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Le Verrier's 1859 Paper on Mercury,

and Possible Reasons for Mercury's Anomalous Precession

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Abstract

Urbain Le Verrier became famous by discovering Neptune from its effect on the orbit of Uranus. He turned his attention to Mercury, publishing a preliminary paper in 1841 and a definitive paper in 1859 on the Theory of Mercury. He discovered an unexplained shift in the perihelion of Mercury, which he attributed to the presence of a small unknown planet he called Vulcan, which was never found. The results were corrected in 1895 by Simon Newcomb, who increased the anomalous shift by about 10% but offered no new explanation for it. Albert Einstein, at the end of his 1916 paper on General Relativity, derived a specific solution for the perihelion shift which exactly matched the discrepancy. Dating from the 1947 Clemence review paper, that explanation and precise value have remained to the present time, being completely accepted by theoretical physicists as absolutely true.

The highly technical 1859 Le Verrier paper was written in French, and astronomers and theorists have gone on from the study of the Solar System to the study of the Universe. The partial translation given here throws light on Le Verrier's analysis and thought processes, and points out that the masses he used for Earth and Mercury are quite different from present day values, possibly leading to a different fit to the old data. A 1924 paper by a professor of Celestial Mechanics critiques both the Einstein and the Le Verrier analyses, and a 1993 paper gives a different and better fit to some of Le Verrier's data. Nonetheless, the effect of errors in planet masses seems to give new condition equations that do not change the perihelion discrepancy by a large amount. The question now is whether or not the excess shift of the perihelion of Mercury has been properly explained in terms of General Relativity, or if there are other reasons for the observations, such as effects from a comet, or the asteroid belt, etc.

Introduction

In his 1912 paper on General Relativity [1], Albert Einstein announced that he had explained the apparent anomalous shift in the perihelion of Mercury, discovered by the French astrophysicist, Urbain Le Verrier [2] in 1859 and corrected slightly by Simon Newcomb [3,4] in 1895. As the accompanying table shows, the total observed precession of 5600 arc seconds per century is made up mostly by a coordinate transformation, whereas the calculation of the effects of other planets remains at roughly the 1895 values, and Einstein's correction is taken as a real effect that exactly explains the difference.

Figure 1 is taken from a 1947 paper by Clemence [5], which is essentially the definitive analysis of all Mercury data taken up to that time. Clemence says, "It is at once evident that the effect can be detected most easily in the motion of Mercury. Indeed, Einstein's announcement of the general theory of relativity in its definitive form was immediately hailed by some astronomers as explaining a previously unaccountable discrepancy between the observed and theoretical motions of this planet. Others were, however, intuitively opposed to relativity, and they directed attention to a small discrepancy yet remaining as evidence that the theory of relativity could not be correct: the relativists contended that the small remaining discrepancy was due to errors either in the observations or in the classical theory of the motion. In justice it should be said that the questions involved are not simple ones, but are complicated by three causes: (1) Observations of Mercury are among the most difficult in positional astronomy. They have to be made in the daytime, near noon, under unfavorable conditions of the atmosphere; and they are subject to large systematic and accidental errors arising both from this cause and from the shape of the visible disk of the planet; (2) The planet's path in Newtonian space is not an ellipse but an exceedingly complicated space-curve due to the disturbing effects of all of the other planets. The calculation of this curve is a difficult and laborious task, and significantly different results have been obtained by different scientists; (3) The observations cannot be made in the Newtonian frame of reference. They are referred to the moving equinox, that is, they are affected by the precession of the equinoxes, and the determination of the precessional motion is one of the most difficult problems of positional astronomy, if not the most difficult. In the light of all these hazards it is not surprising that a difference of opinion could exist regarding the closeness of agreement between the observed and theoretical motions."

Le Verrier became famous by discovering Neptune by its effect on the orbit of Uranus. In analyzing the orbit of Mercury, perhaps he was looking for a similar discovery. Since the original Le Verrier paper is written in French, some of the considerations that led him to his conclusions may have been missed by modern researchers. A partial translation of the paper given below (with some comments in square brackets and a comparison to modern planetary mass data) serves to point out how he did his analysis, and what influenced his conclusions.

