly energetic electron's mass, inside the muon.

Note that there seems to be six different channels of decay of the muon [24]. So the constituents of the muon (supposing that these, acquire their identities inside the muon, at least, prior to the decay), should really depend on these channels. The one we just considered, is the main decay channel [cf. Eq.(33)], the others, as mentioned, bearing very law probabilities of occurrence.

We do not know beforehand how, the energy subtracted from the muon's electron (through the binding process), shall ultimately be accounted by various constituents of the muon.

However, if we were allowed to reason based on the decay data regarding the main decay channel; the mass of the electron in the free muon, can be assessed to be [0.5 x the mass of the free muon] [25].

Thus, it should be the muonic electron alone (and not the muon as a whole), which exhibits a mass deficiency through the binding process of the free muon, to the nucleus in consideration. In other words, we come to expect that the electron's mass, inside the bound muon will decrease as much as the muon's binding energy.

One may check this guess by comparing the "binding energy of the muon to the nucleus" with the "measured energy shift, the electron thrown from the bound muon, delineates, as referenced to the energy of the electron thrown from the free muon" [17]. The match is indeed very satisfactory, chiefly for heavy nuclei; we will present a numerical proof of it, below.

Thus we can conclude that, basically the weakened dynamics of the electron inside the muon, slows down the disintegration of the muon in accordance with Eq.(35).

Now, we can express E_B (the binding energy, or the same the ionization energy of the muon) for the ground state, based on the Bohr-Sommerfeld, or here the same, the general Dirac Model, with the familiar notation;

$$E_B \cong \frac{2\pi^2 m_{\mu\infty} Z^2 e^4}{h^2}\left(1+\frac{1}{4}\alpha^2 Z^2\right) \cong \frac{m_{\mu\infty} c^2 Z^2 \alpha^2}{2}; \tag{36}$$

(muon's binding energy, based on Dirac's Approach)

Z is the atomic number of the nucleus of the hydrogen-like muoatom, and α [not to be confused with that of Eq.(6)], is the fine structure constant.

Note that Eq.(36) is obtained by expending the rigorous result in power of $Z^2\alpha^2$, but the difference in question remains negligible for the region $1 < Z < 85$, within which the experimental data is collected.

The electron's mass in the free muon can be expressed as [$f m_{\mu\infty} c^2$], f following our claim, being 0.5. (Thus $0.5\, m_{\mu\infty}$ is the effective mass of the electron, responsible of the binding of the muon.)

α by definition, is

$$\alpha = \frac{2\pi e^2}{ch} = \frac{1}{137}. \tag{37}$$

The denominator γ, of Eq.(35), thus becomes

$$\gamma = 1 - \frac{E_B}{f m_{\mu\infty} c^2} = 1 - \frac{1}{2f}\alpha^2 Z_0^2\left(1+\frac{1}{4}\alpha^2 Z_0^2\right), \quad f = 0.5\ . \tag{38}$$

(ratio of the bound muon's decay rate to the free muon's decay rate)

Next, we have to take into account the time dilation due to the rotation of the muon around the nucleus (had we presumed so); this is

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cong \frac{1}{\sqrt{1-\frac{4\pi^2 Z_0^2 e^4}{h^2 c^2}}} \quad ; \tag{39}$$

(time dilation factor of the muon due to its rotational motion around the nucleus)

here v the rotational speed of the muon in consideration; it is evaluated through the Bohr-Sommerfeld Model, which should be expected to be quite satisfactory for light nuclei; for heavy nuclei, it is pointed out that, because of "quantum effects" coming into play, Eq. (39) is generally an approximation [17]. To us, the mentioned reason, i.e. (whatever they may be) "quantum effects", are quite unclear. In fact according to our approach, what is wrong in considering v as predicted by the Bohr Sommerfeld Model, in the above relationship, for large Z's, is simply that, this model, just like Dirac Relativistic Model, does not take into account, the mass decrease of the bound electron [2, 10], which is the fundamental problem we aim to elucidate throughout this work.

Anyway, the overall decay half life T_μ of the bound muon, through Eqs. (35), (36), (37) and (38), quite satisfactorily, becomes

$$T_\mu = \frac{T_{\mu\infty}}{\left[1-\frac{1}{2f}\alpha^2 Z_0^2(1+\frac{1}{4}\alpha^2 Z_0^2)\right]\sqrt{1-\alpha^2 Z_0^2}} \quad . \tag{40}$$

(half life of the muon bound to the ground state).

It is interesting to note that this expression does not depend on the muon's mass (but it depends on f).

Thus, if the electron bears any internal mechanism (and we believe it must), the above expression would well tell us how this mechanism would slow down, when the electron is in a bound state (f, though in this case, should be taken as unity).
At this stage we can check our assumption about the fact that "the energy decrease exhibited by the bound muon, should essentially be subtracted from the electron's energy of the free muon". We can validate this assumption, by showing that the energy of the decay electron of the bound muon, in comparison with the energy of the decay electron of the free muon, shifts just as much as the bound muon's ionization energy. For this purpose, we draw Table 1. In this table, by "Actual Shift" we mean [1- (the measured energy of the electron thrown from the bound muon) / (the measured energy of the electron thrown from the free muon)]; on the other hand, f as agreed, is taken to be 0.5. The satisfactory match, chiefly for heavy nuclei validate our initial assumption, i.e. f = 0.5.

