REFORMULATION OF COULOMB AND NEWTON FORCES, EXERTED BY A SOURCE CHARGE, ON A MOVING TEST CHARGE

PART II: NEWTON FORCE EXERTED BY A CELESTIAL BODY, ON A MOVING MASS, LEADING TO ALL END RESULTS OF THE GTR

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ABSTRACT

Previously we have shown that Coulomb Force, or Newton Gravitational Attraction Force *(or in short, Newton Force)* is imposed by the Special Theory of Relativity (STR), though for static *(electric or gravitational)* charges, exclusively. Thus, these forces *(for static charges)* happen to be deep-seated laws of nature. This unfortunately happens to be something totally overlooked chiefly for Newton Force. On the other hand, we have no sign that either force will work, for a moving *test charge,* the source charge being still at rest. In this article we are going to show that, although it remains a must to adopt Newton law, for static masses, as imposed by the STR, still to presume the validity of it, for a *moving test charge,* violates the *relativistic law of conservation of energy.* Thus, herein, we are going to provide the correct expression for Newton Gravitational Attraction Force exerted by a *source charge,* at rest, on a *moving test charge,* which right away leads to all measurable end results of the General Theory of Relativity.

In a previous work, a whole new approach to the derivation of the Celestial Equation of Motion was achieved; this, well led to all crucial end results of the General Theory of Relativity (GTR) *(in a few lines only, and already in an integrated form).*[1,2]

OUR POSTULATE: NOTHING ELSE BUT THE RELATIVISTIC LAW OF CONSERVATION OF ENERGY

We had started with the following postulate, essentially, nothing else but the *"relativistic law of conservation of energy",* thus embodying in the broader sense the *mass & energy equivalence* of the Special Theory of Relativity (STR).[1]

Postulate: The rest energy of an object bound to a celestial body amounts to less than its rest energy measured in empty space, the difference being, as much as the static binding energy coming into play.

The corresponding mass deficiency conversely, via *relativistic,* or *non-relativistic* quantum mechanics *(which ever is appropriate to describe the internal dynamics of the object),* yields the stretching of the size of the object, as well as the weakening of its internal energy, based on a quantum mechanical theorem we proven elsewhere[3,4] *(and which will be reproduced below),* all this, again in full conformity with the relativistic law of conservation of energy.

An easy way to grasp our assertion, is the following. If the given rest energy of the object of concern, is decreased due to binding, then its internal dynamics will necessarily slow down. Thus, any light coming from it, will be fainter, thence the *gravitational red shift.*

Now, if when bound, the object somehow, annihilates, the energy of the resulting electromagnetic radiation will naturally be less than that we would tap from the annihilation of the object in free space. This means *(when the object is bound),* an annihilation radiation with a lower frequency than usual. We again land at the *gravitational red shift.* But if so, the corresponding wavelength must have stretched; thus, a clue that accounts for the corresponding stretching of the size. *(About this, see further our quantum mechanical theorem, restated below.)*

Herein, for simplicity but without any loss of generality, we assume that the binding object is infinitely more massive than the bound object.

BINDING ENERGY AND REST MASS DECREASE

In order to calculate the binding energy of concern, we make use of the *classical Newtonian gravitational attraction law,* yet with the restriction that, it can only be considered for static masses. In a previous article,[5] luckily, we have shown that Newton Gravitational Attraction Force, or in short, Newton force reigning between two static masses m and $\mathcal{M}$, is a *must* imposed by the STR. In short, we have proven that

i) the product $Gm\mathcal{M}$, G being the *gravitational constant,* is Lorentz invariant, and this, as a requirement imposed by the Galilean Principle of Relativity *(which is the underlying principle of the STR),*

ii) then, assuming a spatial dependency for the Newton attraction force between m and $\mathcal{M}$, sitting at rest, at a distance r from each other, in the form of $1/r^n$, n being a priori unknown, the STR imposes that n must be strictly 2, in order for the dipole made of m and $\mathcal{M}$, to cope with the end results of the STR, were this dipole brought to a uniform translational motion.

Briefly, the *Newton force reigning between two static masses* is a requirement shaped by the STR. This of course makes that, this law is a *deep-seated law of nature.* This is unfortunately something totally overlooked. Henceforth, one does not require the *"principle of equivalence"* assumed by the GTR, as a precept, in order to predict the end results of this theory.