Figure 1. Sources of the Precession of Perihelion for Mercury

Amount (arcsec/century)	Cause
5025.6	Coordinate (due to the precession of the equinoxes)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the Sun (quadrupole moment)

42.98±0.04	General relativity
5600.0	Total
5599.7	Observed

The general flavor of Le Verrier's work is a complicated analytical fit to the observational Mercury data over about 50 years. This consisted of deriving, "Condition Equations," containing seven types of terms, some of which are periodic and others time-dependent, plus approximate perturbations of these terms to obtain equations that match the observations with minimal residuals. The final product is a series of Tables predicting when Mercury would be observed in the future. A byproduct of the analysis was the discovery that the contributions of the other planets left an unexplained residual shift of the perihelion by 39 arc-seconds/century.

Le Verrier explored possible reasons for this shift, and concluded that some of the planet masses were possibly incorrect, or that there was an unknown small planet that was the cause. Le Verrier's search for the hypothetical planet Vulcan was unsuccessful, although it is still possible that there is a comet called Vulcan that could have produced the effect.

By comparing the masses Le Verrier used (scaled to the mass of the Sun) with the modern values, it appears that the mass of the Earth he adopted is quite wrong, and the mass used for Mercury is much too large. On the other hand, the other masses seem to be fairly accurate. But it leads to the question, could his unexplained shift be due to these errors?

There is a 1924 critique of Einstein's claim by Charles Lane Poor [4], a professor of Celestial Mechanics, who claims that Einstein's bending of space and time only distorted the problem and did not solve it, saying that his solution actually reduces to the Newtonian solution. Poor also claims that the motion of Mercury is made up of two "wobbles", and Le Verrier could not disentangle them with his approach.

Finally, there is a 1993 reanalysis of some of Le Verrier's data by Takeshi Inoue [5], wherein if a few bad points are omitted, a much better fit is obtained for the excess motion of the ascending node of Mercury's orbit. This suggests that modern computer analysis of old data may lead to quite different results than hand calculations using approximate methods of the past.

None of the above considerations explain the perihelion discrepancy, although they change its value somewhat.

Partial Translation of Le Verrier's 1859 Paper

(From page 19) "The formulas that we assemble here, with few exceptions, are based on the provisory values attributed to various elements in Chapter VII."

"The masses of the planets, in particular, have for their expressions [*which recognizes that they may have to be modified later*]:

Figure 2. Mass Ratios Relative to the Sun

Planet	1859 Planet/Sun	2009 Planet/Sun	Old/New
Mercury, m	3.333E-7	1.66E-7	2.0
Venus, m'	2.4885E-6	2.45E-6	1.015
Earth, m''	2.8174E-6	3.0E-6	0.937
Mars, m'''	3.339E-7	3.09E-7	1.08
Jupiter	9.52381E-4	9.5E-4	1.005
Saturn	2.84738E-4	2.84E-4	1.000

"In comparing the masses of the Earth, Jupiter and Saturn to their volumes, one remarks that the densities of these planets are, to some degree, inverse to their mean distances from the Sun. This rule is not true for Venus and Uranus. With respect to Mercury, in deducing the density of this planet, and thus its mass, we suspect that the accepted radius is too low. One finds then that the mass of the planet should be *a two-millionth* of that of the Sun. I have here reduced this mass to a *three-millionth*, in consideration of the perturbations that have been found by the Comet Encke in its passage on the perihelion in 1838. But, according to Encke, the mass of Mercury should be even smaller. We conclude only that this mass is very small and that it cannot have any sensible influence on the calculation of the major axis of the orbit." [*In fact, the mass used for Mercury is **twice** the currently accepted value, while the **major error is in the mass of the Earth!***]."

"Up to now, in treating the movement of the Sun, we have found that the mass adopted above for Venus has little reason to be modified." [*In fact, the mass used for Venus is reasonably **correct***].

"The mass of Mars is that which we have deduced, in Chapter XIV, Volume IV, on the study of the movements of the Sun. It is smaller than that given in Chapter VII, in the ratio of 0.895 to unity. We have effected this correction, in order to avoid leaving it undetermined, and the theory of Mercury is unable to bring any light on this subject." [*In fact, the mass of Mars should **not** have been corrected*].

(From page 78) "Knowing the values of a and a', one can eliminate $\delta\varepsilon$ between the equations where these quantities are the second members. One is thus led to the relation,

$$2.72\ \delta e + \delta\omega = 10.''27$$

Sensibly one obtains, by elimination of δn between the equations where the second members are equal to b and b',

$$2.72\ e' + \omega' = +0.''392 \quad \text{(per year)}$$

One thus sees that the discussion of the observations of the passages of the planet past the Sun furnishes a precise relation between the eccentricity and the longitude of the perihelion; but only to determine one of these two elements, it shall be indispensible go back to employ the meridian observations."

