Special Relativity Fails to Conserve Momentum

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The article attempts to show that the use of the special theory of relativity, when considering the motion of a closed mechanical system in the inertial reference systems, can lead to non-compliance with the law of conservation of momentum.

1. Introduction

The special theory of relativity can be divided into relativistic kinematics and relativistic dynamics:

Relativistic kinematics establishes a connection (Lorentz transformations) between the coordinates and time of an event, occurring at the point of the space, in one inertial reference system and coordinates and time of the same event in another inertial reference system, and the relationship between the values of projections of speeds of the point (conversion of the speeds) in appropriate times in two inertial reference systems.

Relativistic dynamics, based on the mandatory implementation of the laws of conservation of momentum and energy for the closed system of bodies, whose interaction is instantaneous in nature, in inertial reference systems, establishes the dependences of mass and momentum of a point material body from its speed.

This article proposes:

- to take a closed mechanical system of bodies, whose interaction will be permanent;
- to select two mobile and immobile inertial reference systems with respect to the center of mass of a closed system of bodies;
- to select two points in time in the mobile inertial reference system;
- with the help of the Lorentz transformation to determine the position of bodies in the selected points in time in the mobile reference system;
- using the conversion speeds to determine the projections of speeds of bodies in these moments of time in the mobile reference system;
- to determine the values of the momentum of the bodies at the selected points in time in the mobile reference system, knowing the values of projections of speeds of the bodies, and using the dependencies of mass and momentum of a body on the speed;
- to write the law of conservation of momentum for the two selected points in time in the mobile reference system and determine the conditions for its validity.

2. The Main Dependences of Special Relativity

Assume that there are two inertial reference systems, shown in Fig. 1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$ in which:

- similar the axis of the Cartesian coordinate systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are pairs parallel and equally directed;
- system $O_2x_2y_2z_2$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V along the axis O_1x_1;
- in both systems the start timing (t_1=0 and t_2=0) is selected when the origins O_1 and O_2 of these systems are identical.

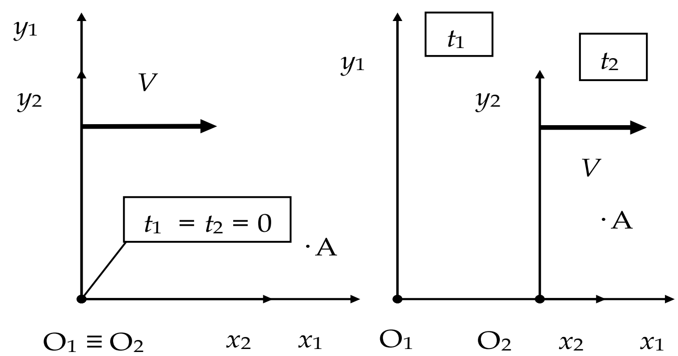

Fig. 1

In the special theory of relativity Lorentz transformations, [1] the relationship between the coordinates x_1, y_1, z_1 of point A at time t_1 in a stationary inertial reference system $O_1x_1y_1z_1$ and coordinates x_2, y_2, z_2 of the same point A in the mobile inertial reference system $O_2x_2y_2z_2$ at the time t_2, correspond to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ as follows:

$$x_1 = \frac{x_2 + (V \cdot t_2)}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{1}$$

$$x_2 = \frac{x_1 - (V \cdot t_1)}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{2}$$

$$y_1 = y_2 \tag{3}$$

$$z_1 = z_2 \tag{4}$$

where c is the speed of light in a vacuum. From Eqs. (1) and (2) we can write the dependence for times t_1 and t_2:

$$t_1 = \frac{t_2 + \frac{V \cdot x_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{5}$$

$$t_2 = \frac{t_1 - \frac{V \cdot x_1}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{6}$$

