Definition of Fundamental Quantities With Respect to Space and Time

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In the SI system, the dimensions of physical quantities are derived from seven base dimensions corresponding to the seven fundamental quantities. This document shows how it might be possible to derive the dimensions of all quantities from space and time only.

1. Introduction

In the SI system, seven base elements constitute the fundamental quantities and their dimension is not derived from another one. This document shows that it seems possible to derive the dimension of all quantities from space and time only. For this, we will present an innovative method using Planck quantities' values in order to discover their dimensions.

2. The Dimensional System

2.1. The Concept of Dimension

The term dimension should be used with care as it has at least three distinct meanings:

- The space-time continuum's four (4) dimensions, which are space, space, space and time
- The "twilight zone" dimensions such as multiple universes and parallel worlds
- The fundamental constitution of physical values based on the seven fundamental quantities.

This study is related to the fundamental constitution of physical quantities and states as an initial hypothesis that the dimension of all physical quantities is derivable from space and time only. This means not only that the SI system's fundamental values can be reduced to two components (space and time), but also that the Planck quantum value of all physical quantities can be easily found from their new dimensional expression. This obviously offers new perspectives on the determination and usage of fundamental quantities.

While this study is performed by using some chosen specific physical quantities such as mass, gravitational constant, Coulomb constant, electric charge, etc., by knowing the relation between quantities, the initial chosen quantities could very well have been different. For example, we heavily use k, the Coulomb constant, but because we know that $k = 1/4\pi\varepsilon_o$ and $c^2 = 1/\mu_o\varepsilon_o$, we could easily use μ or ε in the demonstrations instead of k. On the same idea, only the Gravitic and the electric domains are mentioned and used in demonstrations. However, because all other quantities can be derived from these ones, adjusting the calculation is trivial.

The dimensional system should also be distinguished from the unit system which is a totally different thing. While dimensions reveal the nature and interrelation of physical quantities, units are abstract and related to the measure of these quantities. For example, a Tesla and a Weber can tell us the amount of the magnetic field or the magnetic flux, but nothing about their relation. On the other hand, the dimensional system will ultimately show us that the flux is linked to the field through the square meter ([Flux] = [Field] * L^2). Due to the abstract nature of units, unit coherence does not really mean a lot, while dimensional coherence is an absolute prerequisite to accept a formula.

2.2. Dimensioning Physical Quantities

The SI system defines seven fundamental quantities whose dimensions are the base from which are derived the dimensions of all other physical quantities. These basic dimensions are:

Name	Unit	Symbol
Distance	Meter	L
Mass	Kilogram	M
Time	Second	T
Current	Ampere	I
Light	Candela	J
Heat	Kelvin	K
Concentration	Mole	N

Table 1 Fundamental Dimensions

Any physical quantity Q can then be dimensionally expressed as:

$$[Q] = L^aM^bT^cI^dJ^eK^fN^g \quad (1)$$

With exponents, "a … g", representing the influence of each constituent on the final quantity and the sign of the exponent indicating a direct or inverse (1/x) proportionality. The notation [Q] means "the dimension of Q".

This system allows us to perform dimensional analysis based on the requirement that each side of an equation must be of the same dimension. This is why it became the mandatory tool to validate a formula.

Considering that concentration (Mole) is dimensionless and that heat and light can be derived from energy, it means that it is enough to define mass and electric current in addition to space and time. Furthermore, once the relation between mass and electric current is known, it is only necessary to define one of them.

3. Space-Time Derivation

The idea to derive mass and electric current dimensions from space and time only is not new and J.C. Maxwell already touched the subject in his treatise on electromagnetism [1] when he stated

"… If, as in the astronomical system, the unit of mass is defined with respect to its attractive power, the dimensions of [*M*] are [L^3T^{-2}]. …"

However, we can easily demonstrate from Newton's law of gravitation that it is in fact *G*M* that dimensionally corresponds to L^3T^{-2} because:

$$F = M * a = G * M^2 / r^2 \quad (2)$$

Then:
$$G * M = a * r^2 \quad (3)$$

This leads to:
$$[GM] = LT^{-2} * L^2 = L^3T^{-2} \quad (4)$$

Maxwell did choose to assume, without further explanation, that the gravitational constant *G* is dimensionless and that [*M*] corresponds to L^3T^{-2}. However, the truth is that nothing really supports this assumption and the basic Cartesian product demonstrates that it is invalid, as we will show later. This is the reason why the SI system still uses the mass as a fundamental quantity. If it was so clear and proven that [*M*] = L^3T^{-2}, then the BIPM could immediately remove the physical mass, the one located in Paris, from fundamental quantities (one of their objectives) or at least, provide new ways of determining unit mass' dimension and properties. In addition, there is nothing in current knowledge that allow us to find the dimension of *G* and *M* separately without using a circular reasoning or an initial arbitrary choice.

Concerning the electric charge, its definition is usually based on the mass's definition, but another difficulty arises because apparently two incompatible possibilities exist. The electrostatic version where the Coulomb's constant *k* is assumed dimensionless and [*Q*] = $M^{1/2}L^{3/2}T^{-1}$, and the electromagnetic version where it is [*Q*] = $M^{1/2}L^{1/2}$ because μ, the permeability, is then considered dimensionless. They are so incompatible that the first one incorporates time, while the other does not! Our study will show that one of these options is supported by numerical evidence, while the other is not.

4. The Visual Representation

There is a way to find a correlation between the dimensions of quantities and space-time. We base this demonstration on a matrix presenting the Cartesian product of the Planck space and time sets.

