'Experimentum Crucis' for Magnetic Interaction

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The investigation of the long-standing controversy over the precise formulation of the magnetic interaction between steady currents (or between uniformly moving charges) is re-opened. An experimental technique is proposed whereby the distinction between the Ampere-Weber 'central force' formulation and the Grassmann-Lorentz 'normal force' formulation can be demonstrated decisively within a modestly equipped laboratory. Although no preference is advanced on theoretical grounds, a confirmation of the 'normal force' law would have notable consequences for theory: the third law of motion could only be satisfied through reaction against a background medium, and the possibility of a 'jetless' propulsion system is thereby suggested. It is also argued that the 'longitudinal' portion of the original Ampere force law is nonetheless compatible with, and perhaps necessary for, either experimental outcome.

1. Introduction

Throughout the history of electrodynamics, great efforts have been devoted to formulating a workable, consistent, physically verifiable force law for the magnetic interaction. Although numerous attempts have been undertaken, beginning with Ampere, there is as yet (despite general adoption of the Biot-Savart/Lorentz formulation), no universally accepted rule for this phenomenon among physicists; and though various theoretical arguments have been advanced in support of one or another law, the situation has been characterized by a notable *lack* of agreed-upon experimental discrimination between the various proposals. [1] Some theoreticians have contended that indeed *no such discrimination* is possible, at least for the forces between steady, continuous currents in conductors. Nevertheless, the controversy is maintained into the present time by ever more ingenious constructs. [2]

2. The Issue

The underlying problem concerns two general law types: the 'central force' theory (Ampere-Weber), and the 'normal force' theory (Grassman-Lorentz). The former argues that the force between any two current elements always acts *along their mutual line of separation*; the latter argues for a force that always acts *perpendicular to each respective current element direction*, regardless of the position of the other current element.

For the record, from L. Johansson, reference [3], (amended by writing d^2F for dF on the left side of the equations) where d^2F is the measure of force between current elements $I_1 dl_1$ and $I_2 dl_2$, r_{12} is the distance between them, and $\hat{r}_{12}$ is the unit direction vector, the two force laws may be written:

Ampere Law:

$$d^2F = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[3(dl_1 \cdot \hat{r}_{12})(dl_2 \cdot \hat{r}_{12}) - 2(dl_1 \cdot dl_2)]\,\hat{r}_{12} \tag{1}$$

Grassmann Law:

$$d^2F = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[dl_2 \times (dl_1 \times \hat{r}_{12})] \tag{2}$$

This is also written

$$d^2F = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[(dl_2 \cdot \hat{r}_{12})dl_1 - (dl_1 \cdot dl_2)\hat{r}_{12}] \tag{2a}$$

(Slightly differing versions of these laws appear throughout the literature; results generalized by Whittaker. [4]) Inspection of the Grassmann Law will indicate that a form of the more familiar Lorentz magnetic force law, $d^2\vec{F} = dQ\vec{v} \times d\vec{B}$, may be discerned by substituting $dQ\vec{v}$ for $I_2 dl_2$, and $d\vec{B}$ for $\frac{\mu_0 I_1}{4\pi r^2{}_{12}}(dl_1 \times \hat{r}_{12})$.

Objections to the former arise from the difficulty in ascribing the effects to the standard 'field' conception of the magnetic phenomenon; objections to the latter arise from the difficulty in conforming the effects (especially between moving free charges) to the Third Law of Motion. Complicated calculations, done in support of the contention that *no* distinction can be measured, [5] at least for pairs of continuous currents whose elements all lie within the same plane. In a *GED* paper by Wells [6] an experiment was proposed in which the various predicted directional deflections of an isolated charge moving in the vicinity of a particular current arrangement could be observed to distinguish between the different force laws. The proposal is compromised, however, by several practical considerations: the difficulty in concentrating a sufficient quantity of charge, the weakness of the magnetic fields, and the unaddressed effects of internal currents in the voltage source.

