Geodetic Precession of the Spin in a Non-Singular Gravitational Potential

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Using a non-singular gravitational potential which appears in the literature we analytically derived and investigated the equations describing the precession of a body's spin orbiting around a main spherical body of mass M. The calculation has been performed using a non-exact *Schwarzschild* solution, and further assuming that the gravitational field of the Earth is more than that of a rotating mass. General theory of relativity predicts that the direction of the gyroscope will change at a rate of 6.6 arcsec/year for a gyroscope in a 650 km high polar orbit. In our case a precession rate of the spin of a very similar magnitude to that predicted by general relativity was calculated resulting to a $\frac{\Delta S_{geo}}{S_{geo}} = -5.570{\times}10^{-2}$.

1 Introduction

A new non-singular gravitational potential appears in the literature that has the following form (Williams [1])

$$V(r) = -\frac{GM}{r}\, e^{-\frac{\lambda}{r}}\,, \tag{1}$$

where the constant λ appearing in the potential above is defined as follows:

$$\lambda = \frac{GM}{c^2} = \frac{R_{grav}}{2}\,, \tag{2}$$

and G is the Newtonian gravitational constant, M is the mass of the main body that produces the potential, and c is the speed of light. In this paper we wish to investigate the differences that might exist in the results

2 Geodetic precession

One of the characteristics of curved space is that parallel transport of a vector alters its direction, which suggests that we can probably detect the curvature of the space-time near the Earth by actually examining parallel transport. From non gravitational physics we know that if a gyroscope is suspended in frictionless gimbals the result is a parallel transport of its spin direction, which does not help draw any valuable conclusion immediately. Similarly in gravitational physics the transport of such gyroscope will also result in parallel transport of the spin. To find the conditions under which parallel transport of gyroscope can happen, we start with Newton's equation of motion for the spin of a rigid body. A rigid body in a gravitational field is subject to a tidal torque that results to a spin rate of change given by [2]:

$$\frac{dS^n}{dt} = \varepsilon^{kln} R_{k0s0}\left(-I^s_l + \frac{1}{3}\,\delta^s_l I^r_r\right)\,, \tag{3}$$

where $n, k, l, s, r = 1, 2, 3$. Here R_{k0s0} is the Riemann tensor evaluated in the rest frame of the gyroscope, the presence of which signifies that this particular equation of motion does not obey the principle of minimal coupling, and that the gyroscope spin transport does not imitate parallel transport [1] and the quantity ε^{kqp} is defined as follows $\varepsilon^{123} = \varepsilon^{231} = \varepsilon^{312} = 1$ and $\varepsilon^{321} = \varepsilon^{213} = \varepsilon^{132} = -1$. For a spherical gyroscope we have that $I^s_l \propto \delta^s_l$, then the tidal torque in the equation (3) becomes zero and the equation reads $\frac{dS^n}{dt} = 0$. I^s_l is the moment of inertia tensor defined in the equation below:

$$I^s_l = \int \left(r^2\delta^s_l - x^s x_l\right) dM\,, \tag{4}$$

where δ^s_l is the Kronecker delta. This Newtonian equation remains in tact when we are in curved spacetime, and in a reference frame that freely falls along a geodesic line. Thus the Newtonian time t must now interpreted as the proper time τ measured along the geodesic. In the freely falling reference frame the spin of the gyroscope remains constant in magnitude and direction, which means that it moves by parallel transport.

If now an extra non gravitational force acts on the gyroscope and as a result the gyroscope moves into a world line that is different from a geodesic, then we can not simply introduce local geodesic coordinates at every point on of this world line which makes the equation of motion for the spin $\frac{dS^n}{dt} \neq 0$. In flat space-time the precession of an accelerated gyroscope is called *Thomas Precession*. In a general coordinate system the spin vector in parallel obeys the equation:

$$\frac{dS^\mu}{d\tau} = -\Gamma^\mu_{\nu\lambda} S^\nu \frac{dx^\lambda}{d\tau} = -\Gamma^\mu_{\nu\lambda} S^\nu \dot{x}^\lambda = -\Gamma^\mu_{\nu\lambda} S^\nu v^\lambda\,, \tag{5}$$

where $\Gamma^\mu_{\nu\lambda}$ are the Christoffel symbols of the second kind, and S^μ are the spin vector components (here $\mu, \nu, \lambda = 0, 1, 2, 3$).