Consider a matrix whose horizontal axis corresponds to space, while the vertical axis represents time, as illustrated:

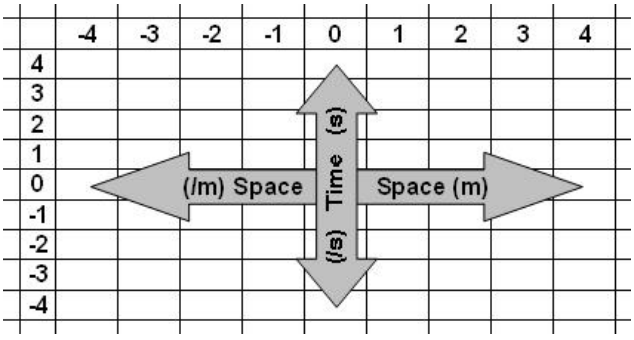

Fig. 1. The Space-Time Matrix

The indices of the matrix directly correspond to the exponents of time and space dimensions. Based on these principles, it is easy to locate the basic elements such as:

	-4	-3	-2	-1	0	1	2	3	4
4									
3									
2									
1					Time				
0					1	Length	Surface	Volume	
-1					Freq.				
-2									
-3									
-4									

Fig. 2. The Basic Elements

If we now replace distance and time by the Planck length and time respectively, we can then directly locate the "Planck quantities":

-1	0	1	2	3	4
	5,3912E-44				
	1	1,6163E-35	2,6123E-70	4,2221E-105	
	1,8549E+43				

Fig. 3. Planck Quantities

We then fill the cells of the matrix with a formula that reflects the relation between a quantity and space-time such as:

$$Q_p = L_p^x * T_p^y \quad (5)$$

With Q_p = Planck quantity, L_p = Planck length, T_p = Planck time, *x* and *y* = exponents of the dimensions of L_p and T_p (or indices on the Matrix).

-2	-1	0	1	2	3	4
2340E-104	5,2269E-139	8,4480E-174	1,3654E-208	2,2068E-243	3,5668E-278	0
,9986E-61	9,6952E-96	1,5670E-130	2,5326E-165	4,0934E-200	6,6160E-235	1,0693E-269
,1127E-17	1,7983E-52	2,9065E-87	4,6977E-122	7,5927E-157	1,2272E-191	1,9834E-226
,0638E+26	3,3356E-09	**5,3912E-44**	8,7136E-79	1,4083E-113	2,2762E-148	3,6790E-183
,8281E+69	6,1872E+34	**1**	**1,6163E-35**	**2,6123E-70**	**4,2221E-105**	6,8240E-140
1006E+112	1,1476E+78	1,8549E+43	**2,9979E+08**	4,8454E-27	7,8314E-62	1,2658E-96
3171E+156	2,1287E+121	3,4405E+86	5,5607E+51	8,9876E+16	1,4526E-18	2,3478E-53
4430E+199	3,9484E+164	6,3817E+129	1,0314E+95	1,6671E+60	2,6944E+25	4,3548E-10
5314E+242	7,3238E+207	1,1837E+173	1,9132E+138	3,0922E+103	4,9977E+68	8,0776E+33

Fig. 4. Filling the Matrix

This means that this matrix represents the Cartesian product, or all combinations, of Planck space and time sets.

	-4	-3	-2	-1	0	1	2	3	4
4	1,2380E-34	2,0009E-69	3,2340E-104	5,2269E-139	8,4480E-174	1,3654E-208	2,2068E-243	3,5668E-278	0
3	2,2963E+09	3,7114E-26	5,9986E-61	9,6952E-96	1,5670E-130	2,5326E-165	4,0934E-200	6,6160E-235	1,0693E-269
2	4,2593E+52	6,8841E+17	1,1127E-17	1,7983E-52	2,9065E-87	4,6977E-122	7,5927E-157	1,2272E-191	1,9834E-226
1	7,9004E+95	1,2769E+61	2,0638E+26	3,3356E-09	**5,3912E-44**	8,7136E-79	1,4083E-113	2,2762E-148	3,6790E-183
0	1,4654E+139	2,3685E+104	3,8281E+69	6,1872E+34	**1**	**1,6163E-35**	**2,6123E-70**	**4,2221E-105**	6,8240E-140
-1	2,7182E+182	4,3932E+147	7,1006E+112	1,1476E+78	1,8549E+43	**2,9979E+08**	4,8454E-27	7,8314E-62	1,2658E-96
-2	5,0416E+225	8,1486E+190	1,3171E+156	2,1287E+121	3,4405E+86	5,5607E+51	8,9876E+16	1,4526E-18	2,3478E-53
-3	9,3519E+268	1,5115E+234	2,4430E+199	3,9484E+164	6,3817E+129	1,0314E+95	1,6671E+60	2,6944E+25	4,3548E-10
-4	#NOMBRE!	2,8036E+277	4,5314E+242	7,3238E+207	1,1837E+173	1,9132E+138	=Lp3 * Tp-2	4,9977E+68	8,0776E+33

Fig. 5. Cell Formula

Each cell being a combination of space and time, therefore any value defined from Planck space and time only must appear on this matrix.

To validate our concept, we must find at specific locations on the matrix some values that we know for sure to be derived only from space and time. For example, we immediately recognize the speed of light which corresponds to $L_p * T_p^{-1}$ and is consequently located at position [1, -1]. But we also find other elements such as the already mentioned [*GM*] = L^3T^{-2} at location (3,-2), and because from the definition of Planck quantities, we have

$$M_p^2 = \hbar c / G \quad (6)$$

So

$$\hbar / M_p = G M_p / c \quad (7)$$

Then, because [*GM*] = L^3T^{-2}, it follows that

$$[\hbar / M_p] = L^2T^{-1} \quad (8)$$

Now, by multiplying both side by GM_p, we also have

$$[\hbar G] = L^5T^{-3} \quad (9)$$

These are interesting values because we know and can prove that their dimension is only defined from space-time, whatever the dimension of their individual terms can be. So we are absolutely sure they must appear on the matrix without any multiplicative factor and at the right place as we can see

