Mogens True Wegener

SOME COSMOLOGICAL MODELS
THEIR TIME SCALES AND SPACE METRICS

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S U M M A R Y

*Accepting clock retardation as an empirical fact, we provisionally adopt Whitrow's derivation of the Robertson-Walker Metric (**RWM**) of Cosmology from the gamma-factor of SR. Recalling that the principle of cosmic isotropy can be used as an argument for the definability of an all-embracing universal time, at least statistically, we propose to reverse this procedure by postulating such time as a regulative idea in the sense of Kant.*

*Taking **RWM** as our formal point of departure, we then investigate the properties of two standard models of modern cosmology: 1) the uniform expansion model of Milne & Prokhovnik, the simplest model of a cosmic "big bang", and 2) the exponential expansion model of Bondi & Gold, (supposed to be) the simplest model of a cosmic "steady state". It is then easy to show that the ideas of "big bang" and "steady state" are not mutually exclusive.*

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1. INTRODUCTION

Taking clock retardation as an empirical fact we provisionally adopt Whitrow's derivation of the standard Robertson-Walker metric *(**RWM**)* from the γ-factor of special relativity *(**SR**)*. According to Milne & Walker, two kinds of *observers* must be distinguished: *fundamental* ones, defining the geometrical structure of the specific cosmological model under consideration by constituting its substratum, and *accidental* ones which are superposed on the substratum in a way that refers the description of their motion to the *substratum* as a universal "frame of rest".

The difference, naturally, is statistical and a matter of degree. Thus it is possible to make sense of a graduation of clocks according to their approximation to the ideal of a universal time: we classify particles by estimating the departure of their distribution from universal isotropy. Recalling the fact that the principle of *cosmic isotropy* can be used as an argument for the definability of an all-embracing *universal time*, at least statistically, we propose to reverse this procedure by postulating such time as *a regulative idea* in the sense of Kant.

Using ***RWM*** as a formal point of departure we investigate the properties of two standard models of modern cosmology: α) the uniform expansion model of Milne & Prokhovnik, which is the simplest model of a cosmic "big bang", and β) the exponential expansion model of Bondi & Gold, supposed to be the simplest model of a cosmic "steady state". Rejecting the so-called "perfect cosmological principle" of the latter, it is easy to show that the ideas of "big bang" and "steady state" are not mutually exclusive after all: a universe starting with a "big bang" at the dawn of creation may very well approximate to a "steady state" in the course of infinite time.

In agreement with our provisional analysis of ***RWM*** we consider the relationship between our choice of time scale for a certain model of the universe and its corresponding space metric. As it turns out, there are at least two important ways of mapping the expansion of the universe: a) that which keeps atomic sizes constant while light is being stretched, and b) that which keeps distances between fundamental observers constant while their atoms are shrinking.

Finally a new "steady state" model of the universe is proposed which deviates from ***RWM*** by allowing atoms to be contracted due to universal dispersion. In this model, spatial curvature is apparently increasing with the distance at which an object is seen by a fundamental observer. The model implies *world map* and *world view* to be identical as regards their formal structure. Being derived directly from $d\tau^2 = dt^2 - ds^2$, it is even simpler than the Bondi-Gold model.

The basic properties of this model and related ones are examined and discussed.

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2. A CLASSICAL ALTERNATIVE TO SR

The hyperbolic formulae corresponding to the standard addition $\alpha' \equiv \alpha - \omega$ are:

$$cosh\alpha' = cosh\alpha\, cosh\omega - sinh\alpha\, sinh\omega$$
$$sinh\alpha' = sinh\alpha\, cosh\omega - cosh\alpha\, sinh\omega$$

With $c_o \equiv unity$, $v \equiv tanh\omega$, the ***LT*** are expressible in temporal coordinates:

$$t' = t\, cosh\omega - x\, sinh\omega \ \ . \ \ x' = x\, cosh\omega - t\, sinh\omega$$

It is interesting that the ***LT*** are derivable from the addition formulae if and only if $\tau \equiv \tau'$ in:

$$t'/\tau' \equiv cosh\alpha' \ \ . \ \ t/\tau \equiv cosh\alpha \ \ . \ \ x'/\tau' \equiv sinh\alpha' \ \ . \ \ x/\tau \equiv sinh\alpha$$

It is natural to identify τ^2 with the ***SR*** invariant: $\mathcal{T}^2 \equiv t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2$.

