

The Long History of the Mass-Energy Relation

by

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Poincaré [1], Kaufmann [2], Abraham [3],[4] und Hasenöhl [5] have shown already before 1905 that the mass of electrons increases during acceleration. The mass increase can easily be derived for the case the energy is transferred through an electromagnetic wave by the help of the Poynting – vector [3][4] as shown below:

$$\frac{dW}{dt} = \int (\vec{E} \times \vec{H}) \cdot d\vec{A} = \frac{1}{\varepsilon\mu} \int (\vec{D} \times \vec{B}) \cdot d\vec{A} = c^2 \int (\vec{D} \times \vec{B}) \cdot d\vec{A} = c^2 \cdot \frac{dm}{dt} \quad (1)$$

m is the mass equivalent of free radiation [4][5]. According to equ. (1), the Poynting-vector \vec{S} , i.e., the flux of radiation, is related to the velocity of light, c , via the expression

$$\vec{E} \times \vec{H} = \vec{S} = \frac{1}{\varepsilon\mu} (\vec{D} \times \vec{B}) = c^2 (\vec{D} \times \vec{B}) \quad (2)$$

The expressions \vec{S}/c and \vec{S}/c^2 have been introduced by Poincaré [1] already in 1900 and by Abraham [4] in 1904 as *the energy density and the momentum of radiation (units: kg/m²s), respectively*. The energy-mass relation, $W = mc^2$, can be derived for the case that the radiation is fully absorbed by a metal plate of cross section A and Mass M . Using equ. (1) the energy of the plate increases by the amount $dW = c^2 dm$, and this leads to an increase of both, velocity and mass of the plate according to the following equation

$$dW = c^2 dm = \frac{d}{dt} (M \cdot \vec{v}) \cdot d\vec{s} = d(M \cdot \vec{v}) \cdot \vec{v} = M \cdot \vec{v} \cdot d\vec{v} + v^2 \cdot dM \quad (3)$$

For the case of total absorption, the mass increase of the plate, dM , must be equal to dm . Using $dM = dm$ and $M = M_o$ for $v = 0$ one obtains by solving equ. (3) the well-known expression for the velocity dependent mass:

$$M = \frac{M_o}{\sqrt{1 - (v/c)^2}} \quad (4)$$

Substituting equ.(4) into equ. (3) yields for the energy transfer dW the expression:

$$dW = c^2 dm = c^2 dM = d \left(\frac{M_o c^2}{\sqrt{1 - (v/c)^2}} \right) \quad (5)$$

And, integrating equ. (5) yields the law of equivalence of energy and mass:

$$W = M c^2 = \frac{M_o c^2}{\sqrt{1 - (v/c)^2}} = M_o c^2 + \frac{M_o v^2}{2} + \frac{3M_o v^4}{8c^2} + \dots \quad (6)$$

In this derivation thermodynamic aspects had been excluded. However, Hasenöhl [5] had pointed out that the internal energy of a body must consist in part of radiation and, hence, the mass of a body will depend in general on its temperature. By studying the problem of a hollow enclosure filled with radiation he calculated similar expressions

$$W = \frac{3}{8} M c^2 \quad \text{and, after some corrections:} \quad W = \frac{3}{4} M c^2 \quad (7)$$

The expressions (6) and (7) indicate that both energy and mass can be transferred via electromagnetic waves and that electromagnetic waves exhibit inertia.

Another equally important conclusion drawn from Hasenöhl's derivation [5] is that the velocity v appearing in the equations (3) through (6) is an absolute rather than a relative velocity and that one has to calculate mass increases by starting off from **an absolute frame of reference** ("ether").

Einstein's 1905 derivation [6] of the energy-mass relation which also neglected thermodynamical effects had been criticized by Planck [7] in 1907 for an un-permissible approximation and for using the principle of relativity, and by Ives [8] in 1952 because of circularity in the derivation. In 1987 Fadner [9] repeated this critique (Fadner's article suffers from not referring to the important work of Abraham [4]). Anyway, Einstein [6] based his derivation on the principle of relativity by using the Lorentz transforms where the square root of $1-v^2/c^2$ also appears. This procedure yielded another paradox of special relativity in addition to the clock paradox. Einstein's derivation suffers from the fact that mass increase depends on the observer platform and does not consider energy transfer. At this point it is important to emphasize that the Lorentz – transforms have been derived by postulating that the one-way speed of light is isotropic in all inertial frames of reference, i.e., by [6]:

$$x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad (8)$$

However, Penzias and Wilson [10], Smoot et al. [11] and Marinov [12], have falsified one of the equations (8) experimentally. Finally, it was Max Planck [7] who presented the first valid and authentic derivation of the mass energy relation [8], by using the argument that the entropy of a body cannot depend on the choice of the platform and by using the principle of the smallest action instead of the principle of relativity. By doing this he found that the inertial mass also depends on temperature.