Also in the special theory of relativity velocity transformations, the relationship between the projections v_{x1}, v_{y1} and v_{z1} of the speed of a point on the axis of the Cartesian coordinates in time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ and similar projections v_{x1}, v_{y1} and v_{z1} of the speed of the same point in the mobile inertial reference system $O_2x_2y_2z_2$ at time t_2, correspond to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ as: [1]

$$v_{x1} = \frac{v_{x2} + V}{1 + \frac{V \cdot v_{x2}}{c^2}} \tag{7}$$

$$v_{x2} = \frac{v_{x1} - V}{1 - \frac{V \cdot v_{x1}}{c^2}} \tag{8}$$

$$v_{y1} = \frac{v_{y2} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v_{x2}}{c^2}} \tag{9}$$

$$v_{y2} = \frac{v_{y1} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x1}}{c^2}} \tag{10}$$

The dependence of the mass $M(v)$ and momentum $\overline{P}(v)$ of a moving body, having rest mass M_o, on the speed $\overline{v}$ in the special theory of relativity are taken in the forms: [1]

$$M(v) = \frac{M_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

$$\overline{P}(v) = \frac{M_o \cdot \overline{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{12}$$

3. A Closed Mechanical System of Bodies

Consider the simplest closed mechanical system of the bodies, which have constant interaction. Assume that there are two inertial reference systems, similar to those of reference systems, shown in Fig. 1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves with speed V parallel to the axis O_1x_1 relative to the system $O_1x_1y_1z_1$.

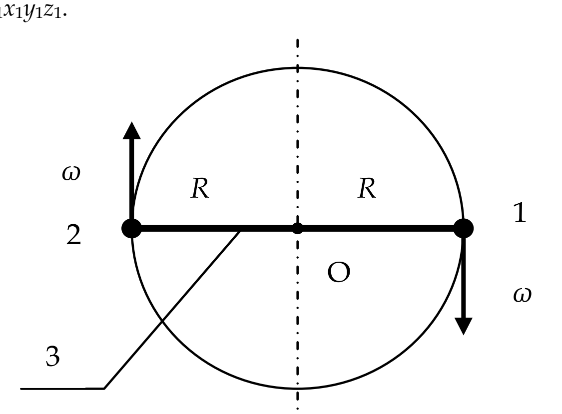

Fig. 2

Suppose there is a closed mechanical system of bodies, shown in Fig. 2, consisting of point bodies 1 and 2, with equal mass M_o at rest, and a string 3. Bodies 1 and 2 are connected by string 3, the mass of which can be neglected because of its smallness. Bodies 1 and 2 rotate with angular speed ω around a common center of mass - the point O. Distance from the point body 1 (body 2) to point O is equal to R.

Let's put the closed mechanical system of bodies 1 and 2 and string 3 in the moving reference system $O_2x_2y_2z_2$, so that point O would be stationary in this reference system and coincident with origin O_2, and the rotation of bodies 1 and 2 around it would occur in a clockwise direction in the plane of $O_2x_2y_2$, as shown in Fig. 3.

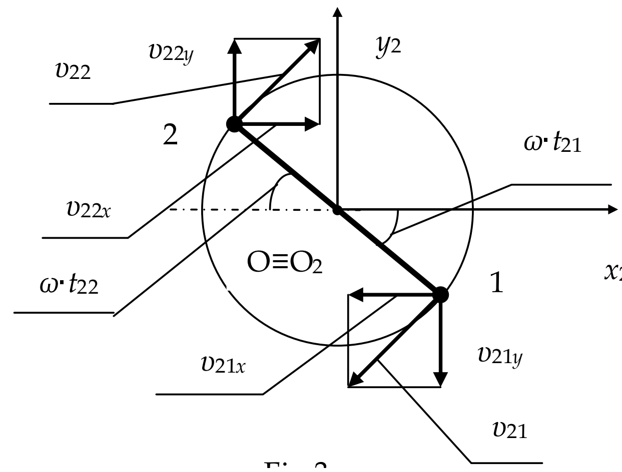

Fig. 3

Also assume that at the start of timing (t_2=0) in reference system $O_2x_2y_2z_2$, bodies 1 and 2 were on the axis O_2x_2 with body 1 on a positive coordinate and the body 2 on a negative one.