The standard (1x3) ***LT*** for three inertial frames, Σ, S & S', in relative motion are:

(1a) $X \equiv x\, cosh\alpha - t\, sinh\alpha = x' cosh\alpha' - t' sinh\alpha'$. $Y \equiv y \equiv y'$

(1b) $T \equiv \ t\, cosh\alpha - x\, sinh\alpha = t' cosh\alpha' - x' sinh\alpha'$. $Z \equiv z \equiv z'$

Consider Σ to be a preferred frame with the privileged observer Ω situated in its *origo*. Let the observers O & O' be situated in the *origos* of S & S', resp., and let the frame times t & t' of S & S' be synchronized to the proper time T of Ω by choosing $T \equiv t \equiv t' \equiv 0$ when O & O' both coincide with Ω. Suppose that an event E occurs at particle P, as observed by Ω, O & O'. Let the standard coordinates of E be (T, X, Y, Z) in Σ, (t, x, y, z) in S and (t', x', y', z') in S' Then, by eliminating the irrelevant frame times t & t' from the expressions for T & X, we get:

$$X = x/cosh\alpha - T\, tanh\alpha = x'/cosh\alpha' - T\, tanh\alpha'$$

Further, using $\omega = \alpha + \alpha' = -\omega'$ to eliminate α or α', we recover ***LT*** for the privileged time T:

(2a) $\underline{x' = \{x\, cosh(\omega - \alpha) - T\, sinh\omega\}/cosh\alpha}$

(2b) $\underline{x = \{x' cosh(\omega' - \alpha') - T\, sinh\omega'\}/cosh\alpha'}$

Finally, introducing non-standard frame-times $\bar{\tau}$ & $\bar{\tau}'$ for frames S & S' defined by means of :

(3) $\underline{T \equiv \bar{\tau}\,\gamma \equiv \bar{\tau}'\gamma'}$

$$\gamma \equiv cosh\alpha \equiv 1/\sqrt{1 - v^2} \ , \ \gamma' \equiv cosh\alpha' \equiv 1/\sqrt{1 - v'^2}$$

and using $w \equiv tanh\omega$, we find the Tangherlini transformations ***(TT)*** as generalized by Selleri:

(4a) $x' = x\,\frac{cosh(\omega - \alpha)}{cosh\alpha} - \bar{\tau}\, sinh\omega = \frac{x\,(1 - wv) - w\bar{\tau}}{\sqrt{1 - w^2}}$

(4b) $x = x'\,\frac{cosh(\omega' - \alpha')}{cosh\alpha'} - \bar{\tau}'\, sinh\omega' = \frac{x'(1 - w'v') - w'\bar{\tau}'}{\sqrt{1 - w'^2}}$

In standard ***SR***, it is always the proper time of a single moving clock which is said to be retarded relative to the slave clocks distributed as a network over the rest frame of the observer; but if we refer the inertial motion of particles to a privileged frame we should use ***TT*** instead. Notice that ***TT*** reduce to ***GT*** if all observations are referred to the frame of the *midway particle*:

(5) $\alpha = \frac{\omega}{2} \ \Rightarrow \ x - x' = \bar{\tau}\, sinh\omega = 2\, T\, sinh\frac{\omega}{2}$

Applying $t \equiv \bar{\tau} - x\, tanh\frac{\omega}{2}$, $t' \equiv \bar{\tau}' - x' tanh\frac{\omega'}{2}$ directly to ***LT***, we get the same result, viz. ***GT*** :

(6) $\underline{\bar{\tau}' = \bar{\tau} \ , \ \omega' = -\omega \ , \ x' = x - \bar{\tau}\, sinh\omega \ , \ y' = y \ , \ z' = z}$

3. THE ROBERTSON-WALKER METRIC

In his monumental *Natural Philosophy of Time* (1961/1980), G.J. Whitrow sketched a method to derive the ***RWM*** of relativistic standard cosmology from the γ-factor of ***SR***. Let:

$$c_o \equiv 1 \;\; . \;\; c_o t_o \equiv r_o \equiv 1$$

and let the *origo* of the comoving standard rest frame S of an observer P be P himself.