	-4	-3	-2	-1	0	1	2	3	4	5	6
5	6,6743E-78	1,0787E-112	1,7439E-147	2,8179E-182	**4,5545E-217**	7,3613E-252	1,1898E-286				
4	1,2380E-34	2,0009E-69	3,2340E-104	5,2269E-139	**8,4480E-174**	1,3654E-208	2,2068E-243	3,5668E-278			
3	2,2963E-09	3,7114E-26	5,9998E-61	9,6952E-96	**1,5670E-130**	2,5326E-165	4,0934E-200	6,6169E-235	1,0693E-269	1,7283E-304	
2	4,2593E-52	6,8841E+17	1,1127E-17	1,7983E-52	**2,9065E-87**	4,6977E-122	7,5927E-157	hbar/Mp 191		3,2057E-261	5,1812E-
1	7,9004E-95	1,2769E+61	2,0638E+26	3,3356E-09	**5,3912E-44**	8,7136E-79	1,4083E-113	2,2762E-148	G*Mp	5,9445E-261	9,6104E-
0	**1,4654E+139**	**2,3685E+104**	**3,8281E+69**	**6,1872E+34**	**1**	**1,6163E-35**	**2,6123E-70**	**4,2221E-105**	**6,8240E-140**	**1,102**	hbar * G **6E-2**
-1	2,7182E+182	4,3932E+147	7,1006E+112	1,1476E-78	**1,8549E+43**	2,9979E+08	**4,8454E-27**	7,8314E-62	1,2659E-96	2,0459E-131	3,3065E-
-2	5,0418E-225	8,1488E+190	1,3171E+156	2,1287E+121	**3,4405E-86**	5,5607E+51	8,9876E+16	**1,4526E-18**	2,3478E-53	3,7944E-88	6,1331E-
-3	9,3518E-269	1,5116E-234	2,4430E+199	3,9494E+164	**6,3817E+129**	1,0314E+95	1,6671E+60	2,6944E+25	4,3548E-10	**7,0385E-45**	1,1376E
-4		2,8036E+277	4,5314E+242	7,3239E+207	**1,1837E+173**	1,9132E+138	3,0922E+103	4,9977E+68	8,0776E+33	1,3055E-01	2,1101E
-5			8,4050E+285	1,3585E+251	**2,1956E+216**	3,5487E+181	5,7356E+146	9,2701E+111	1,4983E+77	2,4216E+42	3,9139E
-6				2,5198E+294	**4,0726E+259**	6,5823E+224	1,0639E+190	1,7195E+155	2,7791E+120	4,4917E+85	7,2598E
-7					**7,5541E+302**	1,2209E+268	1,9733E+233	3,1894E+198	5,1549E+163	8,3316E+128	1,3466E
-8							3,6602E+276	5,9159E+241	9,5616E+206	1,5454E+172	2,4977E+
-9								1,0973E+285	1,7735E+250	2,8665E+215	4,6330E+1
-10									3,2897E+293	5,3169E+258	8,5935E+

Fig. 6. Reference Values

We can also confirm that numerically

$$G * M_p = L_p^3 * T_p^{-2} \quad (10)$$

and

$$\hbar / M_p = L_p^2 * T_p^{-1} \quad (11)$$

and finally

$$\hbar * G = L_p^5 * T_p^{-3} \quad (12)$$

This validates our base formula and it is obvious that all combinations based on Planck space and time will appear on such matrix.

Therefore, if our hypothesis is correct concerning the fact that dimensions of physical quantities can be derived from space and time only, their "Planck value" will appear on this matrix at the location corresponding to their space and time dimensions.

Now, the definite proof that Maxwell's assumption is false lies in the logic and mathematics behind it. By proposing [*M*] = L^3T^{-2}, Maxwell is already in the context that the dimension of mass can be derived from space and time only. In this context, if we study the Cartesian product of the space and time sets as per the set theory, then all physical quantities defined from the initial sets must be into the resulting set. By creating a matrix whose axis are the Planck length and time, we provide the Cartesian product of these two sets. The Planck mass does not show at the (3,-2) location (which corresponds to L^3T^{-2}), indicating that if the dimension of mass is derived from space time, it is not L^3T^{-2}. However, one could argue that the numerical value of the Planck mass is "hidden" behind the *GM* product that appears at L^3T^{-2}. While it is highly suspect, it is still not totally impossible that the result of a multiplication hides one of the terms if the other is dimensionless. But what about all other quantities whose space-time dimension could then be easily determined, but are not found on the numerical matrix at the expected locations? Take for example the Dirac constant $\hbar$ used in the definition of all Planck's values. It must also appear because it can be defined from the mass which is, per our hypothesis, defined from space and time. From the definition of the Planck mass, we know that $\hbar = GMp^2 / c$, so in the Maxwell version, the Dirac constant $\hbar$ becomes

$$[\hbar] = ML^2T^{-1} = L^5T^{-3} \quad (13)$$

The numerical concordance should exist and so we should have

$$\hbar = L_p^5 * T_p^{-3} \quad (14)$$

This is obviously not the case and the same can be said for all Planck values. We can then say that Maxwell's assumption fails because the Cartesian product of the space and time sets invalidates it. While we correctly find the well known values such as *GM* (L^3T^{-2}), $\hbar/M$ (L^2T^{-1}) and $\hbar G$ (L^5T^{-3}) on the matrix, no known Planck value appear at the location predicted by the Maxwell assumption.

5. The Dimension of Mass

As previously mentioned, there is an infinity of possibilities for *G* and *M* and we know now that [*M*] = L^3T^{-2} is false.

This study highlighted the presence of simple multiplicative factors, specific to each domain, whose meaning is still unknown. Hopefully, these factors appear as 1eX, which is neutral in a multiplication and doesn't modify the number itself, but only the power exponent. Therefore, and although it could be a third basic dimension in addition to space and time, they have no influence on the two dimension matrix visual output. They however affect the numerical value, but do not hide them, by only changing the power exponent. Once these factors acknowledged, we will show that it is possible to find all Planck quantities on the matrix, proving that their dimensions are derivable from space and time only.

Note that for a better reading, future figures will only display the values relevant to the discussion.