In the mobile reference system $O_2x_2y_2z_2$, at any time t_2 bodies 1 and 2 will have the speeds v_{21} and v_{22}, equal to v_R :

$$v_{21} = v_{22} = v_R = \omega \cdot R \tag{13}$$

In this case, the projections v_{21x} and v_{21y} of speed of the body 1 and the projections v_{22x} and v_{22y} of speed of the body 2 on the axis O_2x_2 and O_2y_2, respectively, for times t_{21} and t_{22} will equal:

$$v_{21x} = -[v_R \cdot \sin(\omega \cdot t_{21})] \tag{14}$$

$$v_{21y} = -[v_R \cdot \cos(\omega \cdot t_{21})] \tag{15}$$

$$v_{22x} = v_R \cdot \sin(\omega \cdot t_{22}) \tag{16}$$

$$v_{22y} = v_R \cdot \cos(\omega \cdot t_{22}) \tag{17}$$

The relationship between the coordinates x_{21} and y_{21} of the body 1 depending on time t_{21} and the relationship between the coordinates x_{22} and y_{22} of the body 2 depending on the time t_{22} in the mobile reference system $O_2x_2y_2z_2$ can be written:

$$x_{21} = R \cdot \cos(\omega \cdot t_{21}) \tag{18}$$

$$y_{21} = -[R \cdot \sin(\omega \cdot t_{21})] \tag{19}$$

$$y_{22} = -[R \cdot \cos(\omega \cdot t_{22})] \tag{20}$$

$$y_{22} = R \cdot \sin(\omega \cdot t_{22}) \tag{21}$$

Based on the Eqs. (1) and (3), we can write the relationships between:

- coordinates x_{11} and y_{11} of the body 1 at time t_{11} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{21} and y_{21} of the body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_{21} , which corresponds to the time t_{11} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11} = \frac{x_{21} + (V \cdot t_{21})}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{22}$$

$$y_{11} = y_{21} \tag{23}$$

- coordinates x_{12} and y_{12} of the body 2 at time t_{12} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{22} and y_{22} of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_{22}, which corresponds to the time t_{12} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12} = \frac{x_{22} + (V \cdot t_{22})}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{24}$$

$$y_{12} = y_{22} \tag{25}$$

Using Eq. (5), the relationship between the values of the times t_{11} and t_{21}, t_{12} and t_{22} will look like this:

$$t_{11} = \frac{t_{21} + \frac{V \cdot x_{21}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{26}$$

$$t_{12} = \frac{t_{22} + \frac{V \cdot x_{22}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{27}$$

We are interested in the position of bodies 1 and 2 in the stationary reference system $O_1x_1y_1z_1$ at the same time, i.e. where:

$$t_{11} = t_{12} \tag{28}$$

Taking into account Eqs. (18), (20), (26) and (27), Eq. (28) becomes

$$t_{21} + \frac{V \cdot R \cdot \cos(\omega \cdot t_{21})}{c^2} = t_{22} + \frac{V \cdot R \cdot \cos(\omega \cdot t_{22})}{c^2} \tag{29}$$

Now consider two points in time in the stationary reference system $O_1x_1y_1z_1$.

4. Moment of Time t_{1p}

In the mobile reference system $O_2x_2y_2z_2$ when performing the condition (28) it is interesting position of the bodies 1 and 2 at the time t_{2p} when:

$$t_{21} = t_{22} = t_{2p} \tag{30}$$

Substituting condition (30) in Eq. (29) for the case $\omega t_{2p} < \pi$ we obtain:

$$\omega \cdot t_{2p} = \frac{\pi}{2} \tag{31}$$

I.e., as shown in Fig .4, under the terms of (28), (30) and (31) in the moving mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2 are on a line parallel to the axis O_2y_2 and in the stationary reference system $O_1x_1y_1z_1$ the bodies 1 and 2 will be on a line parallel to the axis O_1y_1 at time $t_{11}(t_{12})$, equal t_{1p} and which corresponds to the time t_{2p} in the mobile reference system $O_2x_2y_2z_2$.