Table 1 $E_B/(fm_{\mu\infty}c^2)$, versus the "actual energy shift" displayed by the electron thrown from the bound muon [17, 18]

Binding Nucleus	*Atomic Number*	$E_B/(fm_{\mu\infty}c^2)$ $=\alpha^2 Z_0^2\left(1+(1/4)\alpha^2 Z_0^2\right)$	*Actual Shift*
Iron	26	0.04	0.18
Lead	82	0.40	0.40
Uranium	92	0.50	0.50

We have no explanation for the poor match between our prediction and the data, with respect to iron (cf. the second raw of Table 1). But, the use of f, has anyway little effect, for small Z's, given that, in this range the retardation is minor (cf. Figure 1), and the uncertainty about the measured data is quite broad. We will discuss the matter, in the conclusion.

7. CHECK AGAINST EXPERIMENTAL AND PREVIOUS THEORETICAL RESULTS

We were totally uninformed, in regards to preexisting experimental results, and we are more than happy to discover that our prediction about the bound muon decay, matches quite well with the experimental results [17, 18]. Moreover our prediction, at a first strike, appears to be much better than previous predictions made so far, no matter how sophisticated, also inevitably cumbersome these may be.

The predictions in question, handle the retardation of the decay process through

i) a semiclassical approach, which embodies

- the "phase space effect" (which consists in the reduction of the volume of phase space of the muon decay products, because of the binding),
- the classical "relativistic time dilation effect", and
- "the electron Coulomb effect" (which consists in the attraction exerted by the binding nucleus, on the muonic electron) , and

ii) sophisticated quantum mechanical approaches.

It would be interesting to compare quickly our prediction ($_{\text{Author}}$) [cf. Eq.(40)], with the semiclassical ($_{\text{SC}}$) results, exempt of time dilation effect:

$$\gamma_{SC} \cong 1-\frac{11}{2}\alpha^2 Z^2 \qquad \text{(for light Z)}, \tag{41}$$

$$\gamma_{SC} \cong 0.58\left(1-\alpha^2 Z^2\right)^{5/2} \qquad \text{(for heavy Z)}, \tag{42}$$

$$\gamma_{Author} \cong 1-\alpha^2 Z^2 \qquad \text{(for all Z)}. \tag{43}$$

Other predictions are so complicated that, they bear no easy series expansions.

Note also that, seemingly, there is no one prediction embodying satisfactorily, the entire spectrum of Z's (except ours). Those which are satisfactory for small Z's, are poor for large Z's, and those which appear satisfactory for large Z's, are poor for small Z's.

In Figure 1 we present the experimental data, and the results of previous calculations (decay rate normalized to the decay rate of the free muon, versus the atomic number), achieved to clarify these data.

- Curve A represents a semiclassical calculation including the time dilation effect, along with an exponential muon wave function.
- Curve B represents the same approach, but along with a Gaussian muon wave function.
- Curve C is a semiclassical calculation of the time dilation effect alone.
- Curve D is an interpolation from an anterior calculation achieved by Gilinsky and Mathews [26].
- Curve E is interpolated from the calculations achieved by Huff [18], which happens to be the seemingly the most recent calculations (but still way off the experimental results, chiefly for heavy nuclei).

The experimental results are achieved by Yovanovitch, Barrett, Holmstrom, Keufel, Lederman and Weinrich [27, 28, 29].

Let us note that little more recent data [30, 31], sweeps out some of the anomalously high values, in the iron region [cf. Figure 1].

In Figure 2 we present our prediction, as the denominator of the RHS of Eq.(40), versus the atomic number, together with the corresponding data in hand. We also sketch separately, γ of Eq. (38), versus the atomic number, since this constitutes the basis of our claim.

The match of our prediction with data, indeed appears to be successful.

Analyzing the validity of various proposed contributions, up against that we developed herein, constitutes the topic of a subsequent article.

It would be interesting now, to compare the previous results and ours with the actual data. This is done in Table 2.

Table 2 Comparison of different results (normalized decay rates), and ours with the actual data

	Z=29	*Relative Error with Respect to Data*	Z=53	*Relative Error with Respect to Data*	Z=82	*Relative Error with Respect to Data*
Data	0.95		0.70		0.40	
Curve A	0.82	-0.16	0.60	-0.17	0.35	-0.14
Curve B	x	*very large*	0.50	-0.40	0.35	-0.14
Curve D	0.97	0.02	0.83	0.16	0.70	0.42
Curve E	0.98	0.03	0.90	0.22	0.85	0.53
Author's Prediction	0.92	-0.03	0.80	0.12	0.50	0.20

8. CONCLUSION

As pointed out, right at the beginning of the article, this work was issued from a much broader angle, than the one intercepting the bound muon decay rate retardation.

It was the author's idea that, owing to the law of conservation of energy, the overall internal energy of a bound particle should be weakened as much as the binding energy coming into play, whether it is question of the atomic world or the celestial world. It is all the same nature, and one should not really have to conceive different packages of conception in order to predict different scales of it. Thus, when a particle is bound to any field, based on our approach, a given internal mechanism, the particle may embody, shall be affected accordingly, provided that the particle's inner articulations in relation to each other, are not degenerated via the binding process, i.e. the particle preserves its original characteristics.

Only after this discovery, constituting a totally different approach to the end results of the GTR (sketched throughout the text) that the author looked for a charged particle, with a measurable internal mechanism, bound to an electric field.