Now suppose an event E, taking place at some object O, to be triggered by a light signal which instantaneously released a visible flash. Suppose further that this light signal was sent off by P at the instant τ_1, and that the flash was perceived by P at the instant τ_3, both τ_1 & τ_3 being instants of proper time τ of P as read off his own standard atomic clock C. We then recover the Einstein coordinates of the Cartesian frame S of P by means of the usual definitions:

$$\tau_3 \equiv t{+}r \;\; . \;\; \tau_3' \equiv t'{+}r'$$
$$\tau_1 \equiv t{-}r \;\; . \;\; \tau_1' \equiv t'{-}r'$$

From the standard ***SR*** invariant $d\tau_3 d\tau_1$ we get the γ-factor for the retardation of moving clocks:

$$d\mathcal{T}^2 \equiv d\tau_3 d\tau_1 = dt^2{-}dr^2 \equiv \gamma^{-2}dt^2$$

Whitrow now suggested the substitution: $dr \rightarrow \mathcal{S}(T)\, d\sigma.$ With this move, T is no longer the *private frame time* t, but more like the *public proper time* τ of all fundamental observers, i.e. all observers at rest in the universe, e.g. relative to the cosmic background radiation ***(CBR)***. Putting $T \equiv \tau$, this transforms the standard invariant of ***SR*** into the standard ***RWM*** metric:

$$d\mathcal{T}^2 = \; dt^2 - dr^2 = d\tau^2 - \mathcal{S}^2(\tau)\, d\sigma^2$$

$\mathcal{S}$ is the expansion or scale factor for the universe, and σ a fixed "comoving" coordinate. Now, for *fundamental observers*, $d\sigma = 0$, i.e. $d\mathcal{T} = d\tau$, showing that all fundamental observers participate in the same common ***cosmic time*** $\mathcal{T}$. By implication, any devitation of proper time τ from $\mathcal{T}$ is restricted to non-fundamental or *accidental observers* distinguished by a variable σ. Considering $\mathcal{T} \neq \tau \neq t$, one can ask if all this amounts to more than mere analogy.

According to the standard view, it is always the *proper time* of a "moving" particle which is claimed to be "slow" relative to the *frame time* of a "stationary" observer. So coordinate time, or frame time, is thereby tacitly assumed to represent the "true extended time" of any observer. The cosmic time $\mathcal{T}$ implied by ***RWM*** is seldom taken seriously, but mostly ignored or explained away as being of "statistical origin" and thus "ill defined". Nevertheless, it is the firm stance of the present writer that a fundamental importance should be ascribed to the cosmic time $\mathcal{T}$.

If we define *true time* by the readings of the master clocks of our fundamental observers when they have been properly synchronized - e.g. by letting a definite non-local cosmic event such as the beginning of everything in a socalled "big bang" represent a common time zero - then it is no longer true to say that the master clock of a fundamental observer is slow relative to the frame clocks of another observer, fundamental or not. Much rather it is true to say that it is the clocks of accidental particles that are slow relative to the clocks of fundamental observers. Everything depends on convention in the sense that it follows from a preferred point of view.

Please, notice that this does not involve us in any conflict with the results of experiment. The only conflict at stake is one relating to the standard interpretation of ***SR***.

Hence, if the clocks of fundamental observers show the true time $\mathcal{T}$, then the clock of an accidental particle will be more or less astray. In fact, the greater its distance to that fundamental particle relative to which it is momentarily at rest, and which thus constitutes the natural origo of its own rest frame, the slower its clock will run and the more it will deviate from true time. The natural way of interpreting this retardation of moving clocks is as an effect of gravitation. In this way we have found a natural coupling between the rates of non-fundamental clocks and what seems to be a gravitational potential due to the substratum of fundamental particles.