Alternative theories of gravitation have also been proposed that predict different magnitudes for this effect [3, 4].

3 Gyroscope in orbit

In order to examine the effect of the new non singular gravitational potential has on the gyroscope let us assume a gyroscope in a circular orbit of radius r around the Earth. In real life somebody measures the change of the gyroscope spin relative to the fixed stars, which is also equivalent of finding this change with respect to a fixed coordinate system at infinity. We can use Cartesian coordinates since they are more convenient in calculating this change of spin direction than polar coordinates. The reason for this is that in Cartesian coordinates any change of the spin can be directly related to the curvature of the space-time, where in polar coordinates there is a contribution from both coordinate curvature and curvature of the space-time [2].

Next let us in a similar way to that of *linear theory* and following Ohanian and Ruffini [2] we write the line element ds^2 in the following way:

$$\begin{aligned} ds^2 \cong c^2 \left(1 - \frac{2GM}{rc^2}\, e^{-\lambda/r}\right) dt^2 - \\ - \left(1 - \frac{2GM}{rc^2}\, e^{-\lambda/r}\right)^{-1} \left(dx^2 + dy^2 + dz^2\right) \end{aligned} \tag{6}$$

further assume that our gyroscope is in orbit around the Earth and let the orbit be located in the $x-y$ plane as shown in Figure 1.

In a circular orbit all points are equivalent and if we know the rate of the spin change at one point we can calculate the rate of change of the spin at any point. For that let us write the line interval in the following way:

$$\begin{aligned} ds^2 \cong c^2 \left(1 - \frac{2GM}{rc^2}\, e^{-\lambda/r}\right) dt^2 - \\ - \left(1 + \frac{2GM}{rc^2}\, e^{-\lambda/r}\right) \left(dx^2 + dy^2 + dz^2\right) , \end{aligned} \tag{7}$$

which implies that:

$$\begin{aligned} g_{00} &= \left(1 - \frac{2GM}{rc^2}\, e^{-\lambda/r}\right) , \\ g_{11} = g_{22} = g_{33} &= - \left(1 + \frac{2GM}{rc^2}\, e^{-\lambda/r}\right) . \end{aligned} \tag{8}$$

4 The spin components

To evaluate the spatial components of the spin we will use equation (5), and the right hand symbols must be calculated. For that we need the four-velocity $v^\beta \approx (v_t,\, v_x,\, v_y,\, v_z) = (1,\, 0,\, v,\, 0)$. We also need the S^0 component of the spin, and for that we note that in the rest frame of the gyroscope

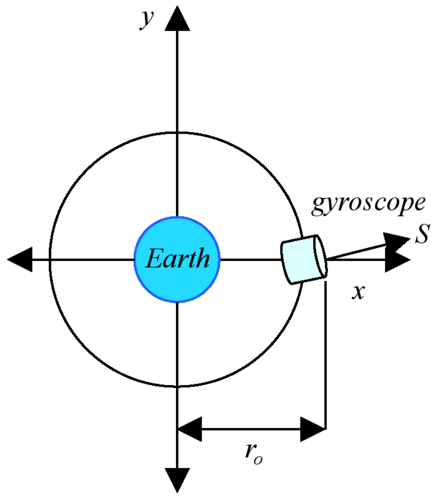

Fig. 1: A gyroscope above a satellite orbiting the Earth, and where its orbital plane coincides with the $x-y$ plane, having at an instant coordinates $x = r_0$, $y = z = 0$, and where S is the spin vector of the gyroscope.