Our basic hypothesis states that the numerical values of *G* and *M* must be on the matrix and our researches show that if we incorporate the dimensionless 1e-67 multiplicative factor, the numerical value of the Planck mass appears at the location [7, -7], which means:

$$[M] = L^7T^{-7} \quad (15)$$

The gravitic domain's multiplicative factors being 1e67 and 1e-67, the constitutive formula of the Planck mass is:

$$M_p = L_p^7 * T_p^{-7} * 1e\text{-}67 \quad (16)$$

This numerically gives:

$$2.1764e\text{-}08 = (1.6162e\text{-}35)^7 * (5.3912e\text{-}44)^{-7} * 1e\text{-}67 \quad (17)$$

We also have a visual confirmation on the matrix (which does not incorporate the multiplicative factor):

	-2	-1	0	1	2	3	4	5	6	7
7										
6										
5										
4										
3			1,5670E-130							
2			2,9065E-87							
1			5,3912E-44							
0		6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105	6,8240E-140	1,1029E-174	1,7826E-209	2,8811E-244
-1			1,8549E+43	2,9979E+08						
-2			3,4405E+86			1,4526E-18				
-3			6,3817E+129							
-4			1,1837E+173							
-5			2,1956E+216							
-6			4,0726E+259							
-7			7,5541E+302							2,1764E+59

Fig. 7. Planck Mass Position

6. The Dimension of G

We know that to comply with $[GM] = L^3T^{-2}$, the dimension of G automatically becomes:

$$[G] = M^{-1} L^3T^{-2} = L^{-4}T^5 \quad (18)$$

By using the 1e67 multiplicative factor, the numerical value of G is then:

$$G = 6.6742e\text{-}11 = (1.6162e\text{-}35)^{-4} * (5.3912e\text{-}44)^5 * 1e67 \quad (19)$$

This is directly confirmed on the Matrix:

	-5	-4	-3	-2	-1	0	1	2	3
7									
6									
5		6,6743E-78				4,5545E-217			
4						8,4480E-174			
3						1,5670E-130			
2						2,9065E-87			
1						5,3912E-44			
0		1,4654E+139	2,3685E+104	3,8281E+69	6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105
-1						1,8549E+43	2,9979E+08		
-2						3,4405E+86			1,4526E-18
-3						6,3817E+129			
-4						1,1837E+173			
-5						2,1956E+216			
-6						4,0726E+259			
-7						7,5541E+302			

Fig. 8. Gravitational Constant Position

7. The Dimension of Force

Quantities associated with the energetic domain (energy, power, momentum, action, etc.) are usually defined from mass, space and time. Therefore, once the position of the mass is found, Newton's second law (F=ma) tells us that the dimension of Force becomes L^8T^{-9}, and accordingly at position [8,-9] on the Matrix we find the expected value of the Planck force. All other Planck related values from the energetic domain also appear as expected. This is also confirmed by the numerical value:

$$1.2103e44 = (1.6162e\text{-}35)^8 * (5.3912e\text{-}44)^{-9} * 1e\text{-}67 \quad (20)$$

From the dimension of Force, it is trivial to derive the dimensions of all energetic, thermic and pressuric domain elements.

	-2	-1	0	1	2	3	4	5	6	7	8	9	10
7													
6													
5													
4													
3			1,5670E-130										
2			2,9065E-87										
1			5,3912E-44										
0		6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105	6,8240E-140	1,1029E-174	1,7826E-209	2,8811E-244			
-1			1,8549E+43	2,9979E+08									
-2			2,4405E+86			1,4526E-18							
-3			6,3817E+129										
-4			1,1837E+173										
-5			2,1956E+216										
-6			4,0726E+259										
-7			7,5541E+302							2,1764E+59			
-8											6,5248E+87	1,0546E+33	
-9											1,2103E+111	1,9561E+76	
-10												3,5283E+119	
-11													

Fig. 9. Energetic Domain Elements

8. The Dimension of Electric Charge

Electric current, seen as the expression of a flow of electric charges, can be derived directly from electric charge ($I = Q / t$). However, the value of the Planck electric charge does not appear directly on the matrix. But the presence of the square of this charge (3.5176e-36 C^2) at location [8,-7] is a clear indication that the dimension of electric charge is:

$$[Q] = L^4T^{-3.5} \quad (21)$$

Considering that for electric and magnetic domains, the multiplicative factors are 1e-30, 1e37, 1e7 and 1e-7, the constitutive formula of the Planck charge is:

$$Q_p = L_p^4 * T_p^{-3.5} * 1e\text{-}30 \quad (22)$$

This gives the following numerical value:

$$1.8755e\text{-}18 = (1.6162e\text{-}35)^4 * (5.3912e\text{-}44)^{-3.5} * 1e\text{-}30 \quad (23)$$

Because the time dimension exponent is not an integer, it implies a non-integer dimension. To represent this on the matrix, it is necessary to use the square root of Planck length and time as bases for the axis of the matrix. Then, the exponents of space and time will take the values 0, 0.5, 1, 1.5 … and the electric charge automatically appears

	-1	-0,5	0	0,5	1	1,5	2	2,5	3	3,5	4
4											
3,5											
3											
2,5											
2											
1,5			1,2518E-66								
1			5,3912E-44								
0,5			2,3219E-22								
0	6,1872E+34	2,4874E+17	1	4,0203E-18	1,6163E-35	6,4978E-53	2,6123E-70	1,0502E-87	4,2221E-105	1,6974E-122	6,8240E-140
-0,5			4,3068E+21								
-1			1,8549E+43		2,9979E+08						
-1,5			7,9885E+64								
-2			3,4405E+86								
-2,5			1,4818E+108								
-3			6,3817E+129								
-3,5			2,7485E+151								1,8755E+12
-4											

Fig. 10. Electric Charge Position

As a proof, this dimension of the electric charge must satisfy either the electrostatic or the electromagnetic versions. Because of the L^2T^{-2} relation between μ and k, the two options are exclusive and cannot be both true. More precisely:

$$k = 1/4\pi\varepsilon_o \quad (24)$$

And

$$\varepsilon_o = 1/\mu_o c^2 \quad (25)$$

It follows that:

$$k = \mu_o c^2/4\pi \quad (26)$$

So:

$$[k] = [\mu] * L^2T^{-2} \quad (27)$$

We can say that if $[K] = 1$ then $[\mu] = L^{-2}T^{2}$ and if $[\mu] = 1$ then $[K] = L^{2}T^{-2}$. We strongly suggest that the electromagnetic version is true, so, the electrostatic is false.

