the general science journal

Revealing the Essence of Planck's Constant

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Abstract

According to traditional classical quantum mechanics theory, due to the prior existence of Planck's constant, considered a universal constant, it is thought that the energy of a photon can be determined if its frequency is known, and the wavelength of a quantum can be determined if its momentum is known ($E = hv$ and $\lambda = h/p$).

In this paper, however, the case is made that logically, since the product of the momentum and wavelength of any photon can be expressed by the constant $p\lambda$, Planck's constant only comes into existence when $p\lambda$ is replaced with h.

In this paper, we show that Planck's constant is not a universal constant but is instead just a usual fundamental physical constant.

Key words: Planck's constant, Universal constants, Fundamental physical constants

PACS codes: 03.65.Sq , 03.65.Ta

1. Introduction

In 1900, when deriving a formula that derived an experimental value of black-body radiation, M. Planck (1858-1947) proposed the quantum hypothesis stating that the energy of a harmonic oscillator with oscillation frequency v would quantize at the integral multiple of hv. This was the first time that Planck's constant h appeared in physics theory [1].

Planck's constant is thus thought to be a fundamental physical constant defined in the realm of quantum theory, but the essence of this constant is generally not well understood.

In this paper, using non-historic reasoning, the true essence of this constant is revealed.

Beforehand, let us verify the following points regarding fundamental physical constants and Plank's constant.

Fundamental physical constants play an essential part in elementary formulas that describe natural phenomena and can be largely divided into universal constants and material constants.

Also, physical quantities and constants are included in fundamental physical constants that belong to one category.

Physical quantities belonging to micro material constants include electron mass m_e, elementary charge e, and electron Compton wavelength λ_C, and include such constants as the fine-structure constant α and the Rydberg constant R_∞.

The Boltzmann constant k and the Avogadro constant N_A are examples of macro material constants.

However, Planck's constant h is thought to be a universal constant representative of quantum mechanics.

Because Plank's constant has an action quantity dimension, it was at first called an action quantum when quantum theory originally emerged. h appears in the inequality $\Delta x \Delta p_x \geqq \mathrm{h}/2$ when W. Heisenberg (1901-1976) discovered the uncertainty principle in 1927.

Planck's constant h, along with the speed of light in vacuum c and the Newtonian constant of gravitation G, also plays an important role when assembling planck units from universal constants.

From the above, Planck's constant is a constant by name, but it has come to be strongly regarded as being the smallest unit of angular momentum.

2. Planck's Constant Derived from Fundamental Physical Constant

Below is Einstein's formula expressing the equality of energy and mass [2].

$$E = mc^2. \qquad (1)$$

Here, m is the mass of a particle and c is the speed of light in a vacuum.

Meanwhile, Einstein's relational expression regarding light quanta is as follows [3].

$$E = h\nu. \qquad (2)$$

The photon's energy E is proportional to its frequency ν, and this constant of proportionality is known as Planck's constant.

Formula (2.1) and Formula (2.2) are traditionally thought to be representative formulas of the special theory of relativity and quantum mechanics, the roots of modern physics, and these two formulas have been thought to have similar importance.

If m_e is the mass of an electron, an electron's mass energy E_0 can be represented by the following formula.

$$E_0 = m_e c^2. \qquad (3)$$

Meanwhile, if ν_C is the frequency of a photon carrying an amount of energy equivalent to E_0, the following is true.

$$E_0 = h\nu_C. \qquad (4)$$

The equation in (2.4) is not based on the assumption that an electron at rest can decay into a single photon, which is in violation of conservation of momentum. The decay cannot occur.

While the energy of naturally existing photons carry a variety of values, this paper happens to use an example of what would happen to the wavelength of a photon if it had the same energy as E_0.

Combining equals from Formulas (2.3) and (2.4), we obtain:

$$m_e c^2 = h\nu_C. \qquad (5)$$

Fundamentally these two types of energy have different characteristics, but from a quantitative perspective, it is possible to combine them as equals.

Thus, a photon's frequency ν_C is expressed as follows.

$$\nu_C = \frac{m_e c^2}{h}. \qquad (6)$$

Next, a photon's wavelength λ becomes:

$$\lambda = \frac{c}{\nu_{\mathrm{C}}} \tag{7a}$$

$$= \frac{h}{m_{\mathrm{e}}c}. \tag{7b}$$

Now, an electron's Compton wavelength λ_{C} is represented by the following formula.

$$\lambda_{\mathrm{C}} = \frac{h}{m_{\mathrm{e}}c}. \tag{8}$$

The wavelength of a photon with energy E_0 is the same as the Compton's wavelength λ_{C} of an electron.

Thus, (2.3) can be transformed as follows.

$$E_0 = m_{\mathrm{e}}c^2 \tag{9a}$$

$$= m_{\mathrm{e}}c\lambda_{\mathrm{C}}\nu_{\mathrm{C}}. \tag{9b}$$

In (2.9b), λ_{C} is the wavelength of a photon, not an electron. However, because the right sides of (2.9a) and (2.5) match, the following relationship holds true in the case of a photon as well.

$$m_{\mathrm{e}}c\lambda_{\mathrm{C}} = h. \tag{10}$$

3. Planck's Constant Derived from the Various Energies of a Photon

The specific energy held by a photon was considered in the previous chapter. This chapter is a more generalized discussion based on a photon having various types of energy.

