Alternative to the Epoch Time Solution Patching Structure of the Cosmological Standard Model in the Friedman Dust Universe with Einstein's Lambda

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April 14, 2010

1 Abstract

The need for the cosmological constant, Lambda, in Einstein's field equations to be an absolute mathematical constant over all the time that they are used to describe some astrophysics process is demonstrated. Only if that condition holds will the conservation laws of mass and momentum hold as in classical physics. The Friedman equations that can be deduced rigorously from general relativity are consequently equally restricted to a constant valued Lambda and for the same reasons. However, the standard cosmological model, is not constructed from one solution of the Friedman equations but rather from at least three different but rigorous solutions patched together at times where they are physically thought to join. This is because the known solutions are thought to represent different conditions of mass movement, highly erratic or thermal at time near the big bang or more particle like and organised into systems at time near now, just to mention two types of activity when there obviously could be a continuous range of activities of mass types. Clearly this idea of how things have evolved after the big bang is very plausible, if the big bang idea is accepted as fact. The apparent need to patch solutions together over time creates great mathematical difficulties for cosmology because the three functions selected have to join smoothly which is the same as saying that they have to be differentiable not once but twice if accelerations are taken into account as they must be if the Friedman equations are to hold through the join. It is not clear whether or not this patching process can be rigorously achieved. However it is clear that the big bang concept does violate Einstein's field equations at $t = 0$ because this concept implies that mass and momentum comes from nowhere. It is shown that all of these problems can be removed by introducing a continuously variable over time structure into the definition of temperature for the dust universe model. This only affects the value of the temperature that is associated with a given time and make no difference to the validity of the dust universe model with regard to it being a rigorous solution to the Einstein Field equations for all time from minus infinity to plus infinity.

Keywords: Cosmology, Dust Universe, Dark Energy, Epoch Time Solutions Matching, Friedman Equations, Standard Model Einstein's field Equations, Lambda

PACS Nos.: 98.80.-k, 98.80.Es, 98.80.Jk, 98.80.Qc

2 Introduction

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [33]. All of this work and its applications has its origin in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7],[25],[3]). The applications can be found in ([23],[24],[33],[37],[35][41]). Other useful sources of information are ([17],[3],[31],[27],[30],[29]) with the measurement essentials coming from references ([1],[2],[11],[38]). Further references will be mentioned as necessary. In the following pages, I shall introduce a simple extension to the dust universe model that greatly enlarges its ability to describe astrophysical phenomena that are conceived as depending strongly on the cosmological temperature. This is particularly relevant to those processes that involve transitions from heat dominated disordered conditions at early epoch time to the cooler mass particle ordered conditions of the present time. I shall first consider the important contribution of the cosmological constant in terms of its contribution to the conservation of mass and momentum as described by the Einstein field equations.

In 1917, Einstein introduced his modified tensor field equations, (2.1),(2.3) and (2.4), with the addition of the so called *Lambda*, Λ, term,

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda &= -\kappa T_{\mu\nu}. && (2.1)\\
G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R && (2.2)\\
G^{\mu}{}_{\nu;\mu} &= 0 && (2.3)\\
T^{\mu}{}_{\nu;\mu} &= 0. && (2.4)
\end{aligned}
$$