The experiments carried out by Penzias und Wilson [10] and Smoot et al. [11], [13] indicate that for properly calculating the masses of the universe a fundamental frame of reference (the former ether, now the cosmic microwave background) should be used. Very recently, T. A. Jacobson and R. Parentani [14] wrote an article (in the December 2005 issue of the Scientific American) showing that by assuming „*the ether of pre-Einsteinian physics*“ many puzzles of black holes can be solved much more satisfactorily than without making this assumption.

The correct explanation of the Michelson/Morley null result [15] is obtained by using phase velocities rather than group velocities for calculating the phase shifts in the two interferometer arms as shown in [16], [17] and in the Appendix. This leads to the conclusion that the Michelson/Morley-interferometer is, in principle, not suited to detect an ether. The authors [15] had obviously been aware of this fact as they suggested in their famous paper in 1887 to measure the velocity of light without returning the ray to the light source, i. e., to perform a

one-way light velocity measurement with mechanical synchronisation of the clocks. Marinov [12] actually did that in his “coupled mirrors experiment” thereby detecting the anisotropy of the light velocity on earth, or, in other words, thereby detecting the cosmic microwave background previously measured by Smoot et al. [13] by another method. Marinov [12] thus carried out the first experimental falsification of special relativity in 1975.

Conclusion

Since all of the derivations of $W=mc^2$ known up to date are more or less incomplete the discovery of the mass-energy-relation should not be ascribed to a single person but rather to a group of scientists including Poincaré, Abraham, Hasenöhlrl, Kaufmann, Einstein und Planck. This new view should be adopted by all Physical Societies.

References

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Appendix

Michelson – Interferometer

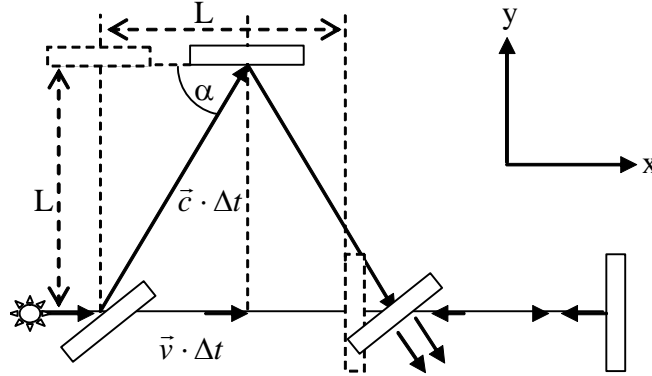


Fig. 1 The Michelson-Morley-Interferometer Experiment

Wave in rest frame S: $\psi = \sin(\omega t - \vec{k} \cdot \vec{r})$ (9)

Phase velocity: $\vec{c} = \vec{k} \cdot \omega / k^2$ (10) $c^2 = \omega^2 / k^2$ (11)

Galilei-Transformation: $\vec{r}' = \vec{r} - \vec{v} \cdot t$ (12)

Wave in moving frame S': $\psi' = \sin[(\omega t - \vec{k} \cdot (\vec{r}' + \vec{v}t)] = \sin[(\omega - \vec{k} \cdot \vec{v})t - \vec{k} \cdot \vec{r}']$ (13)

Classical Doppler shift: $\omega' = \omega - \vec{k} \cdot \vec{v}$ (14)

Phase velocity in moving frame of reference:

$$\vec{c}' = \vec{k} \cdot \omega' / k^2 = \frac{\vec{k}}{k^2} [\omega - \vec{k} \cdot \vec{v}] = \frac{\vec{k} \omega}{k^2} \left(1 - \frac{\vec{k} \cdot \vec{v}}{\omega}\right) = \vec{c} \left(1 - \frac{\vec{c} \cdot \vec{v}}{c^2}\right) \quad (15)$$

This expression is valid for all angles between the light beam and the direction of motion. The two directions of interest are:

longitudinal arm: $c' = c \left(1 \mp \frac{v}{c}\right)$, (16) back and forth: $c' = c \left(1 - \frac{v^2}{c^2}\right)$ (17)

transverse arm $c' = c \left(1 - \frac{\vec{c} \cdot \vec{v}}{c^2}\right) = c \left(1 - \frac{c \cdot v \cdot \cos \alpha}{c^2}\right) = c \left(1 - \frac{c \cdot v \cdot v / c}{c^2}\right) = c \left(1 - \frac{v^2}{c^2}\right)$ (18)

However, in the textbooks one finds a different value for the transit time in the transverse arm because group velocities have been used for calculating the transit times:

longitudinal: $2 \cdot \Delta t = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L \cdot c}{c^2 - v^2} = \frac{2L}{c(1 - v^2/c^2)}$ (19) identical with equ.(17).

transverse: $2 \cdot \Delta t = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c\sqrt{1 - v^2/c^2}}$ (19) **not identical** with equ.(18) due to the

square root, which has been obtained by employing Pythagoras: $v^2 \Delta t^2 + L^2 = c^2 \Delta t^2$ (20)

Wrong conclusion: there is no ether

Correct conclusion: an ether cannot be detected interferometrically