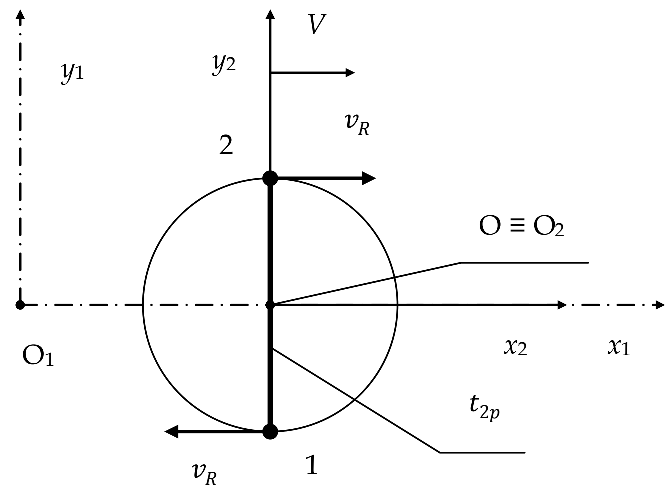

Fig. 4

According to Eqs. (31), (14) - (17) in the mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2, respectively, have the following values of the projections v_{21xp}, v_{21yp} and v_{22xp}, v_{22yp} of speeds of his movement on the axis O_2x_2 and O_2y_2:

$$v_{21xp} = -v_R \tag{32}$$

$$v_{21yp} = 0 \tag{32}$$

$$v_{22xp} = v_R \tag{33}$$

$$v_{22yp} = 0 \tag{34}$$

Then, on the basis of Eqs. (7), (9) and equalities (32) - (35), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections v_{11xp}, v_{11yp} and v_{12xp}, v_{12yp} of speeds of his movement on the axis O_1x_1 and O_1y_1:

$$v_{11xp} = \frac{V - v_R}{1 - \frac{V \cdot v_R}{c^2}} \tag{36}$$

$$v_{11yp} = 0 \tag{37}$$

$$v_{12xp} = \frac{V + v_R}{1 + \frac{V \cdot v_R}{c^2}} \tag{38}$$

$$v_{12yp} = 0 \tag{39}$$

Hence, using Eqs. (11) and (12), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections P_{11xp}, P_{11yp} and P_{12xp}, P_{12yp} of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11xp}=\frac{M_o\cdot v_{11xp}}{\sqrt{1-\frac{v_{11x}^2}{c^2}}} \quad (40)$$

$$P_{12xp}=\frac{M_o\cdot v_{12xp}}{\sqrt{1-\frac{v_{12x}^2}{c^2}}} \quad (41)$$

$$P_{11yp}=0 \quad (42)$$

$$P_{12yp}=0 \quad (43)$$

5. Moment of Time t_{1h}

Also in the mobile reference system $O_2x_2y_2z_2$, when (28) holds, it is interesting that the position of body 2 when finding body 1 on the axis O_2x_2 at t_{21} equals t_{21h} , where:

$$t_{21h}=0 \quad (44)$$

Time t_{22}, when (28) and (44) hold, is denoted t_{22h}, for which Eq. (29) becomes:

$$\omega\cdot t_{22h}=\frac{v_R\cdot V}{c^2}\cdot[1+\cos(\omega\cdot t_{22h})] \quad (45)$$

From Eq. (45), the value of time t_{22h} must be greater than 0.

Under the terms of (28) and (44) in the mobile reference system $O_2x_2y_2z_2$ at time $t_{21h}=0$, body 1 will be located on the O_2x_2 axis. In the stationary reference system $O_1x_1y_1z_1$, body 1 will be located on the O_1x_1 axis at time t_{11} (t_{12}), equal to t_{1h}, corresponding to $t_{21h}=0$ in the mobile reference system $O_2x_2y_2z_2$.

Moreover, in the mobile reference system $O_2x_2y_2z_2$, according to Eq. (45), body 2 cannot lie on axis O_2x_2 at time t_{22}, corresponding to time t_{1h} in stationary reference system $O_1x_1y_1z_1$.