$S'^0 = 0$ and $v'^\beta = (1,\, 0,\, 0,\, 0)$ and therefore $g'_{\mu\nu} S'^\mu v'^\nu = 0$, and also in our coordinate system we will also have that $g_{\mu\nu} S^\mu v^\nu = 0$, using the latter we have that:

$$\begin{aligned} S^0 &= -\frac{1}{g_{00}} \left[S^1 g_{11} \frac{dx^1}{d\tau} + S^2 g_{22} \frac{dx^2}{d\tau} + S^3 g_{33} \frac{dx^3}{d\tau} \right] , \\ S^0 &= -\frac{1}{g_{00}} \left[S_x g_{11} \frac{dx}{d\tau} + S_y g_{22} \frac{dy}{d\tau} + S_z g_{33} \frac{dz}{d\tau} \right] , \end{aligned} \tag{9}$$

substituting for the metric coefficients we obtain:

$$\begin{aligned} S^0 = \frac{\left(1 + \frac{2GM}{rc^2}\, e^{-\lambda/r}\right)}{\left(1 - \frac{2GM}{rc^2}\, e^{-\lambda/r}\right)}\, v S_y \cong \\ \cong \left(1 + \frac{2GM}{rc^2}\, e^{-\lambda/r}\right)^2 v S_y\, . \end{aligned} \tag{10}$$

Next letting $\mu = 1$ and summing over $\nu = 0,\, 1,\, 2,\, 3$ the component of the spin equation becomes:

$$\frac{dS^1}{d\tau} = -\Gamma^1_{0\lambda} S^0 v^\lambda - \Gamma^1_{1\lambda} S^1 v^\lambda - \Gamma^1_{2\lambda} S^2 v^\lambda - \Gamma^1_{3\lambda} S^3 v^\lambda\, , \tag{11}$$

summing over $\lambda = 0,\, 1,\, 2,\, 3$ again we obtain:

$$\begin{aligned} \frac{dS^1}{d\tau} = &- \Gamma^1_{00} S^0 v^0 - \Gamma^1_{01} S^0 v^1 - \Gamma^1_{02} S^0 v^2 - \Gamma^1_{03} S^0 v^3 - \\ &- \Gamma^1_{10} S^1 v^0 - \Gamma^1_{11} S^1 v^1 - \Gamma^1_{12} S^1 v^2 - \Gamma^1_{13} S^1 v^3 - \\ &- \Gamma^1_{20} S^2 v^0 - \Gamma^1_{21} S^2 v^1 - \Gamma^1_{22} S^2 v^2 - \Gamma^1_{23} S^2 v^3 - \\ &- \Gamma^1_{30} S^3 v^0 - \Gamma^1_{31} S^3 v^1 - \Gamma^1_{32} S^3 v^2 - \Gamma^1_{33} S^3 v^3 . \end{aligned} \tag{12}$$

Next we will calculate the Cristoffel symbols of the second kind for that we use:

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda} \left(\frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right) . \tag{13}$$

Since $\Gamma^{\sigma}_{\mu\nu} = 0$ if $\mu \neq \nu \neq \sigma$ equation (12) further simplifies to:

$$\begin{aligned} \frac{dS^1}{d\tau} &= -\Gamma^1_{00} S^0 v^0 - \Gamma^1_{01} S^0 v^1 - \Gamma^1_{10} S^1 v^0 - \\ &- \Gamma^1_{11} S^1 v^1 - \Gamma^1_{12} S^1 v^2 - \Gamma^1_{13} S^1 v^3 - \\ &- \Gamma^1_{21} S^2 v^1 - \Gamma^1_{22} S^2 v^2 - \Gamma^1_{31} S^3 v^1 - \Gamma^1_{33} S^3 v^3 \,. \end{aligned} \tag{14}$$