Other schemes involving isolated charges may be exemplified in the following construct:

Isolated charges are affixed to specific positions on the circumferences of a pair of wheels, whose axes of rotation are mutually perpendicular and fixed within a rigid framework, in the manner depicted in Figure 1. When set in rapid rotation, the directions of the respective charge motions at the upper and lower portions of their courses will conspire to produce Lorentz forces on them which will sum to a net force along the diagonal of the x,y plane. Hence, motion of the entire system should be produced in this direction, if the Grassmann-Lorentz rule is valid. Alternatively, *no such motion* would result if the Ampere-Weber rule is correct. Unfortunately, this proposal is also compromised by practical difficulties: the problem of isolating sufficient charge quantities remains, along with attaining sufficient rotational speeds to produce a measurable effect, in addition to violent vibrations set up by the Coulomb forces.

3. New Experimental Procedure

Despite these disappointing and disconcerting results, probity in examination of the relations enables viable experimental discrimination between the two forms of law, when employing *only* continuous closed currents for the purpose. Two peculiar distinctions of the Lorentz force allow this aim to be realized:

1. Whenever the direction of a current flow coincides with the direction of the magnetic circulation, *no* magnetic force acts on the current element in question.
2. For any non-zero, and non-perpendicular, value for the angles φ_1,φ_2 between the respective current-element directions and their mutual line of separation, there exists an apparent breach of the third law of motion—the respective force elements will add in such manner as to produce a net force on the system.

These statements, in affirmation of the familiar Lorentz force law, insure that such forces act *only* upon those current elements which 'cross' magnetic field lines. It is on this account possible to construct an experimental 'engine' which employs these features with such discretion as to render discrimination between the force laws possible after all:

First, let 'Magnetically Conjugate Currents' be defined. These are composed of two equal currents shaped into identical oval circuits, each threading the other, which lie in planes *mutually perpendicular* to one another. Each current course is empirically plotted to follow a magnetic field line of the other current; in this way, *no Lorentz-type forces* (normal to the current direction) can occur outside the plane of each circuit, since each current nowhere crosses the magnetic field of the other current. (Those forces which do occur from each current crossing its own magnetic field, will on the whole cancel, when the circuits are fixed to a rigid framework.) This arrangement is depicted in Figure 2a.

In Figure 2b, the experimental apparatus has been arranged to accommodate an arc traverse ($bc'd$) by current I_2, across a portion of magnetic field B_1 of current I_1. Meanwhile, the shapes of the remaining current paths are empirically re-adjusted to conform to new magnetically conjugate pathways for these segments of current. The conductors, along with their voltage source and connectors, are affixed to a rigid, insulated framework, which is freely gimbaled in such manner as to permit free rotation about any axis. If the Lorentz-type normal force is valid, rotation about the y-axis should be expected to result from the torque τ manifested from the moments of the forces ($I_2 \Delta s \times B_1$) in the respective opposite x- directions. If *no* rotation is induced, an Ampere-type central force is indicated.

4. Calculation of Results

Due to the complications of the current-path adjustments, precise determination of the net torque magnitude becomes calculatively intractable; however, an order-of-magnitude approximation may be surmised from consideration of a simpler, equivalent arrangement:

If the current oval I_1 were regarded as two concentric circular segments connected by radial straight-line segments, produced from the midpoint of $bc'd$, where the outer circular is just twice the radius of the inner, the current-distance magnitudes may be estimated from the respective arc lengths, and then converted to equivalent charge-velocity magnitudes. If reasonable laboratory proportions are granted, the current arc $bc'd$ for I_2 may be taken at 40 cm, the outer current arc of I_1 also at 40 cm., and the inner arc of I_1 at 20 cm. Then, a one ampere current in I_1 will correspond to a charge velocity (Qv) of roughly 0.4 coulomb meter/second in the outer arc, and 0.2 coulomb meter/second in the inner arc. The approximate magnetic field magnitude at the location of $bc'd$ can be found from the algebraic sum of the I_1 segment contributions (where the radial segment contributions are effectively zero):

$$B_1 \cong \frac{\mu_o}{4\pi}\left[\frac{Q_{v(\text{inner})}}{r^2_{(\text{inner})}} - \frac{Q_{v(\text{outer})}}{r^2_{(\text{outer})}}\right] \cong 6\times 10^{-8} \text{ Newton/Amp-m} \quad (3)$$