I.e., as shown in Fig. 5, body 1 is located on the O_1x_1 axis in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h}, corresponding to time $t_{21h}=0$ in the mobile reference system $O_2x_2y_2z_2$. At time t_{1h}, body 2 cannot lie on axis O_2x_2.

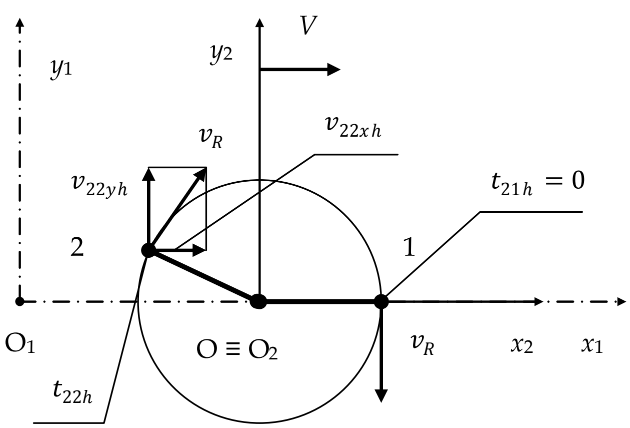

Fig. 5

In the mobile reference system $O_2x_2y_2z_2$ the body 1 at the time $t_{21h}=0$ and the body 2 at the time t_{22h} respectively have projections v_{21xh}, v_{21yh} and v_{22xh}, v_{22yh} of speeds of his movement on the axis O_2x_2 and O_2y_2, so that:

$$v_{21xh}=0 \quad (46)$$

$$v_{21yh}=-v_R \quad (47)$$

Then, by Eqs. (7), (9) and (46), (47), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h}, body 1 and body 2 respectively will have projection values v_{11xh}, v_{11yh} and v_{12xh}, v_{12yh} of their speeds along the axis O_1x_1 and O_1y_1:

$$v_{11xh}=V \quad (48)$$

$$v_{11yh}=-\left(v_R\cdot\sqrt{1-\frac{V^2}{c^2}}\right) \quad (49)$$

$$v_{12xh}=\frac{V+v_{22xh}}{1+\frac{V\cdot v_{22xh}}{c^2}} \quad (50)$$

$$v_{12yh}=\frac{v_{22yh}\cdot\sqrt{1-\frac{V^2}{c^2}}}{1+\frac{V\cdot v_{22xh}}{c^2}} \quad (51)$$

Given Eq. (45), we note that, i.e. the time $t_{22h}>0$, so the projection v_{22yh} of the speed will be the direction of axis O_2y_2. From Eqs. (16) and (17) it follows that:

$${v_{22xh}}^2+{v_{22yh}}^2={v_R}^2 \quad (52)$$

From Eqs. (11) and (12), it may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h}, body 1 and body 2 respectively will have the following values of the projections P_{11xh}, P_{11yh} and P_{12xh}, P_{12yh} of momentums on the axis O_1x_1 and O_1y_1

$$P_{11xh}=\frac{M_o\cdot v_{11xh}}{\sqrt{1-\frac{v_{11xh}^2+v_{11yh}^2}{c^2}}} \quad (53)$$

$$P_{12xh}=\frac{M_o\cdot v_{12xh}}{\sqrt{1-\frac{v_{12xh}^2+v_{12yh}^2}{c^2}}} \quad (54)$$

$$P_{11yh}=\frac{M_o\cdot v_{11yh}}{\sqrt{1-\frac{v_{11xh}^2+v_{11yh}^2}{c^2}}} \quad (55)$$

$$P_{12yh}=\frac{M_o\cdot v_{12yh}}{\sqrt{1-\frac{v_{12xh}^2+v_{12yh}^2}{c^2}}} \quad (56)$$

6. Verification of the Law of Conservation of Momentum

The law of conservation of momentum of a closed mechanical system of bodies, connected with the symmetry properties of space - the homogeneity of space [1], states, that the momentum of a closed mechanical system of bodies (which is not acted upon by external forces) is a constant value, i.e., in any inertial reference system for any point in time the value of the momentum of a closed mechanical system of bodies is a constant value (because there is no external influence).