" The annual movement, $2.72\ e' + \omega' = +0''.392$, should fix in our attention: this quantity being essentially tied to **values admitted for the masses of the planets**. The secular variations of the eccentricity and the perihelion of Mercury have been calculated in attributing to the masses of the planets furnished by considerations outside the theory of Mercury, but which one has reason to believe are strongly exact [*Some of the masses differ from **present** values, especially the Earth!*]. One can thus hope that the discussion of the observations of Mercury simply confirm previous research. Now this is not negligible: we see here that the approximately 3-fold secular movement of the eccentricity, added to the secular movement of the perihelion, gives a sum in which the observations are greater by 39" than those which result from calculation. The part of this sum, due to the action of Venus, is equal to 288" [*The previous given value was 280".6*] given by the calculation based on the solar mass fraction 4.885 E-6: and in consequence, to make the theory agree with the observations of Mercury, **one should augment** the mass used for Venus by about a seventh [14%] of its value!"

(From page 79) "On the other hand, the value adopted for the mass of Venus influences several things. It enters into the expressions for the secular variations for e, ω, θ, e'', and ω'' [*That is, it affects Mercury and Earth*]: it finds itself in the periodic perturbations of the longitude of Mercury and the longitude of the Sun. The preliminary discussion, of which we have written, shows that we cannot confront these actions one at a time."

"The corrections e' and π' of the annual movement of the eccentricity and the perihelion of Mercury are treated as two unknowns immediate and distinct, independent of all considerations of the cause which can render them necessary. We know by now that this applies conveniently to satisfy all the equations. But, thus in virtue of the values thus found for e' and ω', one comes to ask if it is effectively necessary to augment notably the mass of Venus, since it requires us to consider the effect it has on the terms proportional to this mass and which have an effect on the secular variations of θ, e'', and ω'', and therefore in the periodic perturbations of Mercury and the Sun. We thus introduce a term proportional to the correction, υ', of the mass of Venus into the equations."

(From pages 94-96) "This value of υ', deduced from consideration of all the terms that affect the action of Venus, the movement of the perihelion and the eccentricity excepted, is contrary to the supposition that the mass of Venus used up to now should be much too small; at least if one does not diminish the longitude of the node by a quantity that is inadmissible. We shall bind ourselves here to this remark."

"These diverse points being established, the formulas IV, III, II and I, become successively, and with sufficient accuracy, (4 equations). We see whether the meridian observations can be fixed on the two arbitrary quantities, e' and δe', which alone remain undetermined, but which, as we have already remarked, cannot vary more than in very tight limits."

"The equations of condition relative to the longitude can partake in two classes: following the coefficient of correction of the perihelion, devised from the eccentricity, surpassing the coefficient of correction of the eccentricity where it is inferior. The first class of equations fixes first the position of the perihelion: the second class determines principally the magnitude of the eccentricity."

"As it is indispensible to consider the secular movements of the eccentricity and the perihelion, one shall, in the equations written on pages 83 and following, replace δe by δe + e't, and $\delta\omega$ by $\delta\omega$ + ω't. But now, in order to deduce the values of e' and ω', which we want, one devises each of these classes of equations in two groups: one corresponding to the observations effected from 1801 up to 1828, the other from observations done in 1836-1842. Finally, to better judge the actual values of the assignments that we obtain by this route, one has devised each group in two distinct sections: each of them embraces in its half the condition equations, taken from all the epochs."

"Having elsewhere taken the number of observations on which are founded each of the condition equations, one has therefore formed the two systems of equations following: first system, and second system (4 equations each of seven terms)."

"We replace in these equations, $\delta\varepsilon$ and δn by their values as functions of δe and e', and omitting the effect of the small terms dependent on its undetermined values, which are not the point of this objective, we obtain: first and second system (4 equations each of five terms)."

"One can, in the two first equations, for either the first or second system, where $\delta\pi$ and π' have small coefficients, replace the unknowns by their values in δe and e' tied to relations IV' and III'. And finally, we arrive at the following values for e' and δe:

First system $e' = -0''.0743$, $\delta e = -1''.19$ Second system $e' = -0''.0869$, $\delta e = -2''.73$

The accord is good as can be attained; and one should remark in accepting the mean value $e' = 0''.0806$ one is concluding, by the very precise conditions deduced in the discussion of the passages, $\pi' = 0''.60$, a considerable annual value!"

"The determination of the correction for the perihelion and its annual variation, by the two latter equations of each of the systems, does not give very agreeable results. The ensemble of all the meridian equations leads nonetheless to a value of π' which is positive and larger than the previous one."