When multiplied against the equivalent crossing charge-velocities of I_2 for the portions of arc bc' and $c'd$, the force magnitude on each of these is found to be roughly

$$F_x \cong \frac{(Qv)_2 \times B_1}{2} \cong 6\times 10^{-9} \text{ Newton} \quad (4)$$

(where $(Qv)_2 \approx 0.2\,amp.met.$) for one ampere-turn of current in each circuit.

The approximate torque magnitude is then obtained by adding the moments of these forces about the y-axis at their median distance ($R_z \approx 10cm.$):

$$\tau_y \cong 2(R_z \times F_x) \cong 10^{-9}\, newton \times meter \quad (5)$$

Since the magnetic force is proportional to the *product* of the number of respective ampere-turns, if 100 turns of one-ampere current could be managed on each circuit, the respective forces and torques would be multiplied by $100^2 = 10^4$; if the mass of the entire apparatus could be managed at one kilogram with radius of gyration of 10 cm, the resulting division of the torque τ by the moment of inertia M_i will yield:

$$\frac{d}{dt}\omega = \frac{\tau}{M_i} \cong 10^{-3}\,\frac{\text{radian}}{\text{sec}^2} \quad (6)$$

for the order-of-magnitude of the angular acceleration of the apparatus about the y-axis.

Thus a very rough order-of-magnitude estimation of the resulting angular acceleration of the entire framework may be obtained. If the entire mass is one kilogram, for example, whose moment is at 10 centimeters from the y-axis, after 100 seconds of current flow, an angular velocity of roughly 0.1 radian per second should have accumulated. If 1,000 seconds could be managed, optimistically, 1.0 radian per second could be expected.

On the other hand, if the central-force, Ampere-Weber interaction is correct, *no* net motion of the apparatus will occur, due to the mutual action of the force between the respective current elements through their respective centers of mass. It must be noted in this context that the Grassman-Lorentz force results in an apparent breach of the Third Law of Motion—the apparatus apparently self-motivates without any discernible reaction. This breach has been answered by the author in another paper, [7] in which it is postulated that the reaction is found in counter-momentum absorbed by a surrounding ponderable (though invisible) medium, or *ether.*

5. Magnetic Propulsion?

The above proposal indicates a net impulse on the system in a *rotational* sense. Pursuing the hypothesis of reaction against a ponderable medium raises the question of propulsion in the *translational* sense: might an arrangement of currents be possible which would produce a continuous net force in a given direction on the system as a whole?

Such a scheme is developed in association with Figure 3. Instead of maintaining a single current about a modified oval as in Figure 2b, the current is caused to begin at point *e*, proceed to c', where it bifurcates into two symmetric pathways, one leading through b to a, the other through d to a. In this way, the segments ec', $c'b$, and $c'd$ will each experience a force in the *positive* x-direction (out of the page), whereas the remaining segments of I_2, being magnetically conjugate by design, will experience *no* force. The return current path from a to e, in crossing an equal amount of magnetic flux as the combined prior segments, will produce an equal force in the *negative* x-direction, thus yielding *zero* net force on I_2. Meanwhile, the bifurcations of I_2 will combine to produce a resultant magnetic field (B_2) in the positive z-direction along segment ghe of I_1, and in the negative z-direction along segment efg of I_1: these current crossings will produce a net force on $efgh$ in the positive x-direction. Hence, translational magnetic propulsion is expected to follow from the Lorentz force formula in such a design. The magnitude of the force will be comparable to that obtained in the rotational case; a linear acceleration, for a one-kilogram apparatus which employs 100 ampere-turns for each current, could have a magnitude on the order of as much as 10^{-2} cm/sec^2.