The internal mechanism in question, if measurable, once the particle is bound, should be slown down. An electron would be a perfect example. But the electron's internal mechanism is not measurable. One does not even know, whether the electron, classified as a point-like particle, has an internal mechanism. (To us, it sure must have.)

The unstable muon appeared to be a second best candidate to prove the validity of the present approach. By the time the author did not even know, that the bound muon's decay rate was measured, and this long time ago. The data the author discovered only afterwards, had indeed shown that, the bound muon's decay rate retards, and this finding, overlapping to a great extent, with the curve, similar to Figure 2, the author had already drawn, with f = 1 [cf. Eq.(38)]; the match for small Z's were perfect, but not that satisfactory for large Z's.

There was indeed one important point to be clarified. The muon, in a gravitational field, like anything else, is touched on the whole. Yet, is it really touched on the whole, when bound to an electric field? If the muon, exists as a one single peace, prior to the decay, then its charge, can most likely be considered to be uniformly distributed, and that the electric field would affect it, in its entirety. This seems to be what happens for the muon bound to nuclei of small Z's. At least the author's prediction in this range, is well confined within the rather broad measurement uncertainties, whether f is taken to be unity, or 0.5, given that for small Z's, the deflection of the bound muon's decay rate from the free muon's decay rate, remains bearly significant.

Supposing that f, in the range in question, can be considered to be unity, we well arrive to prove our claim, i.e. Theorem 5, applies to the bound muon, and accordingly, the metric is altered nearby the nucleus, vis-à-vis a charged particle, just like it is altered nearby a celestial body, vis-à-vis a mass.

This is already something vital, which would drastically transform our present conception.

Now, chiefly for large Z's, one can conjecture that, just like the alpha particle is considered to exist within its heavy parent, prior to the decay, the electron can well be considered to have been formed within the muon's body, prior to the decay.

If so, a difficulty arises, since that way, not the entire muon will be attracted to the electric field produced by the nucleus, but only the electron will be attracted. The total relativistic energy shared by the electron within the muon, amounting to half of the muon's total relativistic energy, as shown by the data, shall be decreased as much as the electron's binding energy. As a result, the activity of the electron within the muon, will slow down just as much. Thus, it seems that the decay rate of the bound muon is slown down, because of the weakening of the bound electron's activity, within the muon.

At least that is what we assumed throughout, and succeeded to a great extent to predict the data for large Z's too. (Otherwise our prediction along with f = 1, remains little above the data.)

We could anyway validate our stand point, noting that the energy of the electron thrown by the bound muon, as compared to the energy of the electron thrown by the free muon, is shifted just as much as the electron's binding energy (cf. Table 1).

Our assumption then, reduces to the assumption that the bound muon's decay rate will slow down, as much as the electron's energy, within the muon, weakens.

Clearly our prediction's match with data, on the whole, is better than that of other predictions, and constitutes a fundamental explanation to bound muon decay rate retardation.

Our prediction is further extremely simple, in comparison with the all other very cumbersome predictions.

Note that the data embody a peak near iron. Our approach did not predict it. Yet neither could the previous attempts. It is suspected that this may be due to the large background of low energy gamma rays associated with accompanying inelastic muon capture events, though little more recent data show that the anomaly in question is not as important as that delineated by previous measurements [23, 24].

Our original approach, sought for the slowing of the internal mechanism of a charged particle, that would be affected in its entirety, by an electric field, binding it. If there is still discrepancy between our prediction and the data with respect to bound muon decay rate, it is, we believe, basically due to the fact that, the bound muon, chiefly for large Z's, is not affected by the field, in its entirety.

But the plain electron must be.

Thus excitingly enough, we come to state that just like "mass", "electric charge" too, slows down clocks, interacting with the electric field in consideration.

This means that just likewise, say in an atom, the bound electron's mass, should be lighter than the free electron's mass, and this as much as binding energy, coming into play.

This seems quite trivial, but very much against the general wisdom, since neither Dirac nor anyone else after him, has seemingly, given a thought to alter the mass of the bound electron.

Taking it into account, strikingly induces the change of the metric nearby the nucleus [32], just like the metric change nearby a gravitational source.

APPENDIX A

PROOF OF THE FACT THAT COULOMB'S FORCE OR NEWTON'S FORCE REIGNING BETWEEN RESPECTIVELY STATIC CHARGES OR STATIC MASSES, MUST ACT AS $1/r^2$, IS IMPOSED BY THE SPECIAL THEORY OF RELATIVITY: HENCE, BOTH LAWS ALONG WITH THEIR CAST, MUST BE UNIVERSAL

Our claim can be achieved easily by noting that the quantity,

$$\mathcal{H} = [\text{force}] \times [\text{mass}] \times [\text{distance}]^3 \ , \qquad \text{(A-1)}$$

is Lorentz invariant.

In fact, dimensionally speaking, it amounts to the square of the Planck Constant, which in return is Lorentz invariant.

On the other hand, it is known that the electric charges are Lorentz invariant. (If not, say in excited atoms, energetic electrons would exhibit electric charge intensities different than the electric charge intensity of the electrons on the ground level, which is not the case.)

Now suppose we have a dipole, such as a water molecule,[‡‡] which can be represented by charges q and Q, situated at rest, at a distance r_0, from each other; it has the mass m_0, still at rest.