Let then m_0 be the *rest mass* of the object in consideration, at infinity. Its *rest relativistic energy* is m_0c^2. When this object is bound at rest, to a celestial body of mass $\mathcal{M}$ *(assumed, without though any loss of generality, to be much more massive than the object getting bound to it),* the rest energy of the bound object, will be diminished as much as the binding energy coming into play, to become $m(r)c^2$, so that[1,2]

$$m(r) = m_0 e^{-\alpha(r)} , \qquad (1)$$

(mass of the bound object at rest)

where $\alpha(r)$ is

$$\alpha(r) = \frac{G\mathcal{M}}{rc_0^2} \; ; \tag{2}$$

r is the distance of m(r) to the center of $\mathcal{M}$, as assessed by the distant observer.

Note that, as Eq.(1) delineates, the present theory excludes singularities, thus *black holes.* On the other hand, as easy as it was to get to this equation, it still embodies the very interesting property, known to us, as the gravitational red shift, originally drawn by the GTR,[6,7] but in our approach, through solely quantum mechanics.

RED SHIFT, YET CONCLUDED QUANTUM MECHANICALLY

Thus Eq.(1), well means that the object in consideration will delineate a *gravitational red shift,* but in our case, owing to our general quantum mechanical theorem, we paraphrase below.[2.3]

Theorem: Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description is to point to an *internal dynamics* which consists in a *"clock motion",* achieved in a *"clock space",* along with a *"unit period of time".* The description excludes *"synthetic potential energies", which may otherwise lead to incompatibilities with the Special Theory of Relativity.* The object is supposed to embody $\mathcal{K}$ particles, altogether. If then, different masses m_{k0}, k = 1, …, $\mathcal{K}$, involved by this description of the object at rest, are all multiplied by the arbitrary number γ , the following two general results are conjointly obtained: a) The *eigenvalue* or the same, the *total energy* E_0 associated with the given clock's motion of the object, is increased as much, or the same, the unit period of time T_0, of the motion associated with this energy, is decreased as much. b) The *characteristic length,* or the *size* $\mathcal{R}_0$ to be associated with the given clock's motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0},\, k = 1, \ldots, \mathcal{K}) \rightarrow (\gamma\, m_{k0},\, k = 1, \ldots, \mathcal{K})] \Rightarrow [(E_0 \rightarrow \gamma E_0) \text{ or } (T_0 \rightarrow \frac{T_0}{\gamma}), \text{ and } (\mathcal{R}_0 \rightarrow \frac{\mathcal{R}_0}{\gamma})].$$

Thence, a *rest mass decrease* due to binding, well leads to an overall *energy decrease* of the object, thus an energy decrease of any light coming from the object, which is nothing else, but the gravitational red shift, were the object embedded in a gravitational field, and accordingly its rest mass is decreased following Eq.(1). Our major aim, over here was not really to prove this, but this occurred something surely interesting we wanted to report on our way toward the derivation of Newton's force for a moving test mass.

MODIFIED NEWTON'S LAW OF GRAVITATIONAL ATTRACTION OR THE SAME, NEW GRAVITATIONAL EQUATION OF MOTION

Now suppose that the object of concern is in a given motion around $\mathcal{M}$; the motion in question, can be conceived as made of two consecutive steps:

i) Bring the object quasistatically, from infinity to a given location r, on its orbit, but keep it still at rest.

ii) Deliver to the object at the given location, its motion on the given orbit.

The first step yields a decrease in the rest mass of m_0 as delineated by Eq.(1). The second step yields the Lorentz dilation of the rest mass m(r) at r [as expressed by Eq.(1)] , so that the *overall mass* $m_\gamma(r)$, or the same, the *total relativistic energy* of the object in orbit becomes[1,2]

$$m_\gamma(r)c_0^2 = \frac{m(r)c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_0 c_0^2 \frac{e^{-\alpha(r)}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \; ; \tag{3}$$

(overall relativistic energy, or the same, total energy of the bound object, on the given orbit)

v_0 is the local tangential velocity of the object at r.

Here, the reason for which we refer to the distance r, as measured by the distant observer, is that, we consider the Newton's law of gravitational attraction, specifically in that frame, along with the gravitational constant G, still being considered in this frame.