The reason for this dependence is that the deviation of the clock of an accidental particle from true time $\mathcal{T}$ is found by direct comparison with the clock of that fundamental particle with which it momentarily coincides; and the greater the distance of an accidental particle is to the origo of that rest frame to which it belongs, the faster its speed relative to that fundamental particle with which it coincides will appear; this follows from the expansion function $\mathcal{S}(\tau)$. What we have disclosed is the possibility of an influence of the substratum on particles which do not belong to the substratum and which represent deviations from cosmic symmetry.

This supports a conjecture of Whitrow's former master, E.A. Milne. The essential point of his *Kinematic Relativity*, devised as an alternative to Einstein's theories, ***SR*** & ***GR***, is precisely that what we call gravitational effects may emerge from local deviations from cosmic symmetry. Indeed, if elevated to a universal principle, Milne's conjecture amounts to nothing less than an inversion of Mach's principle: where Mach claimed that inertia should be reduced to gravitation, Milne instead held that gravitation should be reduced to inertia - and demonstrated how to do it! But all this is a repetition of ideas presented earlier. With polar coordinates the ***RWM*** becomes:

(7) $$\underline{d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + \lambda^2(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(8) $$\underline{\mathcal{R} \equiv \mathcal{S}(\tau)\rho\ ,\ \mathrm{T} \equiv \int d\tau/\mathcal{S}(\tau) + const.\ ,\ \mathcal{C}(\mathrm{T}) \equiv \mathrm{dT}/d\tau \equiv 1/\mathcal{S}(\tau)}$$

$\mathcal{R}$ is proper distance, $\mathcal{C}$ is an inverse scale function, and T is an auxiliary time parameter.

$$d\rho \equiv d\lambda/\sqrt{1-\kappa\lambda^2} = \begin{cases} d\lambda & \Leftarrow \kappa = 0 \\ d\, arcsin\lambda & \Leftarrow \kappa = 1 \\ d\, arsinh\lambda & \Leftarrow \kappa = -1 \end{cases}$$

$$d\mathcal{T}^2 \underset{\kappa=0}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}$$

$$d\mathcal{T}^2 \underset{\kappa=1}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + sin^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}$$

$$d\mathcal{T}^2 \underset{\kappa=-1}{\equiv} d\tau^2 - \mathcal{S}^2(\tau)\{d\rho^2 + sinh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}$$

$$d\rho \equiv d\lambda/\sqrt{1-\kappa\lambda^2} \equiv d\varrho/(1+\kappa\varrho^2/4) = \begin{cases} d\varrho & \Leftarrow \kappa = 0 \\ d\, arctan(\varrho/2) & \Leftarrow \kappa = 1 \\ d\, artanh(\varrho/2) & \Leftarrow \kappa = -1 \end{cases}$$

$$d\rho^2 + \lambda^2(d\theta^2 + sin^2\theta\, d\phi^2) \equiv \frac{d\varrho^2 + \varrho^2(d\theta^2 + sin^2\theta\, d\phi^2)}{1+\kappa\varrho^2/4} \equiv \frac{d\xi^2 + d\eta^2 + d\zeta^2}{1+\kappa\varrho^2/4}$$

The following versions comprise all possible values of the constant of curvature, κ:

(9a) $$\underline{d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\{d\lambda^2/(1-\kappa\lambda^2) + \lambda^2(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(9b) $$\underline{= d\tau^2 - \mathcal{S}^2(\tau)\{d\varrho^2 + \varrho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}/(1+\kappa\tfrac{\varrho^2}{4})}$$

(9c) $$\underline{= \mathcal{C}^{-2}(\mathrm{T})[\,\mathrm{dT}^2 - \{d\xi^2 + d\eta^2 + d\zeta^2\}/(1+\kappa\tfrac{\varrho^2}{4})]}$$

In the T-scale, the expansion of cosmos is explained by a shrinking of its atoms!