Coulomb's Force F_{C0}, reigns between q and Q. Suppose we assume that this force is as usual expressed as proportional to the electric charges coming into consideration, also to $1/r_0^n$, where though we do not know, a priori, the exponent n, i.e.

$$F_{C0} = \frac{qQ}{r_0^n} \ . \tag{A-2}$$

Suppose now, we bring the dipole to a uniform translational motion of velocity v, along the direction perpendicular to the line connecting the electric poles.

Through the motion, the quantity

$$I = [\text{mass}] \text{ x } [\text{length}] \ , \tag{A-3}$$

remains invariant.
More specifically,

$$I = m_0 r_0 = (\beta m_0)\left(\frac{r_0}{\beta}\right) = \mathit{Constant} \ ; \tag{A-4}$$

β is the usual Lorentz dilation factor, i.e.

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \ . \tag{A-5}$$

It becomes evident that the invariance of

$$\begin{aligned} \mathcal{H} &= [\text{force}] \text{ x } [\text{mass}] \text{ x } [\text{distance}]^3 \\ &= \frac{qQ}{r_0^n} \text{ x } m_0 \text{ x } r_0^3 = \frac{qQ}{r_0^n} \text{ x } I \text{ x } r_0^2 = \frac{qQ}{r_0^n \gamma^{-n}} \text{ x } I \text{ x } r_0^2\, \gamma^{-2} = \mathit{Constant} \, , \end{aligned} \tag{A-6}$$

holds, if and only if n=2, i.e. if Coulomb's Force, behaves as

$$F_{C0} = \frac{qQ}{r_0^2} \quad \text{(c.q.f.d.)} \ . \tag{A-7}$$

Note that the same holds, if "charges", in question, are "gravitational charges"; in this case however, the product of charges should be considered, together with the universal gravitational constant.

---

‡‡ In water molecule, the oxygen atom, attracts the binding electrons of the hydrogen atoms. This makes that the hydrogen atoms get charged positively and the oxygen atom negatively. Thus the water molecule can be represented by a dipole, made of -2e situated nearby the oxygen atom, and +2e situated on the median of the triangle HOH, between the hydrogen atoms.

In other words:

i) The $1/r_0^n$ dependency of Newton's Force F_{N0}, reigning between two static masses m and $\mathcal{M}$, and expressed as

$$F_{N0} = G\frac{m\mathcal{M}}{r_0^n} \quad (n \equiv 2)\ , \tag{A-8}$$

with n being exclusively 2, is imposed by the special theory of relativity.

ii) The quantity $Gm\mathcal{M}$ is Lorentz invariant, thus G, the universal gravitational constant G, is not Lorentz invariant. This result makes that G is not as universal as one may think it is.[§§] By the same token, the quantities $\sqrt{G}m$, or $\sqrt{G}\mathcal{M}$ (bearing the dimension of an electric charge), are well Lorentz invariant.

The fact that the $1/r_0^2$ dependency of Coulomb's Force, is imposed by the special theory of relativity, is certainly correlated with the fact that, this dependency is well furnished as a result of the Klein Gordon Equation, built via replacing the energy and momentum quantities in the relativistic equation

$$p^2c^2 + m_{0\infty}^2c^4 = E^2\ , \tag{A-9}$$

by the corresponding quantum mechanical symbols; here p is the momentum of a moving particle of rest mass $m_{0\infty}$, and E its total energy. With regards to Coulomb's Force, the rest mass $m_{0\infty}$ is taken to be zero.

If not, the Yukawa Force is obtained. Note that here, an extra, but Lorentz invariant exponential attenuation factor takes place in the numerator, next to the product of nuclear charges; otherwise the $1/r_0^2$ dependency still appears, on the background of the force expression.

Evidently, the cast of Newton's Force is the same as that of Coulomb's Force, and we show elsewhere that (see Section 2 of the text), through our approach, based on Newton's Force, but written exclusively for static masses, one is able to arrive at the end results of the general theory of relativity (GTR), up to a measurable sensitivity.

We can conclude that both Eqs. (A-2), i.e. Coulomb's Force Law, and (A-8), i.e. Newton's Force Law (but once again, written for respectively, static charges, and static masses), delineate universal force laws, provided that the special theory of relativity is universally valid (and it seems it is).

[§§] Consider for instance the solar system rotating around the center of the Milky Way. The relatively small rotational motion can be considered as a uniform translational motion. Take the attraction force between the Sun and the Planet Earth. This force, when assessed relative to the center of the galaxy, is not the same force, if assessed relative to the Sun. Let $\mathcal{M}$ be the mass of the Sun, and m the relativistic mass of Earth on its solar orbit, in the Sun's frame of reference. Suppose we define G too, in this frame. The quantity $Gm\mathcal{M}$ [bearing the dimension of (electric charge)^2], is the same, whether we consider it relative to the center of the galaxy, or relative to the Sun. But the masses of concern, are not the same if assessed relative to the center of the galaxy. Thus, the universal gravitational constant does not remain the same when one switches from the first frame of reference, to the second.

This tells that, the weak force law, governing the muon's decay, too, must be somehow built in a similar manner. This result is important to generalize the theorem we will demonstrate in the Appendix B.

APPENDIX B

PROOF OF THEOREM 5 OF THE TEXT

Herein we will prove the Theorem 5 of the text.