Equation (2.2) is the definition of the Einstein tensor, $G_{\mu\nu}$, to be used in the line below it. Equations (2.3) and (2.4) are covariant derivatives of $G_{\mu\nu}$ and the stress energy momentum tensor, $T_{\mu\nu}$, respectively. They are both first order tensor, or equivalently vector equations, because one index of a second order tensor has been contracted out through the differentiation process. They both represent conservation of energy and momentum. The zero character of the covariant derivative of $G^{\mu}{}_{\nu;\mu}$ is an inevitable consequence of the geometrical structure represented by $G_{\mu\nu}$ and the way it is defined. The zero character $T^{\mu}{}_{\nu;\mu}$ is an inevitable consequence of the physical meaning of $T_{\mu\nu}$ and the way it is represented. Thus it follows that with the exception of the Λ term all the terms in that equation, (2.1), satisfy the laws of energy and momentum conservation of classical physics. That is to say Einstein's *original unmodified* equation conforms to classical energy momentum conservation. However even this conclusion is not completely true for the following reason. The time variable that emerged with Einstein's original equations has an unusual property in classical physics experience. The Einstein field equations have a built in singularity at cosmological time zero which of course, we all know about as the big bang origin of the universe. However, proponents of the big bang theory accommodate the massive violation of mass and momentum conservation at the instant $t = 0$ by the caveat that everything began at that instant or just after so that the field equations and the covariant conservation equations can be disregarded at that instant. So ignoring this complication temporarily, at time other that time zero we now need to consider what the situation is when the Λ term is left in the tensor field equation (2.1) as in the modified form.

For the Einstein field equations with the lambda term included not to be in conflict with the energy momentum conservation laws of classical physics it is necessary that the additional term $g_{\mu\nu}\Lambda$ also satisfies the covariant conservation equation,

$$(g^{\mu}{}_{\nu}\Lambda)_{;\mu} = 0. \tag{2.5}$$

That is

$$(g^{\mu}{}_{\nu})_{;\mu}\Lambda + g^{\mu}{}_{\nu}(\Lambda)_{;\mu} = 0 \tag{2.6}$$

$$g^{\mu}{}_{\nu}(\Lambda)_{;\mu} = 0. \tag{2.7}$$

The last equation following because the covariant derivatives of the $g_{\mu\nu}$ are zero. The Cosmological quantity Λ is never taken to depend on the space variables so that the only derivative left to consider is the time variable, x_4 differentiation part in (2.6),

$$g^{4}{}_{\nu}(\Lambda)_{;4} = g^{4}{}_{\nu}(\Lambda)_{,4} = 0, \tag{2.8}$$

because the covariant derivative of a scalar is equal to the normal derivative. There will certainly be at least one of the $g^{4}{}_{\nu}$ elements that is not zero. It therefore follows that the Lambda included version of Einstein's field will only conform with the conservation laws of classical physics provided that the x^4 or time derivative of Λ satisfies

$$\partial\Lambda/\partial t = 0. \tag{2.9}$$

That is to say that the cosmological quantity Λ has to be an absolute constant. If a space or time variable Λ is used in Einstein's field equations their physical mathematical validity is totally compromised. Lambda is rightly referred to as *The Cosmological Constant*. In the next section, I examine how the covariant conservations laws of Einstein's general relativity are reflected in the structure of the cosmological standard model.