Supposing that the object is not influenced by any other effect in space, its total energy in a given orbit [i.e. $m_\gamma(r)c_0^2$] must remain constant, so that for the motion in consideration, one finally has

$$m_\gamma(r)c_0^2 = m_0 c_0^2 \frac{e^{-\alpha(r)}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} = m_\gamma c_0^2 \quad . \tag{4}$$

(total energy written by the author, for the object in motion around the sun)

This is in fact, the integrated form of the general equation of motion, we will furnish right below; it is interesting to note that we have arrived at it, in just *two lines* [i.e. Eqs. (1) and (4)]. Note that this equation is the same as the one* furnished by the GTR,[8] up to a third order of Taylor expansion. But as seen clearly, we tapped it in an incomparably easier way.

The differentiation of the above equation leads to

$$-\frac{G\mathcal{M}}{r^2}\left(1-\frac{v_0^2}{c_0^2}\right)dr = v_0 dv_0 \; . \tag{5-a}$$

[differential form of Eq.(4), *leading to the equation of motion]*

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* The GTR [Eq.(88.9) of Reference 8], more precisely furnishes

$$m_\gamma(r)c_0^2 = m_0 c_0^2 \frac{\sqrt{1-2\alpha(r)}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} \quad . \tag{i}$$

This equation can be put into the form[1]

$$-\frac{G\mathcal{M}}{r^2} m_0 e^{-\alpha_0} \sqrt{1-\frac{v_0^2}{c_0^2}}\,\frac{\underline{r}_0}{r_0} = m_\gamma \frac{d\underline{r}_0^2(t_0)}{dt_0^2} \ , \qquad \text{(5-b)}$$

or, in the form[2]

$$-\frac{G\mathcal{M}}{r_0^2}\left(\frac{e^{-\alpha_0}}{1+\alpha_0}\right)\left(1-\frac{v_0^2}{c_0^2}\right)\frac{\underline{r}_0}{r_0} = \frac{d\underline{r}_0^2(t_0)}{dt_0^2} \ , \qquad \text{(5-c)}$$

[vectorial equation written based on Eq.(5-a), *or the same, equation of motion written by the author, via the energy conservation law, extended to cover the relativistic "mass & energy equivalence"]*

written in terms of the *"proper quantities"* only, via the relationship

$$r = r_0 e^{\alpha} \cong r_0 e^{\alpha_0} \ , \qquad \text{(5-d)}$$

(length stretched in the gravitational field)

as induced by Eq.(1), and the above theorem; $\underline{r}_0$ is the vector bearing the magnitude r_0, and directed outward.

It should be recalled that, though consisting in a totally different set up, than that of the GTR, Eq.(4), thus Eq.(5-a) and (5-c), amazingly yields results identical to those of this theory, within the frame of the second order of the corresponding Taylor expansions, yet in an incomparably easier, but still rigorous manner.[2]

Notice that, Eq.(5-b) is the same relationship as that proposed by Newton, except that the gravitational force intensity is now modified by the factor $e^{-\alpha_0}\sqrt{1-v_0^2/c_0^2}$.

DISCUSSION

We find it remarkable that within the frame of our approach, we are able to treat a *"light photon"* just like any other particle. Thus Eqs. (4), (5-a), (5-b) and (5-c) are perfectly valid for a photon, just like any object affected by the gravitational field.

This has two major consequences:

1) The first one is that the photon as small as this may be, seems to bear a kernel; de Broglie speculated on this point in his doctorate thesis, and calculated the *"rest mass of the photon"* to be 10^{-44} grams, if an electromagnetic wave of wavelength of 1 kilometer, moves with a speed, 1.01 times larger than an electromagnetic wave of wavelength of 30 kilometers.[9]

2) Secondly, there is at any rate, a ceiling to the propagation of the velocity of light in *"empty space"*, yet this is, the speed of a photon of practically *"infinite energy"*. This is the *"Lorentz invariant velocity of light"* c_0, in vacuo, which enters our equations.

At this stage it seems useful to draw the following table displaying the differences between our approach and the standard approach.