The electrostatic version is based on the unsupported assumption that the Coulomb's constant is dimensionless. In fact, the electromagnetic expression is correct because the permeability μ is equal to 4π* 1e-7, where the 4π term corresponds to a spherical integration in space, and the 1e-7 portion is the multiplicative factor of the quantity. Moreover, with their 1e-7 factor, the impedance and Coulomb's constant numerical values appear directly at the [1,-1] and [2,-2] positions, which confirms the parallel between impedance and velocity and the correspondence between the Coulomb's constant and the square of the speed of light as from eq. (27) it follows that

$$k = c^2 * 1e\text{-}7 \tag{28}$$

This highlights the correspondence between c^2 and k. In this context, the 1e-7 term is dimensionally equivalent to μ as it is the result of $\mu_o/4\pi$. As the 1e-7 factor is an arbitrary value and the 4pi part accounts for spatial integration, it suggests that there is a concordance between k and c^2, and so that:

$$[k] = L^{2}T^{-2} \tag{29}$$

Furthermore, by using Planck values for the mass and electric charge, we see that $Qp^2 = Mp * Lp^3 * Tp^{-2}$ is false, while $Qp^2 = Mp * Lp$ is true when introducing the dimensionless multiplicative factors such as:

$$3.5177e\text{-}36 = (2.1764e\text{-}08 * 1.6162e\text{-}35) * 1e7 \tag{30}$$

Also, we see that if $[Q] = L^{4}T^{-3.5}$ and $[M] = L^{7}T^{-7}$ then again $[Q^2] = ML$ is dimensionally verified. When the real dimension of the Coulomb's constant ($L^{2}T^{-2}$) is introduced into the electrostatic version, it then reduces to:

$$Q^2 = F * r^2 / k \tag{31}$$

So $$[Q^2] = ML^{3}T^{-2} / [k] \tag{32}$$

Then $$[Q^2] = ML^{3}T^{-2} * L^{-2}T^{2} = ML \tag{33}$$

Similarly, working with the electron data, we also see that :

$$Q_e = (M_e * R_e * 1e7)^{1/2} = 1.6022e\text{-}19\ C \tag{34}$$

Therefore, the apparent incompatibility vanishes. The electric charge has only one definition and no inconsistency really exists.

9. The Study of Units

The study of electromagnetic units is also a clear indication that the electromagnetic version is correct and so that μ is dimensionless. If we consider the magneto motive force, a magnetic potential, we know that its unit is the "Ampere turn". The "turn" portion is here to reflect the stacking of loops of wire and is dimensionless, so if we consider one loop of wire, the unit of the *MMF* is Ampere. We also know the relation between the *MMF* and the magnetic field:

$$MMF = B * l * \cos(\text{phi}) \tag{35}$$

It then becomes clear that the unit of the magnetic field B, the Tesla, must correspond to Ampere / meter (this is known, but not very often used). But we also know from the relation $B = \mu * H$ that

$$\mu = B / H \tag{36}$$

and as the unit of H, the auxiliary field, is also Ampere / meter, it means that

$$\text{Unit of } (\mu) = (\text{Ampere / meter}) / (\text{Ampere / meter}) \tag{37}$$

As Ampere and meter units also correspond to fundamental dimensions, we can then safely say that μ is dimensionless.

10. Synthesis

All aspects of the debate around the space-time dimension can be summarized into a few possibilities depending on the initial assumptions for the dimension of mass and electric charge. The following table presents all possibilities regarding the proposed solutions:

	[M] = M	**[M] = ?**	**[M] = L^3T^{-2}**	**[M] = L^7T^{-7}**
[Q^2] = ML^3T^{-2}	**A**	**C**	**E**	**G**
	[G] = $M^{-1}L^3T^{-2}$	[G] = $[M]^{-1}L^3T^{-2}$	[G] = 1	[G] = $L^{-4}T^5$
	[Q] = $M^{1/2}L^{3/2}T^{-1}$	[Q] = $[M]^{1/2}L^{3/2}T^{-1}$	[Q] = L^3T^{-2}	[Q] = $L^5T^{-4.5}$
	[K] = 1	[K] = 1	[K] = 1	[K] = 1
[Q^2] = ML	**B**	**D**	**F**	**H**
	[G] = $M^{-1}L^3T^{-2}$	[G] = $[M]^{-1}L^3T^{-2}$	[G] = 1	[G] = $L^{-4}T^5$
	[Q] = $M^{1/2}L^{1/2}$	[Q] = $[M]^{1/2}L^{1/2}$	[Q] = L^2T^{-1}	[Q] = $L^4T^{-3.5}$
	[K] = L^2T^{-2}	[K] = L^2T^{-2}	[K] = L^2T^{-3}	[K] = L^2T^{-2}

Table 2 Synthesis of All Possibilities

As can be seen from the greyed areas, rows correspond to different definitions of the electric charge (electrostatic and electromagnetic), while columns are different definitions of the mass.

[M] = M

The first column means that the mass is considered itself as a dimension. It represents the current situation with the SI system and there is no possible further reduction to space-time. While perfectly coherent and accurate dimensionally, it defines everything with respect to the mass dimension and so, does not help us to find anything as it contradicts our initial hypothesis.

[M] = ?

The second column eventually accepts our initial hypothesis but states that none of the two following proposed solutions are correct and so that the space-time dimension of mass is still unknown. The second column then represents all possible other solutions. In other words, because we know that $[GM] = L^3T^{-2}$, then whatever is chosen for one of them, the second will have to be dimensionally coherent.

If this case is true, it opens two main possibilities:

On one hand, the mass and all other quantities are on the matrix proposed by this study, but with exponents so great that it exceed the capacities of our tool. It could look like $[M] = L^{3856}T^{-}$

93754 for example. Remember that we are in the case where we accept that the mass can be derived from space and time only, but that $[M] = L^3T^{-2}$ and $[M] = L^7T^{-7}$ are false.