3 Standard Cosmology Model

That something very strange happens at cosmological time zero seems to be an unavoidable consequence of Einstein's field equations for general relativity. This feature becomes very noticeable in the consequent Friedman equations and their solutions derived from relativity and used for constructing cosmological models. However, the formulation of these equations has no restriction on the possible $-\infty \to +\infty$ time range. We have all heard about this initial extraordinary event called either the big bang or the initial singularity. The ideas that go along with the big bang concept are expansion of the universe from a zero or very small volume at the same time as a very large quantity of mass possible the whole mass of the universe, M_U is generated within this volume, a very violent *explosion* from a very small volume so that the pressures and temperatures involved must have been enormous, rather like an incredibly large atomic explosion. This image leads to the idea that the initial stages of the universe must have been dominated by radiation with an excess of photon like activity. Conceivable then as expansion proceeded cooling and pressure reductions occurred leading to condensation of matter into particulate forms with very much less kinetic activity. It seems to me that these ideas but with very little supporting evidence from actual observation are the reason that cosmological models starting with the big bang are taken to have three main time phases. They start with inflation, an attempted explanation of the start from nothing, followed by a radiation dominated phase which is followed by a matter dominated phase. Now days the model has to take account of a recently observed accelerated radial expansion of the universe. At some t_c in the evolutional time sequence above an initial deceleration condition due to gravitation attraction is assumed to change into an accelerating condition due to the universe's dark energy content increasing with time. This type of prescription for the time evolution of the universe has for years presented cosmologist with a dilemma because although a number of solutions to the Friedman equations have been found they all seemed to have different characteristics with regard to whether they represented radiation or matter evolving with time. Thus in constructing the standard model for cosmology it seemed imperative that different solution would have to be time wise patched together if the history envisaged above were to be mathematically represented in the standard model. This problem is now complicated by the problem of incorporating the so described mysterious dark energy. So it was that mathematically solutions of the Einstein field equations were time wise patched together to give a model that it is claimed represents the actual time evolution of the universe from time about zero to time in the unlimited distant future, $t = +\infty$. However, there is a down side to this approach which is first of all the big bang concept and the inflationary beginning which obviously breaks Einstein's conservation rules at time zero or thereabouts by the generation of mass from nothing and this is confounded by the inflation section which lasts for a finite time, supposedly starting before time zero and involving a massive value for the cosmological constant which clearly cannot be matched with the small values following at later times and so must violate the time constancy of Λ condition obtained at (2.9). The later time sections also have to be matched and should involve Λ not changing, smooth connections between other parameters and their time derivatives should happen at these times. I do not wish to be dogmatic but it seems to me that this smooth connection scenario is not mathematically demonstrably achieved. Thus ignoring the problem with the inflation section on the grounds that it is inevitable if the big bang concept is true, big problems at later times are still present in the standard model. I shall show in the next section that there is a way of avoiding the uncertainties of the standard model by not using time patching at all.

4 Dust Universe Model

In an early stage of introducing the dust universe cosmological model, I imposed a restriction on the relation between two types of mass into which the universe can be divided, the thermal mass of the cosmic micro-wave background and the rest which I denoted by M_Γ and M_Δ respectively, so that the total non-dark energy mass M_U could be represented as

$$M_U = M_\Delta + M_\Gamma. \tag{4.1}$$

In this model I define M_U, the total mass within the spherical boundary of the universe not including any dark energy mass to be taken as being conserved that is of retaining the constant numerical value throughout the time history of the evolution of the universe for this model. However additionally, I imposed the working condition that both of the masses, M_Δ and M_Γ are to be separately conserved that is their numerical values are to remain constant throughout the time history of the evolution of the universe for this model. I called this restriction on the two mass components the *strong* assumption and in fact I introduced it in the mistaken belief that it made progress with the theory possible which would be otherwise very difficult. The physical consequence of this assumption was that the dust universe model had a history in which total amount of cosmic background mass M_Γ has to remain constant even though its local density can change with time. Clearly this is closely related to the problem of the necessity of time wise patching different radiation or matter dominating solution to get a complete model. Returning to this issue a few years later, I have found that this restriction can easily be removed making no difference to the basic form of the physics or mathematics of the structure. The need to replace this restriction was pointed out to me by Professor C. W. Kilmister some years ago. There is a big bonus earned in making this change. I shall show in the next section that the original model can easily be replaced by model with one rigorous solution to the field equations holding over all time, $-\infty \to +\infty$ but which can change its matter character type as it evolves with time. This is achieved by making a minor addition to the original model related to the definition of temperature. This addition then becomes an input function of a time parameter that users of the theory can chose to fit any theoretical or measured continuous time sequence of mass character that they decide best fits an evolving cosmology universe model. Let us consider the definitions of mass density and temperature from the dust universe model ([23])

$$\begin{aligned}
\rho(t) &= (3/(8\pi G))(c/R_\Lambda)^2(\sinh^{-2}(3ct/(2R_\Lambda)) && (4.2)\\
T(t) &= \pm(M_\Gamma 3c^4/(8\pi a(R_\Lambda)^2 M_U G\sinh^2(3ct/(2R_\Lambda))))^{1/4} && (4.3)\\
T^4(t) &= \pm(M_\Gamma c^2\rho(t)/(aM_U)) && (4.4)\\
& && (4.5)
\end{aligned}$$