Theorem 5: Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an internal dynamics which consists in a "clock motion", achieved in a "clock space", along with a "unit period of time". The description excludes "artificial potential energies" (which may otherwise lead to incompatibilities with the special theory of relativity). It is supposed to be based on $\mathcal{K}$ particles, altogether. If then masses m_{k0}, k = 1, …, $\mathcal{K}$, involved by this description, are overall multiplied by the arbitrary number γ , the following two general results are conjointly obtained:

a) The total energy E_0 associated with the given clock's motion of the object is increased as much, or the same, the unit period of time T_0, of the motion associated with this energy, is decreased as much.

c) The characteristic length, or the size $\mathcal{R}_0$ to be associated with the given clock's motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0},\ k = 1, \ldots, \mathcal{K}) \rightarrow (\gamma\, m_{k0},\ k = 1, \ldots, \mathcal{K})] \Rightarrow [(E_0 \rightarrow \gamma E_0),\ [T_0 \rightarrow \frac{T_0}{\gamma}],\ (\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\gamma})].$$

Let us accentuate that, if the object is, say an atom, then $\mathcal{R}_0$ is (no matter how this is defined) the radius of it; if the object is a diatomic molecule, $\mathcal{R}_0$ is the internuclear distance, etc; $\mathcal{R}_0$, in fact, may be just any length one may pick, within the framework of the object in hand, and Theorem 5, as can be shown, shall still be valid.

Proof of The First Part of Theorem 5

For our purpose, for simplicity, without though any loss of generality, we consider the (time independent) Schrödinger Equation, i.e. with the familiar notation, written for an atomistic or a molecular object composed of J nuclei, of respective masses m_{j0}, j = 1, …, J, and I electrons (altogether), of (the same) mass m_{i0}, i = 1, ..., I:

$$\left(-\sum_{j}\frac{h^2}{8\pi^2 m_{j0}}\nabla_j^2-\sum_{i}\frac{h^2}{8\pi^2 m_{i0}}\nabla_i^2-\sum_{i,j}\frac{Z_{j0}e^2}{r_{ij0}}+\sum_{i,i'}\frac{e^2}{r_{ii'0}}+\sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{r_{jj'0}}\right)\psi_0(\underline{r}_0)$$
$$=E_0\psi_0(\underline{r}_0)\ . \qquad \text{(B-1)}$$

E_0 is the eigenvalue, and $\psi_0(\underline{r}_0)$ the related eigenfunction; Z_{j0} is the atomic number of the j^{th} nucleus; r_{ij0} is the distance between the i^{th} and the j^{th} particles.

Eq.(B-1) already represents a given complexity. Instead, we could have considered straight a much more general quantum mechanical description with respect to a given number of unspecified particles, without making any distinction between these, even those which would be identical. But then the picture we would have to paint would have remained too abstract, and the derivations that would follow, too cumbersome to follow. This is why we have avoided a more general quantum mechanical description, to start with.

On the other hand, in any case, any potential energy term to be input to such a description must be compatible with the special theory of relativity. Otherwise, even a relativistic Dirac description would furnish results which do not consist in Lorentz invariant forms. In Appendix A, above, we have come to show that, in order to be compatible with the special theory of relativity, a potential energy term's spatial dependency must bear the $1/r_0$ dependency, on the background. An extra (Lorentz invariant) exponential term may be added to the numerator, such as that coming into play along with the Yukawa potential. But what is primordial, is that the $1/r_0$ dependency, must appear on the background. (And in Appendix A, we have shown that, this is imposed by the special theory of relativity.) Thus, vis-à-vis our goal, an atomic or molecular quantum mechanical description made of Coulomb Potential energy terms, is a fine basis to work on; furthermore, it well lands itself to further generalization.

*

Thus multiply all, electron masses m_{i0} (i = 1, ..., I), as well as nuclei masses m_{j0} (j = 1, …, J), appearing in Eq.(B-1), by γ; the eigenfunction and the related eigenvalue will accordingly be altered:

$$\left(-\sum_{j}\frac{h^2}{8\pi^2\gamma m_{j0}}\nabla_j^2-\sum_{i}\frac{h^2}{8\pi^2\gamma m_{i0}}\nabla_i^2-\sum_{i,j}\frac{Z_{j0}e^2}{r_{ij0}}+\sum_{i,i'}\frac{e^2}{r_{ii'0}}+\sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{r_{jj'0}}\right)\psi_{new}(\underline{r}_0)$$
$$=E\,\psi_{new}(\underline{r}_0)\ . \qquad \text{(B-2)}$$

This is the same as

$$\left(-\sum_j \frac{h^2}{8\pi^2 m_{j0}}\nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 m_{i0}}\nabla_i^2 - \sum_{i,j}\frac{Z_{j0}e^2}{\frac{r_{ij0}}{\gamma}} + \sum_{i,i'}\frac{e^2}{\frac{r_{ii'0}}{\gamma}} + \sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{\frac{r_{jj'0}}{\gamma}} \right)\psi_{new}(\underline{r}_0)$$

$$= \gamma E \psi_{new}(\underline{r}_0) . \qquad \text{(B-3)}$$

Let now

$$\underline{r}_0 \rightarrow \underline{r} = \gamma \underline{r}_0 \quad , \qquad \text{(B-4)}$$

together with

$$\psi(\underline{r}) \equiv \psi_{new}(\underline{r}_0) . \qquad \text{(B-5)}$$

Since

$$\frac{\partial\psi(r_0)}{\partial u_0} = \frac{\partial\psi(r)}{\partial u}\frac{\partial u}{\partial u_0} \text{ ; } u_0 = x_0, y_0, z_0 \text{; } u = x, y, z; \qquad \text{(B-6)}$$

we have

$$\frac{\partial\psi(r_0)}{\partial u_0} = \gamma \frac{\partial\psi(r)}{\partial u} . \qquad \text{(B-7)}$$