"This circumstance, where the determination of the perihelion and its annual variation by means of the mean of the meridian observations is not very satisfactory, should render us circumspect, in the same with regard to the relative results for the eccentricity, in spite of their apparent precision. And, in consequence, allowing e' to be indeterminate, we are bound to borrow from the meridian observations the fairly exact formula:

$$\delta e = -1''.18 + 10.61\, e'$$

"From these results, approached from those above, we conclude with sufficient accuracy the corrections: (11 equations)."

"These are the corrections which, being added to the values of the elements, beginning in the 2nd Section, furnish the data most precise on which we base our definitive tables on the movement of the planet. They contain the remarkable result, already signaled above, that is to say, the considerable value of the function $2.72\, e' + \omega'$: **a value that seems incompatible with the magnitudes adopted up to now for the masses of the planets, and notably for the mass of Venus**. This consequence of the discussion of the observations of Mercury and their comparison with theory being most grave from the viewpoint of the constitution of our planetary system, it shall be good to think about this and throw an attentive eye on the path already followed, to see if there is anything we can infer on the consequences of this and the significance we should attribute to it."

(From pages 98-101) "The exactness of the observations that have been used being placed outside the cause, one can ask if from the masses of the perturbing planets being used, the secular movement of the perihelion and the eccentricity of the orbit of Mercury have been exactly deduced."

"We must remark in this regard, that outside the determination given in the present work, we have been disposed to refer to a paper published in 1841 on the secular variations of the elements of the orbits of the planets, and having considered elements of the first and third orders. We have reviewed these two determinations, separately, as in the 1841 paper, with terms of various orders, and in consideration of the masses used in the present work."

Figure 3. Secular Movement of the Perihelion of Mercury

Planet	1841 Original	Correction		Total	1859 Total
Venus	287”	-6”	=	281”	280”.6
Earth	86	-3	=	83	83.6
Mars	3	0	=	3	2.6
Jupiter	158	-6	=	152	152.6
Saturn	8	0	=	8	7.2
Uranus	0		=	0	0.1
Total	542	-15	=	527	526.7

Figure 4. Secular Movement of the Eccentricity of Mercury

Planet	1841 Original	Correction		Total	1859 Total
Venus	1”.1	1”.7	=	2”.8	2”.8
Earth	0.3	0.8	=	1.1	1.1
Mars	0	0	=	0	0
Jupiter	-0.7	3.5	=	4.2	4.2

“One sees that there is an agreement between the results, since in the second work we did not borrow anything from the first.”

“One remarks without doubt that the terms of the first order do not give more than 0”.7 for the movement of the eccentricity in a century, while the terms of the third order give +3”.5. That does not impeach the series, where it was used it was convergent. The terms of the first order are more considerable in reality and cancel each other by reason of their positions relative to the perihelion. Finally, we have determined the secular movements of the eccentricity and the perihelion by formulas of interpolation, independent of the coefficients of the series following the powers of the eccentricity and its inclination, and we have obtained the same results as above. It does not seem that, in reporting the exactness of the theory, any powerful doubt exists here.”

“These various points being established, and if we mention the effects which we have given particular attention to that do not introduce progressive errors into the Tables, it is sufficient to consider the smallness of the residues of the 21 equations of condition (page82) to confirm that there are no notable errors, neither in the use of the Tables of the Sun and of Mercury, nor in the calculation of the passages of the planet across the Sun. And thus, the necessity to augment notably the secular movements of the perihelion and the eccentricity being established, it remains to examine if one can change the primitive values attributed to the perturbing masses in a convenient manner; or if one can make recourse to hypotheses of the perturbing causes.”

“The masses of Jupiter and Saturn are well known and the action of Mars is very weak, so that one can sensibly attribute the calculated movements of the perihelion and eccentricity of Mercury by changing the masses of Venus and Earth; this is furnished between the (fractional mass correction) coefficients ν’ and ν’’ dependent on the masses from the relation,

(A) $$288'' \nu' + 87'' \nu'' = 38''.3$$

“It results from measurement of the obliquity of the elliptic, done during a century, where its secular diminution is equal to 45”.76, thus in calculating the average of the masses adopted for the planets, one finds 47”.48. It has led to the condition,

(B) $$0''.53\, \nu + 28''.88\, \nu' + 0''.75\, \nu''' + 1''.72 = 0$$

This condition is that reported in Chapter XIV (Volume IV, page 52) and in which one has diminished the mass of Mars by a *tenth* of its value which has provisionally been attributed in the theory of the Sun.”