The only non-zero Christoffel symbols calculated at $r = r_0$ are:

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2}\, e^{-\lambda/r_0}\right)} \,, \tag{15}$$

$$\Gamma^1_{00} = \frac{GM}{c^2 r_0^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2}\, e^{-\lambda/r_0}\right)} \,, \tag{16}$$

$$\Gamma^1_{11} = -\frac{GM}{c^2 r_0^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\lambda/r_0}}{\left(1 - \frac{2GM}{r_0 c^2}\, e^{-\lambda/r_0}\right)} \,, \tag{17}$$

$$\Gamma^1_{22} = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2}\, e^{-\frac{\lambda}{r_0}}\right)} \,, \tag{18}$$

$$\Gamma^1_{21} = \Gamma^1_{12} = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2}\, e^{-\frac{\lambda}{r_0}}\right)} \,, \tag{19}$$

$$\Gamma^1_{33} = \frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2}\, e^{-\frac{\lambda}{r_0}}\right)} \,, \tag{20}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2}\, e^{-\frac{\lambda}{r_0}}\right)} \,. \tag{21}$$

Thus equation (14) further becomes:

$$\frac{dS_x}{d\tau} = -\Gamma^1_{00} S^0 - \Gamma^1_{22} S^2 v^2 \,, \tag{22}$$

substituting we obtain:

$$\begin{aligned} \frac{dS_x}{d\tau} = -\frac{GM}{r_0^2 c^2} & \frac{\left(1 - \frac{\lambda}{r_0}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2}\, e^{-\frac{\lambda}{r_0}}\right)^2\right] v S_y \,. \end{aligned} \tag{23}$$

Expanding in powers of $\frac{\lambda}{r}$ to first order we can rewrite (23) as follows:

$$\begin{aligned} \frac{dS_x}{d\tau} = -\frac{GM}{r_0^2 c^2} & \frac{\left(1 - \frac{\lambda}{r_0}\right)^2}{\left(1 + \frac{2GM}{r_0 c^2}\left(1 - \frac{\lambda}{r_0}\right)\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2}\left(1 - \frac{\lambda}{r_0}\right)\right)^2\right] v S_y \,, \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{dS_x}{d\tau} \cong -\frac{GM}{r_0^2 c^2} & \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2}\left(1 - \frac{\lambda}{r_0}\right)\right)} \times \\ & \times \left[1 + \left(1 + \frac{2GM}{r_0 c^2}\left(1 - \frac{\lambda}{r_0}\right)\right)^2\right] v S_y \,, \end{aligned} \tag{25}$$

keeping only $\frac{1}{c^2}$ terms and omitting the rest higher powers $\frac{G^2 M^2}{c^4}$ equation (25) can be simplified to:

$$\frac{dS_x}{d\tau} \cong -\frac{2GM}{r_0^2 c^2}\left(1 - \frac{2\lambda}{r_0}\right) v S_y \,. \tag{26}$$

Similarly the equation for the S_Y component of the spin becomes:

$$\frac{dS_y}{d\tau} = -\Gamma^2_{12} v S_x - \Gamma^2_{20} S_y - \Gamma^2_{22} v S_y - \Gamma^2_{32} v S_y \,, \tag{27}$$

which becomes:

$$\frac{dS_y}{d\tau} = -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{\lambda}{r}\right) e^{-\frac{\lambda}{r_0}}}{\left(1 + \frac{2GM}{r_0 c^2} e^{-\frac{\lambda}{r_0}}\right)} \,, \tag{28}$$

can be approximated to:

$$\begin{aligned} \frac{dS_y}{d\tau} &= -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2}\left(1 - \frac{\lambda}{r_0}\right)\right)} = \\ &= -\frac{GM}{r_0^2 c^2} \frac{\left(1 - \frac{2\lambda}{r_0}\right)}{\left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)} v S_x \,. \end{aligned} \tag{29}$$