4. MILNE'S SIMPLE BIG BANG MODEL

In what follows we throw light on the ***RWM*** by discussing some simple world models. One of the simplest is Milne's model of uniform expansion, adopted by Prokhovnik and others:[4]

(10) $$\underline{\mathcal{S}(\tau) \equiv \tau \equiv d\tau/d\mathrm{T} \equiv e^{(\mathrm{T}-\tau_o)/\tau_o} \equiv \mathcal{C}^{-1}(\mathrm{T})}$$

Let us assume that radar signals are being exchanged between a pair of observers, P & Q, in $(1\mathrm{x}1)$ *time-space*. Suppose that a "photon" ϕ is emitted from P at $\tau = \tau_1^p$, reflected by Q at $\tau = \tau_2^q$, and received by P at $\tau = \tau_3^p$. Then, according to the relativity principle as interpreted by Milne, τ_3^p is the same function of τ_2^q as τ_2^q is of τ_1^p - call it $s(\tau) \equiv e^{\sigma}\tau$. Generalizing, and introducing Einsteinian standard coordinates t & r for P (priming those of Q), we at once get:

$$t \equiv \tfrac{1}{2}(\tau_3+\tau_1) \;\Downarrow\; r \equiv \tfrac{1}{2}(\tau_3-\tau_1)$$
$$\tau_3 = \tau e^{\sigma} = t + r \;\;.\;\; \tau_1 = \tau e^{-\sigma} = t - r$$

(11) $$\underline{t = \tau\, cosh\sigma} \;.\; \underline{r = \tau\, sinh\sigma}$$

Let us next assume that σ is not a constant, but a variable; then, by differentiation:

$$dt = d\tau\, cosh\sigma + \tau\, d\sigma\, sinh\sigma$$
$$dr = d\tau\, sinh\sigma + \tau\, d\sigma\, cosh\sigma$$

$$d\mathcal{T}^2 \equiv\; dt^2 - dr^2 = d\tau^2 - \tau^2 d\sigma^2 = e^{2(\mathrm{T}-\tau_o)/\tau_o}(d\mathrm{T} - d\sigma^2)$$

This invariant is easily expanded into a hyperbolic *time-space* of $(1\mathrm{x}3)$ dimensions if we put:

$$d\sigma^2 \equiv d\rho^2 + sinh^2\rho\,(d\theta^2 + sin^2\theta\, d\phi^2) \equiv \{d\xi^2 + d\eta^2 + d\zeta^2\}/(1 - \tfrac{\varrho^2}{4})$$

The standard ***SR*** invariant is thus transmuted into the hyperbolic metric of an expanding universe with expansion function $\mathcal{S}(\tau) \equiv \tau$, which can be transformed into another hyperbolic metric, viz. that of a stationary universe whose atoms all contract in accordance with the Hubble function $\mathcal{C}^{-1}(\mathrm{T}) \equiv e^{(\mathrm{T}-\tau_o)/\tau_o}$, where $\mathrm{T} = \tau_o\{1 + log(\tau/\tau_o)\}$, τ_o being a constant of calibration:

(12a) $$\underline{d\mathcal{T}^2 =\; dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2)}$$

(12b) $$\underline{=\; d\tau^2 - \tau^2\{d\rho^2 + sinh^2\rho\,(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(12c) $$\underset{\tau_o \equiv 1}{\equiv} \underline{e^{2(\mathrm{T}-1)}[\,d\mathrm{T}^2 - \{d\xi^2 + d\eta^2 + d\zeta^2\}/(1 - \tfrac{\varrho^2}{4})]}$$

The **1**st of these metrics, incorporating the universal constancy of the velocity of light, yields an infinity of *private time-spaces*, comprising the flat 3-spaces of fundamental observers. The following two metrics both yield *a public time-space*, each containing a curved 3-space: that of the **2**nd metric being associated with τ-*time*, relative to which atoms keep a constant size while the distances between fundamental observers steadily expand in proportion to $\mathcal{S} \equiv \tau$ (with the consequence that *light is stretched*, as suggested by Prokhovnik), and that of the **3**rd metric being associated with t-*time*, relative to which distances between fundamental observers remain invariant whereas the sizes of their atomic constituents are contracting in proportion to $\mathcal{C}^{-1} \underset{\tau_o = 1}{\equiv} e^{\mathrm{T}-1}$, due to *a secular reduction* of the velocity of light, as explained by Whitrow.