First, by generalizing (2.5) we obtain the following:

$$mc^2 = h\nu. \tag{1}$$

Here, m is not necessarily the entire mass of the electron. The mass of the electron is being gradually reduced due to the emission of photons, and m corresponds to the reduced mass of that electron. (when $0 < m$)

In other words, (3.1) is saying that the reduction in electron energy is equal to the energy of the emitted photons.

The current reduced mass m is defined as follows.

$$m = \mathrm{a}m_{\mathrm{e}}, \quad \text{(when } 0 < \mathrm{a}\text{)}. \tag{2}$$

The momentum of a photon emitted from the electron at this time is expressed as follows.

$$p = mc \quad (3a)$$

$$= \mathrm{a} m_{\mathrm{e}} c. \quad (3b)$$

Also, since there is an inverse proportional relationship between a photon's momentum and wavelength, the wavelength of this photon is can be expressed by the following formula.

$$\lambda = \frac{\lambda_{\mathrm{C}}}{\mathrm{a}}. \quad (4)$$

Thus, the product $p\lambda$ of an emitted photon's momentum and wavelength is:

$$p\lambda = mc\lambda \quad (5a)$$

$$= (\mathrm{a} m_{\mathrm{e}} c)(\frac{\lambda_{\mathrm{C}}}{\mathrm{a}}) \quad (5b)$$

$$= m_{\mathrm{e}} c \lambda_{\mathrm{C}}. \quad (5c)$$

We can see that ultimately, the product $p\lambda$ of the momentum and wavelength of any photon is the same as the constant $m_{\mathrm{e}}c\lambda_{\mathrm{C}}$. That is,

$$p\lambda = m_{\mathrm{e}} c \lambda_{\mathrm{C}} \quad (6a)$$

$$= h. \quad (6b)$$

Considering (3.6), it is possible to logically derive (2.2) from (2.1).

Thus,

$$E = mc^2 \quad (7a)$$

$$= mc\lambda\nu \quad (7b)$$

$$= h\nu. \quad (7c)$$

4. Discussion

We next substitute the following values for physical quantities in $m_{\mathrm{e}}c\lambda_{\mathrm{C}}$ [4].

$$m_{\mathrm{e}} = 9.10938215 \times 10^{-31} \ \mathrm{kg}. \quad (1)$$

$$c = 2.99792458 \times 10^{8} \ \mathrm{m \cdot s^{-1}}. \quad (2)$$

$$\lambda_{\mathrm{C}} = 2.4263102175 \times 10^{-12} \ \mathrm{m}. \quad (3)$$

By doing so, $m_e c\lambda_C$ becomes as follows.

$$m_e c\lambda_C = 6.62606896\times10^{-34}\ \mathrm{J\cdot s}. \qquad (4)$$

Meanwhile, Planck's constant has the following value [4].

$$h = 6.62606896\times10^{-34}\ \mathrm{J\cdot s}. \qquad (5)$$

$m_e c\lambda_C$ and h are a perfect match.

The currently known values for m_e or λ_C were not determined through experimentation.

m_e was determined through precise calculations from Rydberg constant formulas, and λ_C was obtained by substituting m_e in the formula $\lambda_C=h/m_e c$.

Based on measured data from theoretical formulas or experiments designed to represent the fundamental laws of physics, many fundamental physical constants are being adjusted to avoid conflicts from arising between these constants.

Because the formula to determine an electron's Compton wavelength is $\lambda_C=h/m_e c$, naturally the modified version of this Formula (2.10) is true.

Logically, however, Planck's constant should thought of as a constant that only comes into existence once Formula (2.1) is rewritten into Formula (2.2) to include a photon's frequency, and the subsequent recognition that the non-frequency components $mc\lambda$ form a constant which can be replaced by h.

In other words, (2.10) can be interpreted to mean not "$m_e c\lambda_C$ and h are identical" but instead to mean "$m_e c\lambda_C$ is h."

Therefore, this paper does not claim the discovery of any relationship in (2.10).

Rather than naming this constant as Planck's constant h, we can simply regard it as $m_e c\lambda_C= p\lambda$=const.

However, because this constant has been historically used in others of Planck's research, it has taken on the image of being a discovered universal constant.

5. Conclusion

According to existing theory, Formulas (2.1) and (2.2) have been thought to have similar importance. However, according to our discussion, (2.1) is the more fundamental of the two.

Formula (2.2) is Formula (2.1) rewritten to also include frequency. The right side of (3.7), the product of the physical quantities $mc\lambda$ except for frequency, is a steady value.

Regardless of whether $m_e c\lambda_C$ is called Planck's constant h, in this paper we conclude that Planck's constant h only came into existence once it was defined.

However, not being aware of what should have been defined, this task was skipped, and thus Planck's constant was believed to be a discovered universal constant.

Thus, it is valid to regard Planck's constant not as a universal constant but as a physical constant on par with the fine structure constant α or the Rydberg constant R_∞.

Acknowledgements

I would like to express my thanks to the staff at ACN Translation Services for their translation assistance.

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