Eq.(B-3) thus becomes

$$\left(-\sum_j \frac{h^2}{8\pi^2 m_{j0}}\gamma^2\nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 m_{i0}}\gamma^2\nabla_i^2 - \sum_{i,j}\frac{Z_{j0}e^2}{\frac{r_{ij0}}{\gamma}} + \sum_{i,i'}\frac{e^2}{\frac{r_{ii'0}}{\gamma}} + \sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{\frac{r_{jj'0}}{\gamma}} \right)\psi(\underline{r})$$

$$= \gamma E\psi(\underline{r}) . \qquad \text{(B-8)}$$

Dividing by γ^2, and using Eq. (B-4), this yields

$$\left(-\sum_j \frac{h^2}{8\pi^2 m_{j0}} \nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 m_{i0}} \nabla_i^2 - \sum_{i,j} \frac{Z_{j0} e^2}{r_{ij}} + \sum_{i,i'} \frac{e^2}{r_{ii'}} + \sum_{j,j'} \frac{Z_{j0} Z_{j'0} e^2}{r_{jj'}} \right) \psi(\underline{r})$$
$$= \frac{E}{\gamma} \psi(\underline{r}) . \qquad \text{(B-9)}$$

In comparison with Eq.(B-1), we can deduce at once that

$$\frac{E}{\gamma} = E_0 \Rightarrow E = \gamma E_0 \quad \text{(c.q.f.d.)} . \qquad \text{(B-10)}$$

Thus, we have come to achieve the demonstration of the first part of Theorem 5.

Proof of The Second Part of Theorem 5

Next we focus on a size of interest $\mathcal{R}_0$ (i.e. as we just pointed out, the "size of an atom", anyway we would like to define it, or the "internuclear distance" in a diatomic molecule of concern, or whatever), to be associated with the wave like object in hand. $\mathcal{R}_0$ shall be determined based on the solution of Eq.(B-1). Following the mass perturbation, $\mathcal{R}_0$ becomes $\mathcal{R}_{0\text{new}}$, and this latter shall be found based on the solution of Eq.(B-2). According to Eq.(B-4), $\mathcal{R}_{0\text{new}}$ is transformed into $\mathcal{R}$, so that $\mathcal{R}=\gamma\mathcal{R}_{0\text{new}}$. (Note that according to this equation, any distance, say r_0 we would consider, becoming r_{onew} due to the mass change, is transformed into r, so that $r = \gamma\, r_{\text{onew}}$. Thus the derivation presented herein, in fact holds for any distance, thence also for a given specific distance $\mathcal{R}_0$ we would pick up.)

$\mathcal{R}$ is to be determined as the solution of Eq.(B-9). But since this equation is identical with Eq.(B-1) [along with Eq.(10)], the solution of Eq.(B-9) in regards to $\mathcal{R}$ is the "original size" of interest, i.e. $\mathcal{R}_0$.

Hence

$$\gamma\mathcal{R}_{0\text{new}} = \mathcal{R}_0 \ , \qquad \text{(B-11)}$$

or the same

$$\mathcal{R}_{0\text{new}} = \frac{\mathcal{R}_0}{\gamma} \quad \text{(c.q.f.d.)} . \qquad \text{(B-12)}$$

This ends the demonstration of Theorem 5 of the text.

Proof of The Third Part of Theorem 5

Following the multiplication of masses by the factor γ, the total energy E_0 has increased by γ [cf. Eq.(B-10)].

But E_0 can well be written as

$$E_0 = ah\nu_0 = \frac{ah}{T_0} \quad , \tag{B-13}$$

where T_0 is the unit period of time associated with the dynamics in consideration, and the coefficient a, is just a constant insuring the equality [for a quick check one can multiply the right hand sides of Eq.(1) and Eq.(21) of the text, to discover that a, for this case, becomes 1/2].

This makes that

$$(E_0 \rightarrow \gamma E_0) \Rightarrow (T_0 \rightarrow \frac{T_0}{\gamma}) \quad \text{(c.q.f.d.)} \quad . \tag{B-14}$$

One can easily check that the same results can be obtained with respect to a relativistic quantum mechanical description, as well as nuclear potentials input to the description, instead of Coulomb potentials.

ACKNOWLEDGEMENT

The main ideas developed herein, were presented by the author to the International Conference "Physical Interpretations of the Theory of Relativity", which was held in Bauman Moscow State Technical University, through 4 – 7 July 2005, also in Lebedev Institute, the 23rd of July, 2003, and further, the 8th of July, 2005, and were discussed with Russian colleagues, upon the kindest invitation of Professor Vladislav Rozanov, Director of The Laser Plasma Theory Division of Lebedev Institute. The author would like to extend his deep gratitude to Dearest Friend Vladislav for his wonderful patience, understanding and endless support. The author is further indebted to Professor Alwyn Van Der Merwe who has considered this controversial work, with an unequal objectivity and warmth. The author would also like to thank to Dear Friends Dr. Engin Isiksal, Dr. Elman Hasanov, Dr. Alexander Kholmetskii, Dr. Sahin Kocak, Dr. Metin Arik, Dr. Sahin Ozdemir, for very many hours of discussions, which helped to improve considerably the work presented herein. The author would further like to thank to Dear Research Assistant Fatih Özaydin who has drawn the figures.

