TALES OF TOTH

Fairytale of Physics, Number 13

All were hushed and intently riveted their attention on Toth as he ascended the podium to deliver his lecture. The last day in the inevitable time had come. His subject was: *Fallacy in Science.*

Simplicio had finally finished making the introduction before the conclave of the University. He had gone on and on interminably, telling about himself and the wonders of his Astrological Laboratory, of which he was the director. Nobody was going to be allowed to forget that. Really, he was quite a bore. The audience was already tired and fidgety, wishing the whole affair were over, before Toth even began.

"I come to bury Einstein, not to praise him," was Toth's introductory remark.

That brought them to attention! The effect was electric. The assemblage bristled and an angry murmur ran through the audience like a hive of bees that has been shaken up.

I overheard one of the relativists mutter menacingly:

"'Twere best he speak no harm of Einstein here."

"Shh! Let us hear what he has to say," an ungowned citizen insisted, however, and Toth was able to proceed.

"You gentle researchers associated with this great laboratory, you are the scientific leaders of today. This is due to one man's direction, the forever famous Simon Simplicio." There was now a loud clapping of hands that was followed by stomping of feet. The mood of the audience had changed.

Simplicio was pleased by this flattery, quite obviously, and he gestured to his followers to restore order: "Quiet! Let us hear what the god of Reason may have to tell us."

The first relativist immediately changed his tune and commented, "I think there is much reason in his words."

Quite an orator, this Toth! He had spoken but two sentences. With the first he had stirred his audience up emotionally; in the second he had placated it. Already he had it in his paw; even his enemy was won over to his side.

Then, with a seemingly innocent gesture to his dark glasses at the end of his noble, aquiline nose, he mentioned: "Reason is blind."

"Poor soul!" I heard one of the women in the audience remark audibly, "His eyes are blind," and all the rest of the ladies reached into their purses to get a Kleenex to daub away their tears.

"Science was," Toth continued. "We were once scientists," and he also brushed away a tear. I understood him; he was mourning what had occurred since 1905 while he sat on his pedestal. He continued: "Men have lost their reason and it is fled to brutish beasts."

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Another pause ensued until he could regain his composure. I overheard another member of the audience comment nearby, "There's a lot of sense in what he says." Toth was playing on the crowd as if it were a musical instrument.

 n about the third row in front of me I now saw Primitive who was paying the utmost attention to whatever was being said.

Toth went on: "But I would do the relativists no wrong. They are honorable men."

I could see that Simplicio's adherents were again well pleased by this remark and everyone settled back to hear Toth amicably.

He had announced the keynote of his address and now he began to lead out from this theme.

"Remove a grain of truth and go on to do whatever you will. Any set of premises that may be given can be the basis for a system that can be logically developed from them. If the premises are not true to start with, one can with reason lead on to greater fallacies. Let me illustrate this point. Suppose we change only a single entry of the multiplication table; say, instead of $2x2 = 4$ we declare $2x2 = 5$. With all good reason it follows that $2x3 = 7$ since

$$2x3 = 2x(2x1) = 2x2 + 2x1 = 5 + 2 = 7.$$

 oth continued: "It is so easy to withdraw a grain of truth and often it happens very subtly. Thus, let us introduce a transformation T of the (x, y)-plane onto itself, according to

$$T: \begin{matrix} x' = \sqrt{(x^2 + y^2)} \text{ when } |x| \geq |y| \text{ and } x' = x \text{ when } |x| < |y| \\ y' = y \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y' = \sqrt{(x^2 + y^2)} \end{matrix}$$

This permits one to fit round pegs into square holes very easily as all circles around the origin map into squares. The inverse transformation will put square pegs in round holes whenever the need to do so may arise."

A murmur of admiration and applause spread throughout the hall as Toth demonstrated how it was done, drawing a diagram on the blackboard behind him. The relativist sitting in front of me drew out his notebook too and quickly jotted down notes. I read over his shoulder:

Tothentzian transform:- maps square pegs into round holes; and conversely,

followed by the equations that Toth had just given. But on glancing over to where Primitive Thinker was sitting, I saw him turning his head from side to side as if he wanted to know why everyone was so impressed and why one would use the transformation without further justification in the first place. He seemed befuddled and troubled, wondering why a round peg should be forced into a square hole anyway.