Finally the equation for the S_z component becomes:

$$\begin{aligned} \frac{dS^3}{d\tau} &= -\Gamma^3_{00} S^0 v^0 - \Gamma^3_{01} S^0 v^1 - \Gamma^3_{02} S^0 v^2 - \\ &- \Gamma^3_{03} S^0 v^3 - \Gamma^3_{10} S^1 v^0 - \Gamma^3_{11} S^1 v^1 - \\ &- \Gamma^3_{12} S^1 v^2 - \Gamma^3_{13} S^1 v^3 - \Gamma^3_{20} S^2 v^0 - \\ &- \Gamma^3_{21} S^2 v^1 - \Gamma^3_{22} S^2 v^2 - \Gamma^3_{23} S^2 v^3 - \\ &- \Gamma^3_{30} S^3 v^0 - \Gamma^3_{31} S^3 v^1 - \Gamma^3_{32} S^3 v^2 - \Gamma^3_{33} S^3 v^3 \,, \end{aligned} \tag{30}$$

which finally becomes:

$$\frac{dS_z}{d\tau} = 0 \,. \tag{31}$$

Equations (23), (28), (31) are valid at the chosen $x = r_0$, $y = z = 0$ point. These equations can also be written in a form that is valid at any point of the orbit, if we just recognize that all of them can be combined in the following single 3-D equa-

$$S_x(t) = S_0 \left\{ \cosh\left(\frac{\sqrt{2}\,GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right) - \right.$$

$$\left. - \sqrt{2\left(1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}\right)}\, \sinh\left(\frac{\sqrt{2}\,GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right) \right\} \tag{43}$$

$$S_y(t) \cong S_0 \left\{ \cosh\left(\frac{\sqrt{2}\,GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right) - \frac{1}{\sqrt{2}} \sinh\left(\frac{\sqrt{2}\,GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right) \right\} \tag{44}$$

tion in the following way [2]:

$$\frac{dS}{d\tau} = -2v \cdot S\nabla V + vS \cdot \nabla V\,, \tag{32}$$

where $V = -\frac{GM}{r_0} e^{-\lambda/r_0}$ is the non singular potential used. Below in order to compare we can write down the same equations for the spin components in the case of the Newtonian potential.

$$\frac{dS_x}{d\tau} = -\frac{GM}{r_0^2 c^2}\, vS_y \left[\left(1 + \frac{2GM}{r_0 c^2}\right) + \frac{1}{\left(1 + \frac{2GM}{r_0 c^2}\right)} \right] \cong$$

$$\cong -\frac{2GM}{r_0^2 c^2}\, vS_y\,, \tag{33}$$

$$\frac{dS_y}{d\tau} = \frac{GM}{r_0^2 c^2 \left(1 + \frac{2GM}{r_0 c^2}\right)}\, vS_x \cong \frac{GM}{r_0^2 c^2}\, vS_x\,, \tag{34}$$

$$\frac{dS_z}{d\tau} = 0\,. \tag{35}$$

5 Non-singular potential solutions

To find the components of the precessing spin let us now solve the system of equations (26) (29) (31) solving we obtain:

$$S_x(t) = C_1 \cosh\left\{ \frac{\sqrt{2} GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right\} +$$

$$+ C_2 \sqrt{2} \sinh\left\{ \frac{\sqrt{2} GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right\}, \tag{36}$$

$$S_y(t) = C_2 \cosh\left\{ \frac{\sqrt{2} GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right\} +$$

$$+ \frac{C_1}{\sqrt{2}} \sinh\left\{ \frac{\sqrt{2} GM\,(2\lambda - r_0)}{c^2 r_0^3 \sqrt{1 + \frac{2GM}{r_0 c^2} - \frac{2GM\lambda}{r_0^2 c^2}}}\, vt \right\}, \tag{37}$$

$$S_z(t) = \text{const} = D_0\,, \tag{38}$$

since the motion is not relativistic we have that $dt = d\tau$, and the orbital velocity of the gyroscope is $v = \sqrt{\frac{GM}{r_0}}$.