On the other hand, all Planck quantities could also be hidden in non-integer dimensions. We quickly discovered the non-integer dimension of the electric charge thanks to the presence of the square of this value on the integer dimension matrix we had initially. But quantities could derive in fact from weird non integer dimension such as $[M] = L^{4.38}T^{-5.27}$

In both cases, there is an infinity of possibilities …

The first and second columns of the table are different in the way that

$$[G] = M^{-1}L^3T^{-2} \quad (38)$$

This means that the dimension of *G* is equal to a portion of mass combined with a portion of space and a portion of time. Mass is then considered as a fundamental constituent of any physical quantity, what is called a dimension.

While in the second column

$$[G] = [M]^{-1}L^3T^{-2} \quad (39)$$

This means that the dimension of *G* is equal to a portion of "the dimension of mass" combined with a portion of space and a portion of time. The expression "the dimension of mass" is generic and does not imply that mass is in itself a dimension, as did the first column of the table. In this case, the dimension of mass could very well be derived from space and time, but in opposition to the two other proposed solutions (see below), it states that the space-time dimension of mass is still unknown.

$$\boldsymbol{[M] = L^3T^{-2} \text{ and } [M] = L^7T^{-7}}$$

The last two columns represent four different and incompatible possibilities, E, F, G and H.

As for any theory, its merit is evaluated from the demonstrations showing the provability of the initial hypothesis. To summarize it all, if we agree that the dimension of physical quantities can be derived from space-time only, then, by definition, all physical quantities are part of the Cartesian product of the space and time sets. If we build a matrix that presents the Cartesian product of Planck space and time sets ($L_p^x * T_p^y$, with x and y = 0 to infinity), then all Planck values must appear on this matrix.

Based on this reasoning, we have demonstrated that E and F, the Maxwell's version is mathematically impossible. We also showed that the second row, the electro-magnetic version, is supported by numerical evidence, while the electro-static version is not. This leaves us with H as the best possibility to represent reality.

11. A Coincidence?

Considering the intrinsic logic of the matrix, the Planck values had to appear somewhere and we see that they do. So the expected fact that they appear is not a coincidence, nor is the location where they appear as it has to be dimensionally coherent.

There is a possibility that $L_p^{7*}T_p^{-7}$ is just accidentally close to M_p. In this case, M_p should still be on the matrix somewhere else and all Planck quantities should appear at some other locations. Looking at the matrix, it is clear that it is not the case, so L^7T^{-7} is left as the best candidate to "represent" [*M*].

During this research, the matrix was extended to look for other locations where the Planck mass value could be found. It was found that the specific pattern represented by the matrix is repeated over and over with a step of 1e83 in values:

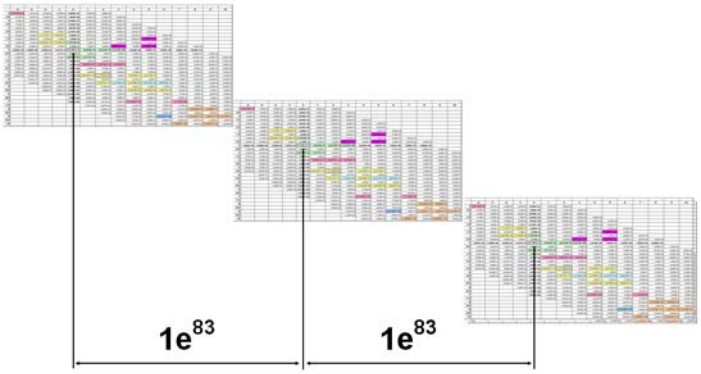

Fig. 11. Expanding the Matrix

However, within this pattern, no other value that could better represent the Planck mass, electric charge or any other Planck quantity was found.

12. Accuracy

The CODATA value for the Planck mass being M_p = 2.176 44(11) e-8 kg, it means, when applying the uncertainty, that the Planck mass is estimated between 2.17633 e-8 Kg and 2.17655 e-8 Kg.

On the other hand, using CODATA values L_p = 1.616 252 1e-35 m and T_p = 5.391 24 1e-44 s, we find

$$L_p^{7*}T_p^{-7} = 2{,}176\ 42\ e{+}59 \quad (40)$$

Taking the multiplicative factor (1e-67) into account, the discovered value is then absolutely compatible with the CODATA value as it falls perfectly into the uncertainty range.

Based on the presented method, dimensions of all physical quantities were derived from space and time only in a perfectly numerically and dimensionally coherent system. The main physical quantities, their dimensional properties as well as their Planck values are presented in Table 3.

This study also produced the Planck matrix in figure 12.

Conclusion

It was shown that by organizing Planck quantities according to the proposed configuration, it is possible to dimensionally define all physical quantities with respect to space and time. This perspective offers numerous possibilities related to the understanding and the determination of physical quantities. The electronic version of the presented matrix can be found at: http://www.losangeinformatique.com/physics/matrix.xls

References

[1] J. C. Maxwell, “A Treatise on Electricity and Magnetism”, chapter 5.