Due to the fact, that the mechanical system of the bodies 1 and 2 (and string 3) is closed, the law of conservation of momentum can write the following equations for the moments of times t_{1p} and t_{1h}:

$$P_{11xp} + P_{12xp} = P_{11xh} + P_{12xh}$$

$$P_{11yp} + P_{12yp} = P_{11yh} + P_{12yh}$$

or:

$$\frac{M_o \cdot v_{11xp}}{\sqrt{1-\frac{v_{11xp}^2}{c^2}}} + \frac{M_o \cdot v_{12xp}}{\sqrt{1-\frac{v_{12xp}^2}{c^2}}} = \frac{M_o \cdot v_{11xh}}{\sqrt{1-\frac{v_{11xh}^2 + v_{11yh}^2}{c^2}}} + \frac{M_o \cdot v_{12xh}}{\sqrt{1-\frac{v_{12xh}^2 + v_{12yh}^2}{c^2}}} \quad (57)$$

$$0 = \frac{M_o \cdot v_{11yh}}{\sqrt{1-\frac{v_{11xh}^2 + v_{11yh}^2}{c^2}}} + \frac{M_o \cdot v_{12yh}}{\sqrt{1-\frac{v_{12xh}^2 + v_{12yh}^2}{c^2}}} \quad (58)$$

By inserting the projections v_{11xp}, v_{12xp}, v_{11xh}, v_{11yh} and v_{12xh}, v_{12yh} of speeds of Eqs. (36), (38), (48) - (51) in Eqs. (57) and (58) and using the Eq. (52), we obtain:

$$\frac{M_o \cdot (V - v_R)}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} + \frac{M_o \cdot (V + v_R)}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} =$$

$$= \frac{M_o \cdot V}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} + \frac{M_o \cdot (V + v_{22xh})}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} \quad (59)$$

$$0 = -\frac{M_o \cdot v_R}{\sqrt{1-\frac{{v_R}^2}{c^2}}} + \frac{M_o \cdot + v_{22yh}}{\sqrt{1-\frac{{v_R}^2}{c^2}}} \quad (60)$$

or:

$$V - v_R + V + v_R = V + V + v_{22xh} \quad (61)$$

$$0 = -v_R + v_{22yh} \quad (62)$$

From Eqs. (61) and (62), we obtain the necessary conditions (the values of the projections v_{22xh} and v_{22yh} of speeds), which in this example will validate the law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$:

$$v_{22xh} = 0 \quad (63)$$

$$v_{22yh} = v_R \quad (64)$$

Substituting conditions (63) and (64) in Eqs. (16) and (17), we obtain:

$$t_{22h} = t_{21h} = 0 \quad (65)$$

And substituting Eq. (65) into Eq. (45):

$$\omega \cdot 0 = \frac{v_R \cdot V}{c^2} \cdot [1+1] \quad (66)$$

will be another condition to validate the conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$. For example, consider:

$$0 = \frac{1}{c^2} \quad (67)$$

But since the speed of light c is not infinite, condition (67) is not feasible, and therefore in this case, the law of conservation of momentum cannot be validated.

I.e., we can conclude, that in the stationary inertial reference system $O_1x_1y_1z_1$ the application of the special theory of relativity to describe the motion of a closed mechanical system of bodies, considered in this example, leads to non-compliance of the law of conservation of momentum.

7. Conclusion

It can be concluded, that the use of the special theory of relativity in dealing with individual examples may lead to non-compliance with the law of conservation of momentum for a closed mechanical system in the inertial reference systems.

Given, that the law of conservation of momentum associated with the homogeneity of space [1], we can assume, that the failure of the law of conservation of momentum will lead to non-compliance with conditions of symmetry of space and time, on which is based the special theory of relativity.

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References

[1] Яворский Б.М., Детлаф А.А., Справочник по физике, Наука, Москва (1980).