6 Newtonian gravity solutions

Next we can compare the solutions in (36), (37), (38) with those of the system (33), (34), (35) which are:

$$S_x(t) = C_1 \cos\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2}\, vt \right) -$$

$$- C_2 \sqrt{2} \sin\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2}\, vt \right), \tag{39}$$

$$S_y(t) = C_2 \cos\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2}\, vt \right) +$$

$$+ \frac{C_1}{\sqrt{2}} \sin\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2}\, vt \right), \tag{40}$$

$$S_z(t) = \text{const} = D_0\,.$$

If we now assume the initial conditions $t = 0$, $S_x(0) = = S_y(0) = S_0$ we obtain the final solution:

$$S_x(t) = S_0 \left\{ \cos\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2} vt \right) - \right.$$

$$\left. - \sqrt{2} \sin\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2}\, vt \right) \right\}, \tag{41}$$

$$S_y(t) = S_0 \left\{ \cos\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2} \sqrt{\frac{GM}{r_0}}\, t \right) - \right.$$

$$\left. - \frac{1}{\sqrt{2}} \sin\left(\frac{\sqrt{2}\,GM}{c^2 r_0^2} \sqrt{\frac{GM}{r_0}}\, t \right) \right\}. \tag{42}$$

Similarly from the solutions of the non-singular Newtonian potential we obtain (43) and (44).

Since numerically $c^2 r_0^3 \gg 2GMr_0^2 - 2GM\lambda r_0$ the above equations take the form (45) and (46).

7 Numerical results for an Earth satellite

Let us now assume a satellite in a circular orbit around the Earth, at an orbital height $h = 650$ km or an orbital radius

$$S_x(t) = S_0\left\{\cosh\left(\frac{2\lambda GM}{c^2 r_0^3}\left(1-\frac{r_0}{2\lambda}\right)\sqrt{\frac{2GM}{r_0\left(1+\frac{2GM}{r_0c^2}-\frac{2GM\lambda}{r_0^2c^2}\right)}}\,t\right) - \right.$$
$$\left. -\sqrt{2\left(1+\frac{2GM}{r_0c^2}-\frac{2\lambda GM}{r_0^2c^2}\right)}\sinh\left(\frac{2\lambda GM}{c^2r_0^2}\left(1-\frac{r_0}{2\lambda}\right)\sqrt{\frac{2GM}{r_0\left(1+\frac{2GM}{r_0c^2}-\frac{2GM\lambda}{r_0^2c^2}\right)}}\,t\right)\right\} \tag{45}$$

$$S_y(t) = S_0\left\{\cosh\left(\frac{2\lambda GM}{c^2 r_0^3}\left(1-\frac{r_o}{2\lambda}\right)\sqrt{\frac{2GM}{r_0\left(1+\frac{2GM}{r_0c^2}-\frac{2GM\lambda}{r_0^2c^2}\right)}}\,t\right) - \right.$$
$$\left. -\frac{1}{\sqrt{2}}\sinh\left(\frac{2\lambda GM}{c^2r_0^3}\left(1-\frac{r_0}{2\lambda}\right)\sqrt{\frac{2GM}{r_0\left(1+\frac{2GM}{r_0c^2}-\frac{2GM\lambda}{r_0^2c^2}\right)}}\,t\right)\right\} \tag{46}$$

$V(r)$	$\Delta S_x/S_x$	$\Delta S_y/S_y$	Geodetic precession S (arcsec/year)	$\Delta S_{geo}/S_g$
Newtonian	-4.30×10^{-5}	2.00×10^{-5}	-6.6	
Non-Singular	-4.80×10^{-5}	2.40×10^{-5}	-6.289	-5.570×10^{-2}

Table 1: Changes of the spin components and final geodetic precession of an orbiting the Earth satellite t an altitude $h = 650$ km.