5. THE FIRST STEADY STATE MODEL

Passing from Milne's world model to that of Gold & Bondi - and of Hoyle - the expansion function $\mathcal{S}(\tau)$ is changed from τ to e^{τ}, to which all "steady state" models must approximate. So:

(13) $$\underline{\mathcal{S}(\tau) \equiv e^{\tau} \equiv d\tau/\mathrm{dT} \equiv \tfrac{1}{1-\mathrm{T}} \equiv \mathcal{C}^{-1}(\mathrm{T})}$$

$\mathcal{R} \equiv e^{\tau}\rho \equiv tanh\, r$ is a candidate to the proper distance between fundamental particles, just as $e^{t-\tau} \equiv 1/\sqrt{1-\mathcal{R}^2} = cosh\, r$ is a plausible relationship of frame time t to proper time τ. Hence, if Bondi & Gold, and Hoyle, want to retain $d\tau^2 - e^{2\tau}d\rho^2$ as a fundamental invariant of their model, in face of the definitions $e^{t-\tau} \equiv coshr$ & $e^{\tau}\rho \equiv tanhr$, they have to accept:

(14) $$\underline{d\tau = dt - tanhr\, dr} \;\; \Downarrow \;\; \underline{e^{\tau}d\rho = dr - tanhr\, dt}$$
$$d\mathcal{T}^2 \;\equiv\; \tfrac{dt^2-dr^2}{cosh^2 r} = \underline{d\tau^2 - e^{2\tau}d\rho^2} = \tfrac{\mathrm{dT}^2 - d\rho^2}{(1-\mathrm{T})^2}$$

Generalizing these to (1x3) dimensional time-space we find the following three metrics, of which the first one is closest to represent the private 3-spaces of the standard frames of ***SR***, whereas the second comprises the public flat 3-space of a universe expanding with $\mathcal{S}(\tau) = e^{\tau}$ and the third encompasses the public flat 3-space of atoms shrinking in step with $\mathcal{C}(\mathrm{T}) = 1-\mathrm{T}$:

(15a) $$\underline{d\mathcal{T}^2 \;=\; [\,dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}]\tfrac{1}{cosh^2 r}}$$

(15b) $$\underline{=\; d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\} =}$$

(15c) $$\underline{= [\,\mathrm{dT}^2 - \{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\tfrac{1}{(1-\mathrm{T})^2}}$$

$d\mathcal{T}^2 = [\,dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}]/cosh^2 r$ does not compete well with the standard invariant of ***SR*** which is the much simpler one $d\mathcal{T}^2 = dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2)$. This rather serious problem is caused by the external factor $cosh^{-2} r$ which is much less akin to the ***LT*** of ***SR*** than to the Voigt transformations ***(VT)*** of some competing ether theory.

Maybe, when evidence accumulates, we shall have to recur to the ether hypothesis again. However, since neither ***LT*** nor ***SR*** have yet been finally disproved, and since the "steady state" assumption does not necessarily exclude the conjecture that the universe is of finite age and originated in a "big bang", it is worth while to search for alternative "steady state" models that are not at variance with $d\mathcal{T}^2 = dt^2 - dr^2$. As a step on the way I shall propose the model below which follows from the first line by means of the definitions stated first in this section:

(16a) $$\underline{d\mathcal{T}^2 \;=\; dt^2 - \{dr^2 + sinh^2 r(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

(16b) $$\underline{= [\,d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\tfrac{1}{1-e^{2\tau}\rho^2}}$$

(16c) $$\underline{= \tfrac{1}{(1-\mathrm{T})^2}[\,\mathrm{dT}^2 - \{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\tfrac{1}{1-\rho^2/(1-\mathrm{T})^2}}$$

This, at the very least, is compatible with the standard invariant $d\mathcal{T}^2 = dt^2 - dr^2$ of ***SR***. However, when interpreted by means of $e^{t-\tau} \equiv cosh\, r$, $e^{\tau}\rho \equiv tanh\, r$, the model seems flawed: τ is not a genuine cosmic time, since the external contraction factor $\frac{1}{1-e^{2\tau}\rho^2}$ also applies to $d\tau$.

This clearly shows that the model does not conform to the standard ***RWM***.