Toth then went on to another example of the mindless use of transformations. It was an old story. A certain physicist was travelling across a desert and came upon a lion. Wishing to capture the lion, he erected a circular cage somewhere in the desert but, of course, the lion did not go into this cage as he wanted it to. So the physicist went after it scientifically, performing an inversive transformation with respect to the center of the cage:

$$T: (\vartheta, r) \to (\vartheta, \frac{1}{r})$$

The lion was instantly mapped into the cage. I had heard the story before but not the sequel to it, which was that the physicist forgot to lock himself in the cage where he belonged before performing his magic. He wound up inside the cage with the lion and there was an unhappy ending to this affair so that nobody has since tried to repeat it.

A titter passed through the audience, but Primitive did not understand the joke. When he did, finally, I could see that he thought the physicist suffered the fate he deserved for believing that one could catch a real, live lion that way. Certain of the relativists, however, took Toth seriously and out came the notebooks once again with appropriate directions jotted down on how to trap lions logically, with the warning that one must be inside the cage before applying the inversive transformation. Modern physics is purely mathematical, you know.

Toth left the topic but picked up the subject of fallacy that can be introduced when transformations and vectors are combined. He began by writing down the simplest of transformations of the plane, which is that of a translation of coordinates by a constant

$$T: \begin{matrix} x + a \to x' \\ y + b \to y' \end{matrix}$$

and this he also wrote vectorially as

$$T: \mathbf{X} + \mathbf{A} \to \mathbf{X}'$$

On differentiating this, assuming that $\mathbf{X} = \mathbf{X}(t)$ represents the locus of a moving object in the plane, one gets an equation relating the velocity vectors in the two coordinate systems:

$$\mathbf{V} = \mathbf{V}'$$

because $\mathbf{A}$ is a constant. Thus, one learns in introductory vector analysis that the velocity of a moving object is a free vector independent of the reference scheme and one simply writes $\mathbf{V}$, dropping the prime and ignoring where its origin is.

But Toth then showed that if the transformation is time-dependent, the rule is a fallacy. As I now recall, he gave the example

$$T: \mathbf{X} + \mathbf{A}t \to \mathbf{X}'$$

and he then differentiated this to give

$$T: \mathbf{V} + \mathbf{A} \to \mathbf{V}'$$

which is the same as the classical composition of velocities. Toth told about the cop who saw a motorist whom he believed was speeding, but his radar device only indicated 50 mph. He ignored the fact that his squad car was travelling at 40 mph. He thought his radar device had gone wrong because the faster he pursued the motorist, the slower the other car went; when he slowed down the other car speeded up. Finally, he came to a stop and then discovered the motorist was doing 90 mph but he could not catch him. The speedster got away.

Then Toth talked about the velocity of light emitted by a source that was moving at velocity v. He pointed out that modern science declares that

$$c + v = c$$

based on the rule. What was even more remarkable, Toth commented, was that if the transformation vector were a function of time, $\mathbf{A} = \mathbf{A}(t)$ with

$$T: \mathbf{X} + \mathbf{A} = \mathbf{X}'$$

then

$$T: V + \mathring{A} \rightarrow V'$$

"By neglecting $\mathring{A}$, we are led on to novel and revolutionary new scientific theories," he remarked.

On this comment, the notebooks all came out again and *'Develop new and revolutionary scientific theories by ignoring the Å term'* was jotted down. Toth's lecture was being well received, even enthusiastically, in fact. But I do not think he realized that the ironic vein of it was being lost.

The effect on poor Primitive was terrible. He was obviously very much upset, as he was not yet ready to accept fallacy as the method of science. His education was incomplete and he had not yet progressed far enough that he was prepared for this step. He was thoroughly confused. I saw him glance this way and that wondering at the beaming faces around him. His professors evidently understood what he could not and were impressed. The lecture was going to go down in the annals of Princeville forever as one shaping the direction that science took. I could see that even Simon Simplicio knew it and was excited. But Primitive Thinker was not; instead, he was distressed and perturbed. He squirmed about unhappily in his seat, rose out of it as if to leave the auditorium, then reseated himself. The most pitiful look imaginable was on his open countenance. Here was the god of Reason, himself, proving that 2x3 = 7 and he did not understand it, while everyone else around him seemed to.