6. Field Pressure and the Ampere 'Longitudinal Force'

While inspection of the Ampere force law reveals that the sine product portion of φ_1, φ_2 (first term on the right of the equation) of the force *decreases* as the angle between current elements decreases, it also reveals that the cosine product portion (second term on the right) *increases* with smaller angles. Thus, a purely 'longitudinal force' develops between collinear current elements, which is repulsive when they are in the same direction. [8] Remarkably, this part of the equation is not necessarily incompatible with the Lorentz force; it could be explained by the type of 'Faraday Pressure' exerted by the like magnetic fields of the current elements against each other. [7] (Hence its detection would not be conclusive proof of Ampere's general formula.) This pressure was found necessary to account for energy conservation in various electrodynamic systems; once again it can account for the gain in purely magnetic field energy resulting from the system of a moving charge closing the distance between itself and a leading charge, in like motion. Unless there is a repulsive magnetic force between the charges, (in addition to the electrostatic force) requiring extra work to overcome, the increase in magnetic field energy would violate energy conservation. Hence it is naturally to be expected that collinear current elements will also manifest a repulsive force along the length of the conductor. Thus a conceptual alternative to the classic Ampereian interaction is suggested by this hypothesis.

7. Figures

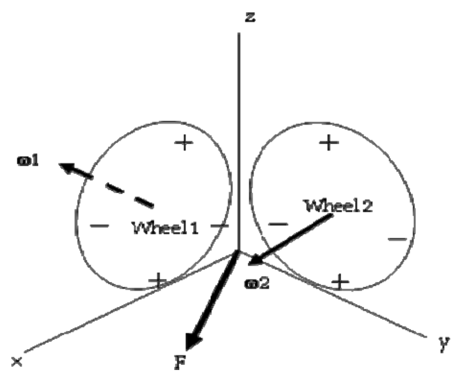

Fig. 1. System of rotating charged wheels (mounted within rigid framework).

When wheel 1 (in the xz plane) rotates with axis pointing in negative y-direction, and wheel 2 (in the yz plane) rotates with axis pointing in positive x direction, fixed charge concentrations on the wheels will mutually interact with magnetic fields caused by their motions to produce net Lorentz force F in direction of the xy plane diagonal, as depicted. Compliance with the Third Law of Motion is presumed through reaction of system against a fixed ponderable medium.