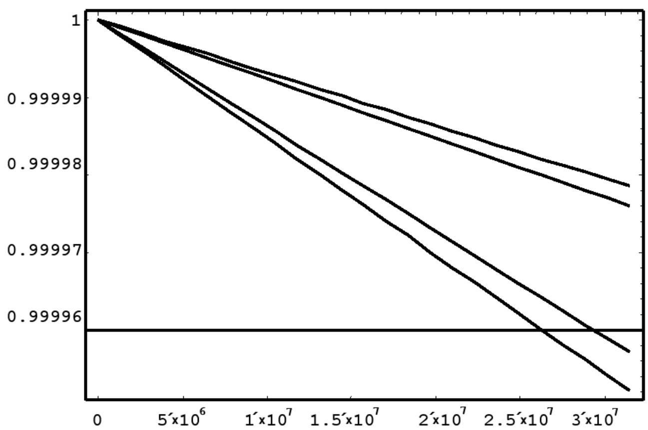

Fig. 2: Gyroscope spin components (S_x, S_y). Newtonian and non-singular potential change in the gyro pin components for a satellite orbiting the earth for a year. Abscissa axis means time.

$r_0 = 7.028\times10^6$ m, then $\lambda = 4.372\times10^{-3}$ m, $v = 7.676$ km/s, $t = 1$ year $= 3.153\times10^7$ s using (41) and (42) we obtain:

Newtonian potential

$$S_x = 0.999957\,S_0\,, \quad S_y = 1.000020\,S_0\,, \tag{47}$$

and from (45) and (46) we obtain:

Non-Singular Potential

$$S_x = 0.999952\,S_0\,, \quad S_y = 0.999976\,S_0\,. \tag{48}$$

For a gyroscope in orbit around the Earth we can write an expression for the geodetic precession in such a non-singular potential to be equal to:

$$S_{geo} = \frac{3}{2}\nabla\Phi\times v = \frac{3GM}{2c^2r_0^2}\sqrt{\frac{GM}{r_0}}\left(1-\frac{\lambda}{r_0}\right)e^{-\frac{\lambda}{r_0}}\,, \tag{49}$$

substituting values for the parameters above we obtain that:

$$S_{geo} = 1.01099\times10^{-12}\ \text{rad/s}\,, \tag{50}$$

$$S_{geo} = 6.289\ \text{arcsec/year}\,. \tag{51}$$

8 Conclusions

We have derived the equations for the precession of the spin in a case of a non-singular potential and we have compared them with those of the Newtonian potential. In the case of the non-singular gravitational potential both components of the spin are very slow varying functions of time. In a hypothetically large amount of time of the order of $\sim10^5$ years or more spin components S_x and S_y of the non-singular potential appear to diverge in opposite directions, where those of the Newtonian potential exhibit a week periodic motion in time.

In the case of the non-singular potential we found that $\frac{\Delta S_x}{S_x} = -4.80\times10^{-5}$ and $\frac{\Delta S_y}{S_y} = -2.40\times10^{-5}$ where in the case of the Newtonian potential we have that $\frac{\Delta S_x}{S_x} = -4.30\times10^{-5}$ and $\frac{\Delta S_y}{S_y} = 2.00\times10^{-5}$. The calculation has been performed using a non-exact *Schwarzschild* solution. On the other hand the gravitational field of the Earth is not an exact *Schwarzschild* field, but rather the field of a rotating mass. Compared to the Newtonian result, the non-singular potential modifies the original equation of the geodetic precession by the term $\left(1-\frac{\lambda}{r_0}\right)e^{-\frac{\lambda}{r_0}}$ which at the orbital altitude of $h = 650$ km contributes to a spin reduction effect of the order of 9.99×10^{-1}. If such a type of potential exists its effect onto a gyroscope of a satellite orbiting at $h = 650$ km could probably be easily detected.

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