Dom	Quantity	Symbol	Dimension SIMKS	Dimension LT	Factor	Value Unit
Dynamic	Length	l	L	L	1	1,6163E-35 m
	Surface	s	L2	L2	1	2,6123E-70 m2
	Volume	v	L3	L3	1	4,2221E-105 m3
	Time	t	T	T	1	5,3912E-44 s
	Frequency	f	T-1	T-1	1	1,8549E+43 /s
	velocity	c	LT-1	LT-1	1	2,9979E+08 m /s
	acceleration	a	LT-2	LT-2	1	5,5607E+51 m /s2
	Energy	N	ML2T-2	L9T-9	1E+67	1,9561E+09 J
	Force	F	MLT-2	L8T-9	1E+67	1,2102E+44 N
	Power	P	MLT-3	L9T-10	1E+67	3,6282E+52 W
	Power distribution (Poyting)	S	ML-1T-3	L7T-10	1E+67	1,3889E+122 W/m2
	Momentum	M	MLT-1	L8T-8	1E+67	6,5247E+00 Js/m
	Action	A	ML2T-1	L9T-8	1E+67	1,0546E-34 Js
	Energy density	U	ML-1T-2	L6T-9	1E+67	4,6329E+113 J/m3
	Energy flux	Φ	ML3T-2	L10T-9	1E+67	3,1615E-26 Jm
Energetic	Mass	M	M	L7T-7	1E+67	2,1764E-08 Kg
	Mass density	ρ	ML-3	L4T-7	1E+67	5,1548E+96 Kg/m3
	Gravitational constant	G	M-1L3T-2	L-4T5	1E-67	6,6743E-11 Nm2/Kg2
	Field	g	LT-2	LT-2	1	5,5607E+51 m /s2
	Potential	ψ	L2T-2	L2T-2	1	8,9875E+16 m 2/s2
	Mass flow	m	MT-1	L7T-8	1E+67	4,0370E+35 Kg/s
	Mass flux	Φ	ML-2T-1	L5T-8	1E+67	1,5454E+105 Kg/m2/s
Gravitic	Volume	v	L3	L3	1	4,2221E-105 m3
	Flow	Q	L3T-1	L3T-1	1	7,8314E-62 m 3/s
	Pressure	P	ML-1T-2	L6T-9	1E+216	4,6329E+113 Pa
	Heat	Q	ML2T-2	L5T1,5	1E+216	1,3807E-23 K
	Boltzmann's constant	K	ML2T-2	L5T1,5	1E+216	1,3807E-23 J/K
	Heat flow	Φ	ML2T-3	L5T0,5	1E+216	2,5610E+20 K/s
	Heat flux	Φq	MT-3	L3T0,5	1E+216	9,8039E+89 K/m2s
	Temperature	T	K	L4T-10,5	1E+283	1,4167E+32 K
	Capacity	C	ML2T-2K-1	LT11,5	1E-67	4,8164E-388 W/mK
	conductivity	c	MLT-3K-1	T10,5	1E-67	4,1969E-466 J/K
Thermic	Conductance	G	ML2T-3K-1	LT10,5	1E-67	7,7846E-423 W/K
	Resistance	R	M-1L2T3K	L-1T-10,5	1E-67	1,2845E422 K/W
	Charge	Q	TA	L4T-3,5	1E+30	1,8755E-18 C
	Charge distribution	D	L-2TA	L2T-3,5	1E+30	7,1797E+51 C/m2
	Charge density	ρ	L-3TA	LT-3,5	1E+30	1,1604E+17 C/m3
	Current	I	A	L4T-4,5	1E+30	3,4789E+25 A
	Current distribution	J	L-2A	L2T-4,5	1E+30	1,3317E+95 A/m2
	Dipole moment	M	LTA	L5T-3,5	1E+30	3,0313E-53 Cm
	Potential	U	ML2T-3A-1	L5T-5,5	1E+37	1,0429E+27 V
	Field	E	MLT-3A-1	L4T-5,5	1E+37	6,4528E+61 V/m
	Flux	Φ	ML3T-3A-1	L6T-5,5	1E+37	1,6656E-06 Vm
	Resistance	R	ML2T-3A-2	LT-1	1E+07	2,9979E+01 Ω
Electric	Resistivity	r	MLT-3A-2	L2T-1	1E+07	1,8549E+36 Ωm
	Coulomb's constant	K	MLT-4A-2	L2T-2	1E+07	8,9875E+09 Ωm/s
	Permeability	μ	MLT-2A-2	1	1E+07	1,0000E-07 H/m
	Inductance	L	ML2T-2A-2	L	1E+07	1,6163E-42 H
	conductance	G	M-1L-2T3A2	L-1T	1E-07	3,3556E-02 S
	Conductivity	σ	M-1L-3T3A2	L-2T	1E-07	2,0638E+33 S/m
	Permittivity	ε	M-1L-3T4A2	L-2T2	1E-07	1,1127E-10 F/m
	Capacity	C	M-1L-2T4A2	L-1T2	1E-07	1,7983E-45 F
Magnetic	Potential	A	A	L4T-4,5	1E+37	3,4789E+18 Tm
	Field	B	L-1A	L3T-4,5	1E+37	2,1524E+53 T
	Auxiliary field	H	L-1A	L3T-4,5	1E+30	2,1524E+60 A/m
	Flux	Φ	LA	L5T-4,5	1E+37	5,6227E-17 Am
	Dipole moment	M	L2A	L6T-4,5	1E+30	9,0877E-45 Am2