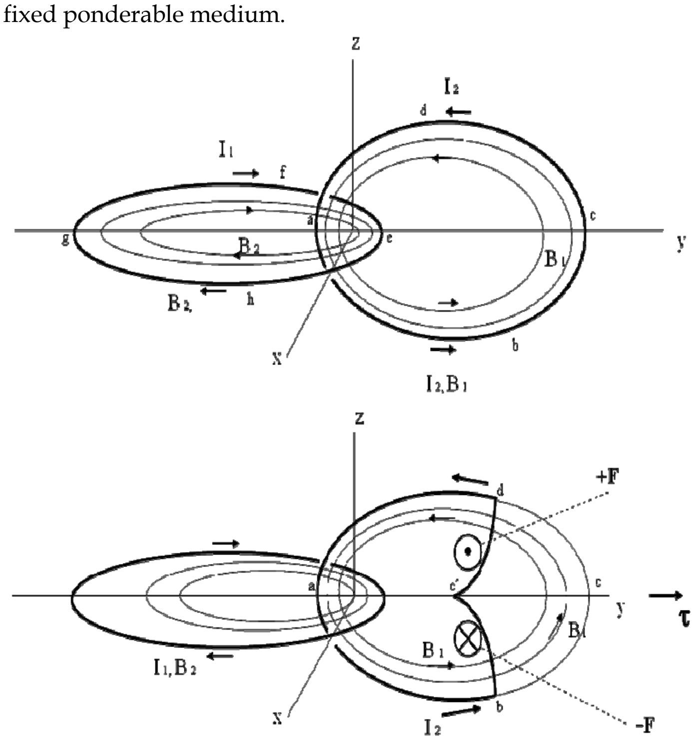

Fig. 2a,b. Magnetically conjugate currents

In Fig 2a, current oval I_1 lies in the *xy* plane, along magnetic field line *efgh* from current I_2. Current oval I_2 lies in the *yz* plane, along magnetic field line *abcd* from current I_1. No Lorentz forces act between the currents.

In Fig 2b, current oval I_1 lies in the *xy* plane. Magnetic field path abcd for B_1 of I_1 lies in the *yz* plane. Modified oval for I_2 along path *abc′d* lies in the *yz* plane.

Minor empirically determined mutual adjustments in the courses of currents I_1 and I_2 exclusive of arcs bc′ and c′d are necessary to conform these portions to conjugate pathways (i.e., each current of these portions follows the pathway of the magnetic field of the other current). Since no crossing of magnetic field of one current by the other current occurs along these portions, no Lorentz forces act between them.

Due to Lorentz forces +*F*, -*F* along *x*-axis on respective arc segments of I_2, a net torque τ will act about *y*-axis on the system if the 'normal force' theory of magnetic interaction is valid. Thus, if the system of conductors and voltage source (battery and connectors not shown) is fixed within a rigid, insulated framework, free to turn about the y-axis, rotation of the entire apparatus should be induced in the positive sense thereabout. Apparent breach of the Third Law of Motion is answered by a theoretical counter-torque reaction against a ponderable pervading medium ('ether'). A null-rotation result in this experiment would validate a 'central force' theory of magnetic interaction, where equal and opposite force elements would cancel.

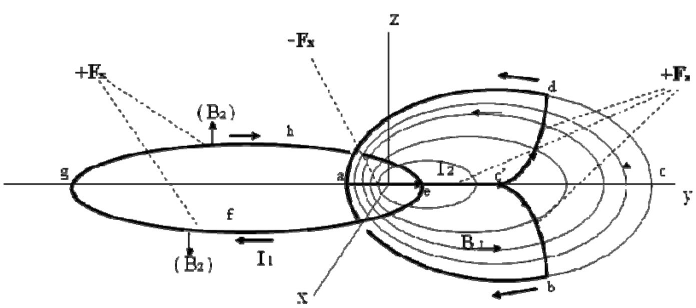

Fig. 3. Translational propulsion arrangement

In Fig. 3, current oval I_1 lies in the *xy* plane. Magnetic field path *abcd* for B_1 of I_1 lies in the *yz* plane. Bifurcated current path *ec′bae* and *ec′dae* for I_2 along path *abc′d* lies in the *yz* plane. Resultant positive *z*-component of magnetic field B_2 exists along segment *ghe*. Resultant negative *z*-component of magnetic field B_2 exists along segment *efg*.

Current paths have been adjusted to produce these effects: Crossing of B_1 by I_2 produces positive *x*-force on segments *ec′*, *c′b*, and *c′d*, while an equal negativeforce is produced on segment ae. A net positive *x*-force on the entire system results from I_1 crossing B_2.

8. Conclusion

An experimental means has been devised to distinguish conclusively between the Ampere-style central force law and the Lorentz-style normal force law for magnetic interaction between steady currents. Since an affirmation of the Ampere law would require much revision of presently accepted electrodynamic formulations, and an affirmation of the Lorentz would indicate the existence of a reactive *ether*, along with the possibility of 'jetless' propulsion, either experimental outcome would be of utmost significance for theory. Hence it is of great importance that efforts be undertaken to perform the experiments outlined above.

References

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