“The discussion of the observations of the third ascension of the Sun has already furnished a relation between the coefficients ν’ and ν’’’ (Chapter XIV, Volume IV, page 95), the four relations (IV), (V), (VI) and (VII), deduced from the periodic inequalities. The equations (V) and (VII) serve to furnish for ν’’’ the value that we are led to for the mass of Mars, at which we have stopped. By means of this value of ν’’’, the equations (IV) and (VI) which depend very little on the mass of Mars, become simply,

(C) $$8''.00\, \nu' + 0''.00 = 0$$

$$8''.00\, \nu' - 0''.07 = 0$$

In these relations, the coefficient of ν’ has been tied to the average value of the periodic perturbations produced by Venus. The former has been tied to the observations of the Sun from 1750 to 1810, the second of the discussion of observations from 1811 to 1850.”

“These are the conditions that one possesses for the determination of the mass of Venus. One can here deduce the value,

(D) $\nu' = -0.0228$

founded on the same discussion of the observations of Mercury, and having respect to all the terms that Venus introduces in the theories of Mercury and Earth, whose secular movements of the perihelion and the eccentricity of Mercury have been excluded." [*The corrected mass of Venus is fairly close to the modern value*]

"Now, how can one, in taking account of the uncertainties in the observations, consider these diverse observations as compatible between themselves?" [*There follows a page of discussion*]

(From page 102) "We do not insist on any of these considerations, awaiting we repeat, that having reviewed all the elements of the discussion, each can produce some knowledge of the cause, and then we adopt the conclusions which seem most sure. It then remains to us only, for the case where one believes that the mass of Venus cannot be augmented, to examine to which consequences one shall be led to by the necessity to make an increase in the movement of the perihelion of Mercury by the **action of unknown masses**. It remains, that we do not need to review the research of all the causes that could produce this result. We should be content to indicate those that appear most probable, in reason of our actual knowledge about the physical constitution of our planetary system."

"A planet, or if one wishes, a group of small planets circulating in the shadow of the orbit of Mercury, would be susceptible to produce the anomalous perturbation proved by the previous discussion. We shall examine now the effect of a single perturbing mass: which can easily be that of an ensemble of bodies."

"The troubling mass, if it exists, does not have any sensible effect on the movement of the Earth. We ignore that it has some effect on Venus; and in awaiting this point to be clarified, we admit that this action is insensitive or at least weak except on Mercury. In this hypothesis, the mass sought should find itself below the orbit of Mercury. If in addition one wants its orbit to not entangle with that of Mercury, it is required that the aphelion distance does not exceed *eight-tenths* the average distance of Mercury, that is to say *three-tenths* the average distance of the Earth to the Sun." [*Hereafter comes a discussion of the* ***hypothetical planet Vulcan*** *which has never been observed. Le Verrier was famous for discovering Neptune from discrepancies in the orbit of Uranus, and postulated a similar situation for Mercury*].

1924 Critique by Charles Lane Poor [5]

"Einstein deduces a law of motion for the planets about the Sun; and this law of motion apparently differs from Newton's law of motion by a single very minute term. And this little term seems to fit into a kink in Mercury's orbit, and to explain or account for certain observed motions of that planet. But, in deducing his law of motion, in traversing the complicated maze of mathematics, the relativist meets difficulty after difficulty, and somehow surmounts them all. For each new difficulty some new mathematical device is evolved, and many of these devices are so intricate and complicated that it is extremely difficult to follow them through all their ramifications. Some of these devices seem to be ordinary approximations, but are called by Einstein substitutions or transformations of coordinates. One such transformation; or approximation, which is used in many portions of the theory, involves the method of measuring the distance between two particles of matter. Instead of using the exact distance between the centers of such particles, the relativist adds a small, a very small factor to this distance; and, in his formulas, uses this modified distance as if it were the true distance between the bodies. In ordinary terms this would be called an approximation, and any result derived there-from would be termed approximate. But to the Einsteinian, such a procedure is a transformation and the result is called *exact."*

"Further, the relativity conception of time differs from that of classical mechanics. From the earliest days of scientific thought, time has been regarded as independent of everyone and everything; the same for all portions of space, for all bodies, whether in motion or at rest: a minute is a minute the world around and everywhere in space. But this identity of time is denied by Einstein: according to the relativity theory time depends upon motion, and the internal of time, known to us as a minute, varies from place to place; it is different for a person at rest and for an aviator. Thus each planet has its own particular system of relativity, or "proper" time, and even this special time changes as the planet changes its speed in different portions of its orbit."

