An Inconsistency in Special Relativity

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Abstract

This note presents an argument based on Einstein's original paper on special relativity, showing that the theory requires that each of two clocks in uniform relative motion actually works slower than the other. This shows an internal inconsistency in the special theory.

Key words: special relativity

About thirty years ago Dingle claimed that Einstein's special theory of relativity was internally inconsistent. In 1972 he published a book[1] describing his attempts to convince the scientific community that his argument was sound. The basis of his argument was his claim that the special theory required that if there were two clocks in uniform motion relative to one another, each one would actually work more slowly than the other, and that is physically impossible. The scientific community did not accept Dingle's claim of an inconsistency, one of the arguments used against him being that the special theory only required each clock to *appear* to work more slowly than the other; for example, Stadlen[2] wrote "What Einstein really said was that each clock would appear to run slow to an observer moving with the other."

Although there have been many other published claims that Einstein's special theory is untenable, the scientific community as a whole has not accepted those claims. In the author's opinion, the matter has not been satisfactorily resolved, and the purpose of the present note is to show, as simply as possible and using arguments taken directly from Einstein's original paper on the subject, that the special theory of relativity does indeed require that if there are two clocks in uniform relative motion, each one actually works slower than the other.

In his original paper on special relativity, Einstein[3] stated that if two clocks were together and synchronized and if one of them went around a polygonal line and returned to the other clock, then the one that had gone around the polygonal line would on its return show an earlier time reading than the one that had stayed behind. Clearly, that result cannot be merely a matter of appearance or observation; an observer who had gone around the polygonal line with one clock and an observer who had stayed behind with the other clock would agree that the difference in readings actually occurred.

Einstein then went on to assume that the result proved for a polygonal line is also valid for a continuously curved line and thence concluded that a clock at the equator would work more slowly than an exactly similar clock at one of the poles. It is again clear that the predicted slowing in this case is an actual slowing and not merely an effect of observation; observers attached to both clocks would agree that there was a progressive retardation of the equatorial clock, as compared with the polar clock, as the process continued. The fact that Einstein later added a footnote excluding the case of a pendulum clock emphasizes the reality of the phenomenon. (Although that footnote has been attributed to Sommerfeld, the present writer has argued elsewhere[4] that it was, in fact, written by Einstein himself.)

Now, taking a step backwards in the argument by which Einstein deduced that the equatorial clock would work more slowly than the polar one, suppose that the "equatorial" clock travels at uniform linear velocity along the perimeter of a square having the "polar" clock at its center. If X denotes the clock at the center of the square, and Y denotes the clock traveling along the square path, then, by Einstein's argument, Y would work more slowly than X. By symmetry Y would obviously experience the same amount of lag while traveling along each of the four sides of the square; furthermore, there would be no discontinuities of Y's reading at any corner of the square. Those facts could be verified by repeated radio transmissions from Y to X, transmitting Y's reading as Y passed each corner of the square; the differences between successive transmitted readings would be of equal magnitude, denoted by T_Y, and the differences between successive readings of X's clock when the transmissions were received would also be of equal magnitude, denoted by T_X. According to Einstein's theory, T_Y would be smaller than T_X; in other words, Y would run steadily slower than X while traveling along any one side of the square.

Now consider a third clock, denoted by Z, traveling through space at uniform velocity relative to X in such a way that for a small portion of its journey it travels beside Y along one side of the square. Since it must keep time with Y as it travels beside it, then Z must also work steadily slower than X, even though its velocity relative to X is perfectly uniform.

In other words, if Y runs slower than X, then Z runs slower than X. Now, using the principle of relativity, it can with equal validity be shown that X runs slower than Z. These results, both of which are deduced from Einstein's original presentation of the theory, are inconsistent with one another.

In other words, if Einstein's prediction that a clock at the equator would work more slowly than a clock at one of the poles is valid, then each of two clocks in uniform relative motion must work more slowly than the other. This supports earlier claims that the special theory of relativity contains an internal inconsistency.

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Résumé

Cette lettre présente une discussion fondée sur l'article original d'Einstein sur la relativité spéciale, montrant que la théorie exige que chacun de deux horloges en mouvement relatif uniforme marche plus lentement que l'autre. Ceci montre une incohérence intrinsèque à la théorie.

References

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