

# Matrix Stress Equation

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## Abstract

In porous medium, matrix and fluids bear forces together. For analyzing strength and deformation in porous medium, matrix stresses must be calculated. After Terzaghi advanced matrix stress for saturated soil in 1925, many people have used matrix stress in their researches. It is found that there is mistake in Terzaghi's equation. A new matrix stress equation is established.

## Introduction

In porous medium, matrix and fluids bear forces together. For analyzing strength and deformation in porous medium, matrix stresses must be calculated. After Terzaghi advanced matrix stress (effective stress) for saturated soil<sup>1</sup> in 1925, many people have taken part in research of effective stress for porous medium. In petroleum engineering textbooks<sup>1-4</sup> and monographs, Terzaghi equation is still in use.

It is found that there is mistake in Terzaghi's equation. A new matrix stress equation is established.

## Matrix Stress Equation

It is defined in textbooks and monographs that overburden pressure at some point equals the weight of matrix and fluid in porous medium:

$$p_{ob} = gh[(1-\phi)\rho_m + \phi\rho_f] \quad (1)$$

or

$$p_{ob} = g \int_0^h [(1-\phi)\rho_m + \phi\rho_f] dh \quad (2)$$

where,  $p_{ob}$  = overburden pressure;  $g$  = acceleration of gravity;

$h$  = depth of point from surface;  $\phi$  = porosity;  $\rho_m$  = density of matrix;  $\rho_f$  = density of fluid.

Matrix and fluids bear overburden pressure together. The portion that is not acting on fluids is called matrix stress.

Matrix stress advanced by Terzaghi for saturated soil is<sup>1</sup>

$$\sigma'_z = p_{ob} - p_f \quad (3)$$

where,  $\sigma'_z$  = matrix stress in vertical direction;  $p_f$  = formation pressure.

Terzaghi's equation does not consider the effect of porosity. It is not right theoretically.

Because matrix and fluids bear overburden pressure together, thus

$$p_{ob} = (1-\phi)\sigma'_z + \phi p_f \quad (4)$$

and

$$\sigma'_z = \frac{p_{ob} - \phi p_f}{1-\phi} \quad (5)$$

## Structural Matrix Stress

If total structural stresses are expressed as  $\sigma_x, \sigma_y, \sigma_z = p_{ob}$ , then matrix stresses  $\sigma'_x, \sigma'_y, \sigma'_z$  are

$$\left. \begin{aligned} \sigma'_x &= \frac{\sigma_x - \phi p_f}{1-\phi} \\ \sigma'_y &= \frac{\sigma_y - \phi p_f}{1-\phi} \\ \sigma'_z &= \frac{\sigma_z - \phi p_f}{1-\phi} \end{aligned} \right\} \quad (6)$$

In formation fracturing prediction and oil enhanced fracturing etc, matrix stress (3) is used. These technologies should be rebuilt, such as research in reference 5.

## Conclusion

Terzaghi's matrix stress equation (3) is wrong. True matrix stress equation is (5).

## References

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## Authors' Biographies



**Zifeng Li** is a professor of Yanshan University, China, Technical Editor of SPE Drilling & Completion. Before joining Yanshan University, he was first a professor of Daqing Petroleum Institute and then a professor of China University of Geosciences. He has published 80 papers and 4 books on drill string, casing, rod pumping mechanics & well bore stability. He holds a BS degree in drilling engineering and an MS degree in machinery engineering from Daqing Petroleum Institute and a PhD degree in petroleum development engineering from Petroleum University, China.