"Thus the formula, or law of motion, as deduced by the Einsteinians, does not represent the motion of a planet about the sun in the ordinary astronomical units of distance and of time. It represents that motion in a special system, in the relativity system of time or coordinates. Now it can readily be shown that this Einstein formula of motion, this formula which has *aptly* been called the essence of the relativity theory, can be derived directly from the Newtonian formula by merely changing the system in which the motion is measured, and by changing from the astronomical to the relativity method of measuring time. By omitting the special, or approximate systems of measurement, and using the ordinary astronomical measures of time and distance, the Einstein formula becomes identical with that of Newton: on the other hand, if in the Newtonian formula the relativity system of measurement be used, then this formula becomes identical with Einstein's. The two formulas thus apparently represent the same motion of a planet, but give that motion in different units; just as a stated amount of gold may be expressed as so many francs, or so many dollars."

"Yet this formula of motion, this formula which can be derived from that of Newton by a simple change in the method of measuring time or distance, is used by Einstein as proof conclusive of his theory. And this so-called proof depends upon certain minute and little understood motions of the planet Mercury. This planet is exceedingly difficult to observe; its motions are rapid and it is never far from the glare of the sun. Its path is not a simple curve: the large planets, - Venus, the Earth, and Jupiter - pull and haul at it, and under these pulls Mercury writhes and squirms along a difficult and tortuous path. Le Verrier calculated the effects of the various pulls of the six larger planets upon unfortunate Mercury, and found that these planets do not fully account for all the writhing of Mercury's orbit. He found a very slight discrepancy: he found that some other force, some other very minute pull, was affecting the motions of Mercury. And this discrepancy in the motions of Mercury consists of a combination of two small" wobbles," one of which is the celebrated motion of the perihelion, or rotation of the orbit in space. But the motion detected by Le Verrier is not a simple motion of the perihelion; it is a combination effect, a combination which Le Verrier himself could not disentangle into its separate parts. Within limits any value could be assigned to the perihedrial motion, and to each such value there would be a definite wobble of the eccentricity. Le Verrier gave, however, 38 seconds of arc per century as the most probable motion of the perihelion, which corresponds to a very small change in the eccentricity. Some years later, in 1895, Simon Newcomb confirmed these general results of Le Verrier; but he made the motion of the perihelion slightly larger and the change in the eccentricity correspondingly smaller, and at the same time he found several other small discrepancies, or wobbles, in the motions of Mercury and in the motions of other planets as well."

1993 Reanalysis of Le Verrier's Data, by Takeshi Inoue [6]

"We have reexamined the twenty-one observations of second and third contacts during transits of Mercury across on the disk of the Sun - the same Le Verrier used himself (Le Verrier 1859) - to check the reality of the excess of the observed motion of the longitude of the ascending node of Mercury's orbit over the theoretically predicted one. To this, we numbered these observations chronologically from 1 to 13 for the transits on November and from 14 to 21 for those on May."

"The results are as follows:
(1) An initial analysis showed that three of the observations (Nos. 2. 8 and 21) might have been erroneously recorded. If the signs of observations Nos. 2 and 8 are changed and if the value for the observation No. 21 is changed from -1."03 to -2."52, the sum of the squares of the residuals is reduced by almost 60%.
(2) Leaving out these observations altogether, as well as observations Nos. 10, 17 and 18 -- which leaves us 15 observations to analyze - the sum of the squares of the residuals is drastically reduced to 5% of its original value (but now, there are of course also fewer observations contributing to this sum), and we obtain an excess motion of 16."7 ± 3."4 (sec.) per century. This shows that the appropriate choice of observations will indeed produce an estimate for the excess motion of the node which exceeds its formal standard error by a factor of 5.
(3) Eliminating eight additional observations and thus utilizing only the seven observations, viz. Nos. 1, 9, 13, 14, 16, 19 and 20, the least squares adjustment of the remaining seven condition equations in the six adjustment unknowns produces an estimate for the excess motion of the ascending rode of Mercury's orbit. This time it gives 15."2 ± 0." 1 per century, with the sum of the squares of the residuals now reduced to the order of 10-5, even with only even summands contributing, clearly an unrealistic result in view of the precision of observations attained in Le Verrier's time."

"The only conclusion one can draw from the data is thus that **they do not contribute to a decision as to whether the actual motion of the ascending node of the orbit of Mercury exceeds that predicted by the theory**."

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