Toth took another direction in his lecture. First, he wrote down an array of numbers on the blackboard that I here reproduce:

									limit
1	1	1	1	1	1	1	. . .	→	1
1	1	1/2	1/2	1/2	1/2	1/2	. . .	→	1/2
1	2	1	1/3	1/3	1/3	1/3	. . .	→	1/3
1	2	3	1	1/4	1/4	1/4	. . .	→	1/4
1	2	3	4	1	1/5	1/5	. . .	→	1/5
1	2	3	4	5	1	1/6	. . .	→	1/6
1	2	3	4	5	6	1	. . .	→	1/7
.	.	.	.	.	.	.	limit ↘ 1		.
.	.	.	.	.	.	.			.
.	.	.	.	.	.	.			.
↓	↓	↓	↓	↓	↓	↓			limit limit → 0
limit 1	2	3	4	5	6	7			limit limit → ∞

He pointed out to us that each row and each column had a limit but that the limits of these limits were not the same:- one tended to zero while the other became unbounded. Also, the principal diagonal which was made up of all ones had a limit but it was unity, still different from either of the limits of the limits.

"When one has something of this sort, it is an easy thing to do to withdraw a grain of truth, and get almost any answer one wishes and the problem calls for," he said. "There are a great number of ways to hide what one is doing, too. This is a very excellent form of reasonable, fallacious argument.

"Suppose we want to calculate the integral

$$\int_{-\infty}^{\infty} \cos x \, dx$$

There are two limiting processes involved here: one is obviously in the bounds of the integral, which ought to be written as

$$\lim_{A,B\to\infty} \int_{-A}^{B} \cos x \, dx$$

The other is notationally hidden in the summation process of the integration and no one notices it.

"Let us just subtly change the last formula to

$$\lim_{A\to\infty} \lim_{B\to\infty} \int_{-A}^{B} \cos x \, dx$$

and then make use of the additivity of the summation limit and write

$$\lim_{A\to\infty} \int_{-A}^{a} \cos x \, dx + \int_{a}^{b} \cos x \, dx + \lim_{B\to\infty} \int_{b}^{B} \cos x \, dx$$

which is the same as the above.

Everyone knows that

$$\int_{p}^{p+2\pi} \cos x \, dx = \sin (p + 2\pi) - \sin p = \sin p - \sin p = 0$$

for any p so we can write

$$\lim_{B\to\infty} \int_{b}^{B} \cos x \, dx = \sum_{n=0}^{\infty} \int_{b+2n\pi}^{b+2(n+1)\pi} \cos x \, dx = 0$$

Similarly

$$\lim_{A\to\infty} \int_{-A}^{a} \cos x \, dx = 0$$

Therefore:

$$\int_{-\infty}^{\infty} \cos x \, dx = 0 + \int_{a}^{b} \cos x \, dx + 0$$

$$= \sin b - \sin a$$

"If we take $b = \pi/2$ and $a = 0$ then the answer is 1; however, if we take $b = \pi/2$ and $a = -\pi/2$, the answer is 2.

"The logical conclusion is thus that 1 = 2."

I agreed that this was so because Sommerfeld used almost the same argument to prove that the phase velocity of electric signals was equivalent to the c-velocity, and as Toth had said, relativists are all honorable men. Only Sommerfeld used the Laplace transforms of functions which were the same as the integrand between A and B but zero beyond these points; then he let A, B tend to infinity. In this way he proved what he wanted.

There passed a murmur of approbation through the hall, that grew into applause. Notebooks were pulled out again and eagerly the audience copied down what Toth had shown them how to do.

He had been talking for less than fifteen minutes by then and all countenances beamed. He was making a tremendous impression on that group.

"Now, I would like to play what I call tensor tiddlywinks with you . . .," as he was interrupted by a wail, almost a shriek, from someone in the audience.

Some member of the audience had taken ill. The person left his seat and rushed from the room in evident pain and anguish. I could not see who it was in the darkened hall immediately, but as he passed down the corridor near where I was seated, I discovered it was Primitive Thinker.

To be continued.