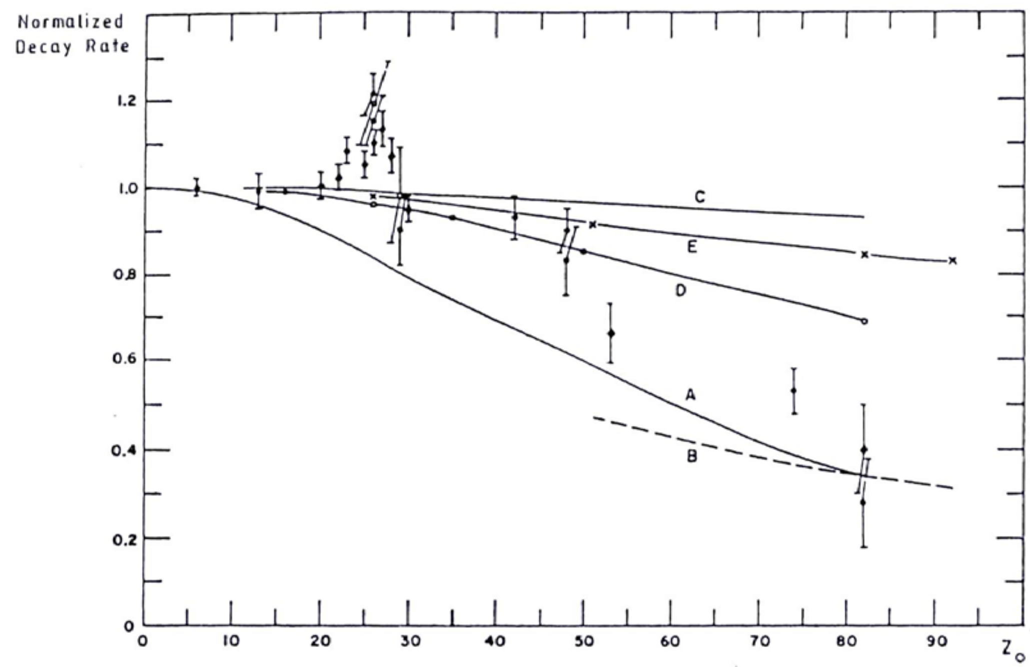

Figure 1 Preexisting experimental results and theoretical predictions

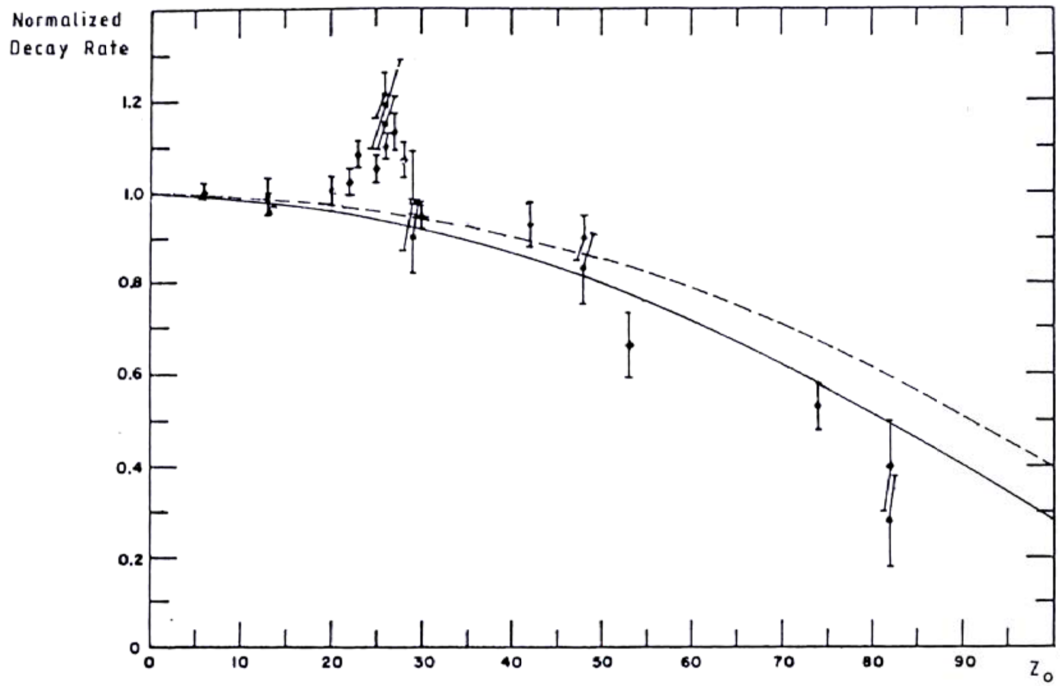

Figure 2 Preexisting experimental results and our predictions:

——— : the overall decay rate

– – – – : the decay rate without the relativistic time dilation effect

REFERENCES

[1] T. Yarman, A Novel Approach to The End Results of The General Theory of Relativity Via Just The Special Theory of Relativity and Quantum Mechanics, Communication to Professor C. Marchal, Director of Research, ONERA (May 1999). Invited Talk, Lebedev Institute, Moscow, Federation of Russia, July 6, 2007. The 20th Internatıonal Congress of Jangjeon Mathematıcal Socıety, August 21-23, 2008, Bursa, Turkey.