6. A NEW STEADY STATE MODEL

Let us start from scratch by adopting $d\mathcal{T}^2 = dt^2 - dr^2$ of ***SR***. However, as we need not accept that standard frames are flat, we are free to assume that the world is better described, when referring to frame time t, by taking the geometry of frames, or 3-spaces, to be hyperbolic:

(17) $$\underline{d\mathcal{T}^2 \;\equiv\; dt^2 - \{dr^2 + sinh^2 r\,(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$ (**1**st metric)

Now the shortcoming alluded to in §5 can be remedied by adopting the following definitions:

(18) $$\underline{\rho \equiv sinh\, r\, e^{-t} \equiv tanh\, r\, e^{-\mathrm{t}} \equiv 2\, tanh\tfrac{r}{2}\, e^{-\tau} \equiv [2] R\, e^{-\tau}}$$

Please, notice that we are free to choose $R \equiv tanh\frac{r}{2} \to 1$ or $R \equiv 2\, tanh\frac{r}{2} \to 2$

From the above definitions we immediately derive the following relationships:

(19) $$e^t d\rho = dr\, cosh\, r - dt\, sinh r = (dr - d\mathrm{t}\, tanh r)\, cosh^{-1} r = dr - d\tau\, sinh\, r$$

From these we further obtain $d\tau = dt - dr\, tanh\frac{r}{2} = \mathrm{dt}\,(1 - tanh^2 r) + dr\, tanh\, r\, tanh\frac{r}{2}$; then, for $d\rho = 0$ (*fundamental observers*) we get $v \equiv dr/dt \equiv tanh r$, $w \equiv dr/d\tau \equiv sinh r$, whence:

(20) $$d\tau = dt - dr\, \frac{1 - \sqrt{1 - tanh^2 r}}{tanh\, r} = dt / cosh\, r = dt/\gamma_r$$

Applying $\rho \equiv sinh\, r\, e^{-t} \equiv tanh\, r\, e^{-\mathrm{t}}$ to our **1**st metric, we obtain the steady-state like metric:

(21) $$\underline{d\mathcal{T}^2 = [\mathrm{dt}^2 - e^{2\mathrm{t}}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\frac{1}{1 - e^{2\mathrm{t}}\rho^2}}$$ (**2**nd metric)

Applying $e^{\mathrm{t}} \equiv \mathrm{dt}/d\mathrm{T} \equiv \frac{1}{1-\mathrm{T}}$ to the **2**nd, we get a metric for a static universe of shrinking atoms:

(22) $$\underline{d\mathcal{T}^2 = [\mathrm{dT}^2 - \{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\frac{1}{(1-\mathrm{T})^2 - \rho^2}}$$ (**3**rd metric)

The ***SR***-like metric $d\mathcal{T}^2 = dt^2 - dr^2 - sinh^2 r(..)$ is thus changed into two other metrics: the **2**nd depicting a universe in exponential expansion, and the 3rd depicting the same universe as stationary, but with shrinking atoms; however, none of these metrics is conformal to ***RWM***. Applying $e^{\tau - \mathrm{t}} = 1 + tanh^2\frac{r}{2} = 1 + \frac{1}{4}e^{2\tau}\rho^2$ to our **2**nd metric, we derive this non-***RWM***:

(23) $$\underline{d\mathcal{T}^2 = [d\tau^2\{1 - \tfrac{1}{4}e^{2\tau}\rho^2 - \tfrac{1}{2}e^{2\tau}\rho\tfrac{d\rho}{d\tau}\}^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\}]\,\frac{1}{(1 - \frac{1}{4}e^{2\tau}\rho^2)^2}}$$ (**4**th m)

This metric holds for fundamental particles only. Noticing that eq. (23), for $d\theta = d\phi = 0$, is reducible to the identity $d\mathcal{T} = d\tau$, both in the case of $d\rho = 0$, or $dR \propto R$, following a single fundamental particle on its course outwards in the line of sight, and in the case of $d\rho = -\rho\, d\tau$, or $dR = 0$, following a series of fundamental particles passing an imaginary border $R =$ *const.*, we have tested our basic assumption, viz. that the master clocks of fundamental particles always keep the same cosmic rhythm, $d\mathcal{T} =$ *invar*. Postulating $d\mathcal{T} = d\tau$ in general, eq. (23) becomes:

(24) $$1 = [\{1 - \tfrac{1}{4}e^{2\tau}\rho^2 - \tfrac{1}{2}e^{2\tau}\rho\tfrac{d\rho}{d\tau}\}^2 - e^{2\tau}\{(\tfrac{d\rho}{d\tau})^2 + \rho^2((\tfrac{d\theta}{d\tau})^2 + sin^2\theta(\tfrac{d\phi}{d\tau})^2)\}]\,\frac{1}{(1 - \frac{1}{4}e^{2\tau}\rho^2)^2}$$

which is clearly the equation, expressed with τ, ρ, θ, ϕ, for an invariant *cosmic hyperbola*.