Table 3 Physical Quantities and Their Dimensions

	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
5	6,6743E-78	1,0787E-112	1,7435E-147	2,8179E-182	4,5545E-217	7,3613E-252	1,1898E-286								
4,5	2,8745E-56	4,6459E-91	7,5090E-126	1,2136E-160	1,9615E-195	3,1704E-230	5,1241E-265	8,2818E-300							
4	1,2380E-34	2,0009E-69	3,2340E-104	5,2269E-139	8,4480E-174	1,3654E-208	2,2068E-243	3,5668E-278							
3,5	5,3318E-13	8,6175E-48	1,3928E-82	2,2511E-117	3,6384E-152	5,8806E-187	9,5045E-222	1,5362E-256	2,4828E-291						
3	2,2963E+09	3,7114E-26	5,9986E-61	9,6952E-96	1,5670E-130	2,5326E-165	4,0934E-200	6,6160E-235	1,0693E-269	1,7283E-304					
2,5	9,8897E+30	1,5984E-04	2,5835E-39	4,1755E-74	6,7487E-109	1,0908E-143	1,7629E-178	2,8494E-213	4,6053E-248	7,4433E-283					
2	4,2593E-52	6,8841E+17	1,1127E-17	1,7983E-52	2,9065E-87	4,6977E-122	7,5927E-157	1,2272E-191	1,9834E-226	3,2057E-261	5,1812E-296				
1,5	1,8344E+74	2,9649E+39	4,7920E+04	7,7450E-31	1,2518E-65	2,0232E-100	3,2700E-135	5,2852E-170	8,5422E-205	1,3806E-239	2,2315E-274				
1	7,9004E+95	1,2769E+61	2,0638E+26	3,3356E-09	5,3912E-44	8,7136E-79	1,4083E-113	2,2762E-148	3,6790E-183	5,9461E-218	9,6104E-253	1,5533E-287			
0,5	3,4026E+117	5,4994E+82	8,8884E+47	1,4366E+13	2,3219E-22	3,7528E-57	6,0654E-92	9,8033E-127	1,5845E-161	2,5609E-196	4,1390E-231	6,6897E-266	1,0812E-300		
0	1,4654E+139	2,3685E+104	3,8281E+69	6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105	6,8240E-140	1,1029E-174	1,7826E-209	2,8811E-244	4,6566E-279		
-0,5	6,3113E+160	1,0201E+126	1,6487E+91	2,6647E+56	4,3068E+21	6,9609E-14	1,1251E-48	1,8184E-83	2,9390E-118	4,7501E-153	7,6773E-188	1,2409E-222	2,0055E-257	3,2414E-292	
-1	2,7182E+182	4,3932E+147	7,1006E+112	1,1476E+78	1,8549E+43	2,9979E+08	4,8454E-27	7,8314E-62	1,2658E-96	2,0458E-131	3,3065E-166	5,3441E-201	8,6374E-236	1,3960E-270	2,2563E-305
-1,5	1,1707E+204	1,8921E+169	3,0581E+134	4,9426E+99	7,9885E+64	1,2911E+30	2,0868E-05	3,3728E-40	5,4513E-75	8,8108E-110	1,4240E-144	2,3016E-179	3,7200E-214	6,0124E-249	9,7176E-284
-2	5,0418E+225	8,1488E+190	1,3171E+156	2,1287E+121	3,4405E+86	5,5607E+51	8,9876E+16	1,4526E-18	2,3478E-53	3,7946E-88	6,1331E-123	9,9126E-158	1,6021E-192	2,5894E-227	4,1852E-262
-2,5	2,1714E+247	3,5095E+212	5,6723E+177	9,1679E+142	1,4818E+108	2,3949E+73	3,8708E+38	6,2561E+03	1,0111E-31	1,6343E-66	2,6414E-101	4,2692E-136	6,9001E-171	1,1152E-205	1,8025E-240
-3	9,3519E+268	1,5115E+234	2,4430E+199	3,9484E+164	6,3817E+129	1,0314E+95	1,6671E+60	2,6944E+25	4,3548E-10	7,0385E-45	1,1376E-79	1,8386E-114	2,9717E-149	4,8031E-184	7,7629E-219
-3,5	4,0277E+290	6,5097E+255	1,0521E+221	1,7005E+186	2,7485E+151	4,4422E+116	7,1797E+81	1,1604E+47	1,8755E+12	3,0314E-23	4,8994E-58	7,9187E-93	1,2799E-127	2,0686E-162	3,3434E-197
-4		2,8036E+277	4,5314E+242	7,3238E+207	1,1837E+173	1,9132E+138	3,0922E+103	4,9977E+68	8,0776E+33	1,3055E-01	2,1101E-36	3,4104E-71	5,5121E-106	8,9090E-141	1,4399E-175
-4,5		1,2075E+299	1,9516E+264	3,1542E+229	5,0980E+194	8,2397E+159	1,3317E+125	2,1524E+90	3,4789E+55	5,6227E+20	9,0878E-15	1,4688E+49	2,3740E-84	3,8369E-119	6,2015E-154
-5			8,4050E+285	1,3585E+251	2,1956E+216	3,5487E+181	5,7356E+146	9,2701E+111	1,4983E+77	2,4216E+42	3,9139E+07	6,3259E-28	1,0224E-62	1,6525E-97	2,6709E-132
-5,5			3,6199E+307	5,8506E+272	9,4561E+237	1,5283E+203	2,4702E+168	3,9925E+133	6,4528E+98	1,0429E+64	1,6857E+29	2,7244E-06	4,4034E-41	7,1170E-76	1,1503E-110
-6				2,5198E+294	4,0726E+259	6,5823E+224	1,0639E+190	1,7195E+155	2,7791E+120	4,4917E+85	7,2598E+50	1,1734E+16	1,8965E-19	3,0652E-54	4,9541E-89
-6,5					1,7540E+281	2,8349E+246	4,5819E+211	7,4055E+176	1,1969E+142	1,9345E+107	3,1267E+72	5,0535E+37	8,1677E+02	1,3201E-32	2,1336E-67
-7					7,5541E+302	1,2209E+268	1,9733E+233	3,1894E+198	5,1549E+163	8,3316E+128	1,3466E+94	2,1764E+59	3,5177E+24	5,6854E-11	9,1891E-46
-7,5						5,2583E+289	8,4987E+254	1,3736E+220	2,2201E+185	3,5882E+150	5,7995E+115	9,3735E+80	1,5150E+46	2,4486E+11	3,9576E-24
-8							3,6602E+276	5,9159E+241	9,5616E+206	1,5454E+172	2,4977E+137	4,0370E+102	6,5248E+67	1,0546E+33	1,7044E-02
-8,5							1,5764E+298	2,5479E+263	4,1180E+228	6,6557E+193	1,0757E+159	1,7387E+124	2,8101E+89	4,5418E+54	7,3407E+19
-9								1,0973E+285	1,7735E+250	2,8665E+215	4,6330E+180	7,4880E+145	1,2103E+111	1,9561E+76	3,1615E+41
-9,5								4,7259E+306	7,6383E+271	1,2345E+237	1,9953E+202	3,2250E+167	5,2123E+132	8,4245E+97	1,3616E+63
-10									3,2897E+293	5,3169E+258	8,5935E+223	1,3889E+189	2,2449E+154	3,6283E+119	5,8642E+84

Dynamic
Gravitic
Thermic
Pressuric
Electric
Magnetic
Energetic

Fig. 12. - The Numeric Planck Matrix