Table 1 Differences Between the *"Standard Approach"* and *"Present Approach"*, Based on the Pair of Sun and Planet

	Standard Approach	**Present Approach**
Force Between the Sun and the Planet, Altogether at Rest	$G\frac{\mathcal{M}m_0}{r^2}$	$G\frac{\mathcal{M}m_0 e^{-\alpha}}{r^2}$ *(as assessed, by the distant observer)*
Total Energy of the Statically Bound Planet	$m(r_0)c_0^2 = m_0c_0^2 - \frac{G\mathcal{M}}{r}$ *(Newtonian Approach)* $m(r_0)c_0^2 = m_0c_0^2\sqrt{1-2\alpha}$ *(Einsteinian Approach)*	$m(r_0)c_0^2 = m_0c_0^2e^{-\alpha}$
Total Dynamic Energy	Rest Energy + Potential Energy + Kinetic Energy *(Newtonian Approach)*	The concept of *"potential energy"*, as considered classically, is misleading.
Total Dynamic Energy of the Sun and the Planet in Motion	$m_\gamma(r_0)c_0^2 = m_0c_0^2\frac{\sqrt{1-2\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}$ *(The Einsteinian total energy)*	$m_\gamma(r_0)c_0^2 = m_0c_0^2\frac{e^{-\alpha}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}$
Force Between the Sun and the Moving Planet	$G\frac{\mathcal{M}m_0}{r^2}$ *(Newtonian Approach)*	$\frac{G\mathcal{M}}{r^2}m_0e^{-\alpha_0}\sqrt{1-\frac{v_0^2}{c_0^2}}$

Note that, our Newtonian force appears to be decreased [cf. Eq.(5-b)]. Yet one, by the same token can well think, in the following way. The modification of the force strength, via dividing both sides of Eq.(5-b), by the inverse of the Lorentz factor, comes to the fact that, now, with the Classical Newton Force strngth, one has a *stronger equivalent acceleration.*

This is actually, according to the present theory, what is responsible of the precession of Mercury's perihelion, nearby the Sun. Along the same line, this is also what makes bend the light nearby the sun, practically as much as twice that the bending angle predicted by the classical Newtonian approach. All that is presented in Reference 2.

Note how ever that, although our metric too changes, through the above quantum mechanical theorem, in the vicinity of the gravitating source, we have a whole different setup than that of the GTR, and accordingly a different metric. Furthermore, we do not have any curvature.[2]

Note further that, we arrived to solve the two-body problem as well,[10,11] since our approach did not have to be restricted to just one gravitating source, and we can well consider many gravitating sources, something which the GTR, seemingly, cannot.

Our approach furthermore can well lead to the quantization of the gravitational field, otherwise hindered by the GTR [12,13]

CONCLUSION

Our primary aim here was not really to present an alternative theory of gravitation to the GTR. It was simply to prove how the Newton's law of gravitation, in short *Newton Force,* is to be altered, if the *test mass* is in motion. Once we did it though, it became somehow necessary to provide evidences, to cross check the modification we brought to Newton's force. In effect, it was almost straightforward to get to the end results of the GTR, right after we knew how to write Newton Force for a moving test mass, and this solely based on the *relativistic law of conservation of energy.*

In any case, in a previous work we have shown that the Classical Newton Force, though for static gravitational charges, exclusively, is imposed by the STR.[5]

If so, Newton Force, for static masses, must be a *deep-seated* law of nature. This is unfortunately something totally overlooked. Thus, we have well, to recognize this force at the very fundamental level it deserves to belong to. Herein, we had to elaborate though, about what happens, if one of the masses in question moves as referred to the other.

Thus we have shown that, to presume the validity of Classical Newton Force for a moving test mass, violates the relativistic law of conservation of energy.

Accordingly, we have provided the correct expression for Newton Force exerted by a source mass, at rest, on a moving test mass. It is that, the usual Newton Force in the latter case, is modified by the inverse of the Lorentz coefficient coming into play. We could check the validity of our disclosure, via getting based on it, quickly, to all measurable end results of the GTR.

Our approach furthermore, removes the blockade toward a unification of fields, since we do not make use of the *principle of equivalence* of the GTR, and accordingly, on the contrary to the GTR, our approach is not restricted to gravitation, at all.

This makes that a clock bound to *(not only a gravitational field, but),* any field it can interact with, will retard, just like it retards in a gravitational field. The bound muon decay rate retardation[14] may be considered as an experimental proof of this prediction.

Our approach furthermore can well lead to the quantization of the gravitational field, otherwise hindered by the GTR.[12,13]

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