The third equation above arises from the first two and gives the temperature at time t in terms of the density at time t. The total mass of the cosmic microwave background radiation in the universe, M_Γ, in this formula is taken to be a constant in that theory. Thus we were able to write the ratio of the temperature at two different times t_1 and t_2 as equal to the ratio of the fourth roots of the density at the same two times as,

$$\left(\frac{T(t_1)}{T(t_2)}\right)^4 = \left(\frac{\rho(t_1)}{\rho(t_2)}\right). \tag{4.6}$$

On the other hand, if M_Γ is some arbitrary function of time, $M_\Gamma(t)$, say, then we have to replace the formula (4.6) with(4.7),

$$\begin{aligned}\left(\frac{T(t_1)}{T(t_2)}\right)^4 = \left(\frac{M_\Gamma(t_1)\rho(t_1)}{M_\Gamma(t_2)\rho(t_2)}\right) &= \Gamma(t_1,t_2)\left(\frac{\rho(t_1)}{\rho(t_2)}\right) && (4.7)\\ \Gamma(t_1,t_2) &= \frac{M_\Gamma(t_1)}{M_\Gamma(t_2)} && (4.8)\\ \Gamma(t_1,t_2) &= \Gamma^{-1}(t_2,t_1). && (4.9)\\ \Gamma(t_2,t_1)\left(\frac{T(t_1)}{T(t_2)}\right)^4 &= \left(\frac{\rho(t_1)}{\rho(t_2)}\right). && (4.10)\end{aligned}$$

From (4.2) and (4.3) it can be seen that making M_Γ time dependent can have no effect on $\rho(t)$ as M_Γ only occurs in the temperature, (4.3). Also, the temperature is not used to define any other quantities in the theory so that the transition from the old theory to its modification can be read off from (4.10) as replace all occurrences in the original theory of the temperature times ratio in (4.7) with the modified ratio form as indicated as below

$$\left(\frac{T(t_1)}{T(t_2)}\right)^4 \rightarrow \Gamma(t_2,t_1)\left(\frac{T(t_1)}{T(t_2)}\right)^4. \qquad (4.11)$$

The only restriction on choosing the function of time $M_\Gamma(t)$ for making this transition is that it will have to conform to (4.1) as below

$$M_U = M_\Delta(t) + M_\Gamma(t). \qquad (4.12)$$

That is to say that because M_U is an absolute constant M_Δ will have to depend on t and

$$\begin{aligned} M_\Gamma(t) &\le M_U \ \forall t && (4.13)\\ M_\Delta(t) &= M_U - M_\Gamma(t) \ \forall t. && (4.14)\end{aligned}$$

Otherwise the mass type conditioning function with time, $M_\Gamma(t)$, is *arbitrary*. This structure with its infinite time range continuous adaptability is obviously a great advance over the standard model time patching scheme, only applicable in about three finite time ranges, with very difficult mathematical problems associated with joining time sectors.

5 Conclusions

The mathematical uncertainties of the joining of differently physically characterised time range solutions of the Friedman equations used in the standard cosmological model can be bypassed using a single solution suitable adapted with regard to its definition of temperature. The single solution used to make the case for this change is the *dust universe model with Einstein's Lambda* which is a rigorous solution to Einstein's Field equations over all time, $-t_\infty \rightarrow +t_\infty$. The mass and momentum conservations laws hold at all times for this model so that additionally to any phase matching avoidance at time other than zero the conceptual and mathematical problems of the inflation phase of the standard model are also avoided.

Acknowledgements

I am greatly indebted to Professors Clive Kilmister and Wolfgang Rindler for help, encouragement and inspiration

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