[2] T. Yarman, V. B. Rozanov, The Mass Deficiency Correction to Classical and Quantum Mechanical Descriptions: Alike Metric Change and Quantization Nearby an Electric Charge, and a Celestial Body, Part I: A New General Equation of Motion for Gravitationally, or Electrically Bound Particles, International Journal of Computing Anticipatory Systems, Volume 19, December 2006.

[3] T. Yarman, V. B. Rozanov, The Mass Deficiency Correction to Classical and Quantum Mechanical Descriptions: Alike Metric Change and Quantization Nearby an Electric Charge, and a Celestial Body, Part II: Quantum Mechanical Deployment for Both Gravitationally, International Journal of Computing Anticipatory Systems, and Electrically Bound Particles, Volume 19, December 2006.

[4] L. de Broglie, Recherches Sur La Théorie Des Quanta, Annales de Physique, 10^{e} Série, Tome III, 1925.

[5] H. Poincaré, Arch. Néerland. Sci. 2, 5, 252-278 (1900).

[6] H. Poincaré, Electricité et Optique, §351 (1901).

[7] H. Poincaré, Compt. Rend. 140, 1504-1508 (1905).

[8] A. Einstein, Ann. Phys., 17, 891, 1905.

[9] A. Einstein, Ann. Phys., 20, 627-633, 1906.

[10] The General Equation of Motion via the Special Theory of Relativity and Quantum Mechanics, Annales de La Fondation Louis de Broglie, Volume 9 (3), 2004 (459).

[11] T. Yarman, The End Results of the General Relativity Theory Via Just Energy Conservation and Quantum Mechanics, Foundations of Physics Letters, Volume 19, December 2006.

[12] P. A. M. Dirac, General Theory of Relativity, Princeton University Press, 1975.

[13] T. Yarman, A Novel Approach to the Bound Mon Decay Rate Retardation: Metric Change Nearby the Nucleus, Physical Interpretations of the Theory of Relativity Conference, Bauman Moscow State Technical University, 4 – 7 July 2005.

[14] R. B. D. Evans, The Atomic Nucleus (Chapter 1), McGraw-Hill Book Company, 1955.

[15] Gunnar Kallen, Elementary Paticle Physics, Addison – Wesley Publishing Company, Inc., 1964.

[16] T. Yarman, Tachyonic Interaction, or The Same, de Broglie Relationship, As Imposed By The Energy Conservation Law, Part I: Electrically Bound Particles, Paper under Review, in Journal of Russian Laser Research.

[17] T. Yarman, Tachyonic Interaction, or The Same, de Broglie Relationship, As Imposed By The Energy Conservation Law, Part I: Electrically Bound Particles, Paper under Review, in Journal of Russian Laser Research.

[18] T. Yarman, Invariances Based on Mass and Charge Variation, Manufactured by Wave Mechanics, Making up The Rules of Universal Matter Architecture, Chimica Acta Turcica, Vol.27, 1999.

[19] T. Yarman, Elucidation of the Complete Set of H_2 Electronic States' Ground Vibrational Data, International Journal of Hydrogen Energy, 29, 2004 (1521).

[20] T. Yarman, An Essential Approach to the Architecture of Diatomic Molecules. Part I. Basic Theory, Optics and Spectroscopy, Volume 97 (5), 2004 (683).

[21] T. Yarman, An Essential Approach To The Architecture of Diatomic Molecules, Part II: How Size, Vibrational Period of Time And Mass Are Interrelated?, Optics and Spectroscopy, Volume 97 (5), 2004 (691).

[22] A. Einstein, The Meaning of Relativity, Princeton University Press, 1953.

[23] T. Yarman, A Novel Approach to The End Results of The General Theory of Relativity and to Bound Muon Decay Retardation, DAMOP 2001 Meeting, APS, May 16-19, 2001, London, Ontario, Canada.

[24] R. W. Huff, Decay Rate of Bound Muons, Annals of Physics, 16: 288-317 (1961).

[25] F. Herzog, K. Adler, Decay Electron Spectra of Bound Muons, Helvetica Physics Acta, Vol. 53 (1980).

[26] V. Gilinsky, J. Mathews, Phys. Rev. 120, 1450 (1960).

[27] D. D. Yovanovitch, Phys. Rev. 117, 1580 (1960).

[28] W. A. Barrett, F. E. Holmstrom, J. W. Keufel, Phys. Rev. 113, 661 (1959).

[29] L. M. Lederman, M. Weinrich, Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956, Vol. 2 (427).

[30] J. V. Allaby, A. Chisholm, J. Eades, B. M. Townes, Phys. Rev., Vol. 125, 2077 (1962).

[31] M . Blair et al, Proc. Phys. Soc., Vol. 80, 938 -944 (1962).

[32] T. Yarman Mass Deficiency Correction To The Relativistic Quantum Mechanical Approach: Metric Change Nearby The Nucleus, Division of Particles and Fields, APS Meeting, August 27 – 31, 2004 Riverside, California, USA