So τ is a genuine ***cosmic time***, although eq. (23) does not conform to the standard ***RWM***. In line with Selleri [1997f.], we finally suggest that the proper transformations to be used locally for the inertial motion of an accidental particle relative to a fundamental observer are:

(25) $$\underline{r' = \gamma\,(r - v\,\bar{\tau})\;.\; \mathcal{T} = \tau = \bar{\tau}\,\gamma = \bar{\tau}'\gamma'\;.\; r = \gamma'(r' - v'\bar{\tau}')}$$

7. CONCLUSION

Any world model with an ***RWM*** type of metric can be described in three different ways: The **1**st description is based on the idea that the average light speed is a universal constant. We claim the metric common to all observers, fundamental or accidental, to be reducible to:

$$d\mathcal{T}^2 = dt^2 - dr^2 \text{ for } d\theta = d\phi = 0$$

However, this is not the case for that of the first "steady state" model; its metric is not ***SR***-like. The safest way to ensure that the claim is fulfilled is, of course, to start with the ***SR*** metric.

As pointed out, this timespace is *private* as a consequence of the retardation of clocks and the contraction of rods or **-** with a single expression **-** the relativization of our metrical units.

The ***SR***-like metric in itself says nothing about cosmic expansion or atomic contraction. In spite of Whitrow's claim to have derived ***RWM*** from the relativistic γ-factor which bears an affinity to an ***SR***-like metric, it is hard to see more than a mere analogy between these two:

(26) $$\underline{d\mathcal{T}^2 = \gamma^{-2}dt^2 = dt^2 - dr^2}$$

(27) $$\underline{d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\, d\sigma^2 = \mathcal{C}^{-2}(\mathrm{T})\{\mathrm{dT}^2 - d\sigma^2\}}$$

Further, if the former applies to accidental and fundamental observers without distinction while the latter represents the structure of an expanding substratum of fundamental observers, we shall obviously need some rules of interpretation which can take us from the former to the latter by explicitly narrowing the perspective, thus giving privilege to fundamental observers. Our presentation of the "big bang" model of Milne & Prokhovnik, of the "steady state" model of Bondi & Gold, and of our own alternative to the latter, offers precisely such rules.

The **2**nd and **3**rd ways of description treat the universe as a spatial totality unfolding in a common world time, thereby invoking the idea of *public timespace*:

a) According to the **2**nd way, the spatial extension of the substratum is assumed to expand relative to the extension of its material content which is determined by the structure of its atoms, i.e., distances between fundamental observers are increasing relative to their internal structure. So the proper distance between two fundamental observers is given by $\mathcal{R} \equiv \mathcal{S}(\mathcal{T})\sigma$, where $\mathcal{S}$ is the scale function, $\mathcal{T}$ is cosmic time, and σ is a fixed coordinate of a fundamental observer.

b) According to the **3**rd way, the spatial extension of the substratum is taken to be stationary whereas the dimensions of its contents contract secularly in pace with the reduction of atoms, i.e., the inner structure of fundamental observers is shrinking relative to their proper distances. This shrinking takes place in step with $\mathcal{C}^{-1}(\mathrm{T})$ where $\mathcal{C}$ is a contraction function, the inverse of the expansion function $\mathcal{S}$, T being an auxiliary time scale defined by: $\mathrm{T} \equiv \int d\tau/\mathcal{S}(\tau) + const.$

But common to all possible descriptions is that our invariant $d\mathcal{T}$ of *time-space* becomes the universal element of a ***cosmic super-time***, as suggested by my mentor André Mercier.

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