Relativity and the Formation of Black Holes

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In order to form Black Holes, matter has to move across the Schwarzschild radius. We demonstrate here that according to Einstein general relativity, matter cannot have the time to form a Black Hole when we consider either the proper time or the Schwarzschild time. Black Holes are incompatible with the time-limited Big Bang cosmology.

1. Introduction: Star Formation.

During the formation of a star, matter falling under the force of gravity increases the amount of matter in the star. Gas and dust contract and therefore the star temperature is increased. Planck radiation is then emitted from the surface of the hot star. Then, the star contracts still more, and consequently more heat is generated again due to gravitational energy.

That process can go on for periods ranging from tens of millions of years to a few billion years depending on the size of the star. If the mass of the star is too small, its contraction stops after a while because the gravitational force is not strong enough to counteract electric and nuclear forces in matter. If the amount of matter M in the star is larger than 1.4 times the solar mass, the gravitational pull is strong enough to make the star reach such a high concentration that relativistic effects become important, as predicted in the model of neutron stars and pulsars. The radius of the star then decreases to a value approaching the Schwarzschild radius. We will consider here the relativistic conditions produced when matter reaches a point located just above the Schwarzschild radius before a Black Hole can be formed. This discussion is important since the existence of Black Holes has never been clearly confirmed (*S&T* 1989) in observations.

2. Contraction of stars.

Black Holes will exist only if no mechanism prevents their formation. We show here that such a mechanism exists. It is related to time. In Schwarzschild coordinates (Misner *et al.* 1973), the Schwarzschild radius R equals $2M$. Therefore, when the infalling mass is located at a distance r just above the Schwarzschild radius R, strong relativistic effects are produced.

The relativistic conditions existing just above the Schwarzschild radius are well known and can be found in several textbooks. Full relativistic considerations have not always been used (Waldron 1983) in the past, but they are absolutely necessary in the case of Black Holes. Here we refer to the well known book *Gravitation* by C.W. Misner, *et al.* (1973). In chapter 25 the equations describing the relativistic orbits of particles in a strong gravitational field above the Schwarzschild radius are given. The Schwarzschild radius is the radius at which nothing, not even light, can escape from the star.

We calculate both the Schwarzschild time t and the proper time t, taken by a mass falling gradually in a radial orbit on a massive star (in

Schwarzschild coordinates). The distance of an element of falling matter to the center of the star as a function of time is plotted on Fig. 1

The Schwarzschild time is the one recorded by a distant observer. As explained by Misner *et al.* (1973, eq. 25.39b) when the Schwarzschild time t varies from $-\infty$ to $+\infty$ the distance r passes from $r = \infty$ to $r = 2$M according to the equation:

$$r = 2M + rM\left[e^{\frac{8}{3}} \times e^{\frac{t-a}{2M}}\right] \tag{1}$$

where $t = a$ is a particular instant related to proper time as will be seen below. In Fig. 1A, the distant observer sees that the falling object, that started at a very low speed and very far away will never get closer to the star than $r = 2M$ after an infinite time ($t = +\infty$).

In the case of the proper time t of matter falling on the star, the radial distance r (Misner *et al.* [1973, Eq. 25.39a) reached by this moving mass is:

$$r = \left(\frac{9M}{2}\right)^{\frac{1}{3}} (t - a)^{\frac{2}{3}} \tag{2}$$

Where $t = a$ at the proper time of final catastrophe ($r = 0$). One has $t \ll a$ when the particle is far away and approaching very slowly. This curve is shown on Fig. 1B.

Figure 1 is a well known diagram in astrophysics. It is important to notice that the time taken to reach a given radius r/M differs whether one considers Schwarzschild or proper time. However the real distance r/M of a falling mass to the center of the star is common to both reference systems. For example if a mass is at a distance $(r/M) > 2$, both observers (external and orbiting) can agree about that radius since general relativity requires that the same reference length

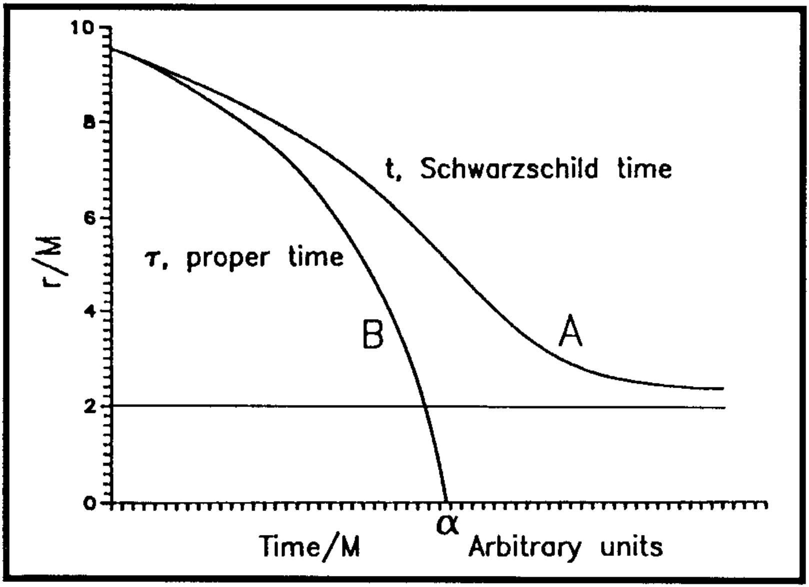

Figure 1
Curve A illustrates Schwarzschild time as given by Eq. 1
Curve B illustrates proper time as given by Eq. 2

(*i.e.*, a standard meter carried by both observers) and the same speed of light c is common to both observers.

3. Paradox

Falling matter just above the Schwarzschild radius leads to a paradox, as illustrated in Fig. 1. An observer located far from the star will see an object falling toward the surface of the star. That object will never reach a radius r smaller than $R = 2M$ as given by Eq. 1. On the other hand, an observer falling on the star along with the falling matter will cross the Schwarzschild radius R very rapidly in his proper time t_p when:

$$t_p = a - \sqrt{\frac{16}{9M}} \quad (3)$$

The falling matter can even reach (apparently) the center of the star $r = 0$ at $t = a$ in agreement with Eq. 2. It is paradoxical that the same object never reach the center of the star when seen by a faraway observer but rapidly reach that center when seen by a moving observer? Can the falling mass actually ever reach $r = 0$ or not?

4. Solution

A better illustration of this paradox is shown on Fig. 2. It shows an expanded view of Fig. 1 between (r/M)=2 and 3 with a logarithmic scale on the time axis.

The plot on Fig. 2 corresponds to a near Black Hole formed in 10 million years in proper time. A typical curve corresponding to the Schwarzschild time is plotted as calculated by Eq. 1.

First we synchronize the proper time t and the Schwarzschild time t. We know that the Schwarzschild time and the proper time are the same for large values of r. Consequently both times are easily synchronized at that large r value. In Fig. 2, when mass A is at a large radius r_A the Schwarzschild time t and the proper time t are practically the same and the clocks are synchronized. We know (Misner *et al.* 1973, page 660) that the proper time is related to Schwarzschild time by the time dilatation relation $(1 - 2M/r)^{-1}$. Consequently, increasingly longer periods in Schwarzschild time correspond to shorter and shorter periods in proper time when a traveler approaches $r = 2M$ ($Y = 2$).

When the synchronized clocks in proper time and in Schwarzschild time are at a radius r_A (corresponding to Y_A) (Fig. 2) both observers record the same age; both will be about 2 million years old. When the falling mass is at a radius r_B (Fig. 2), the observer in

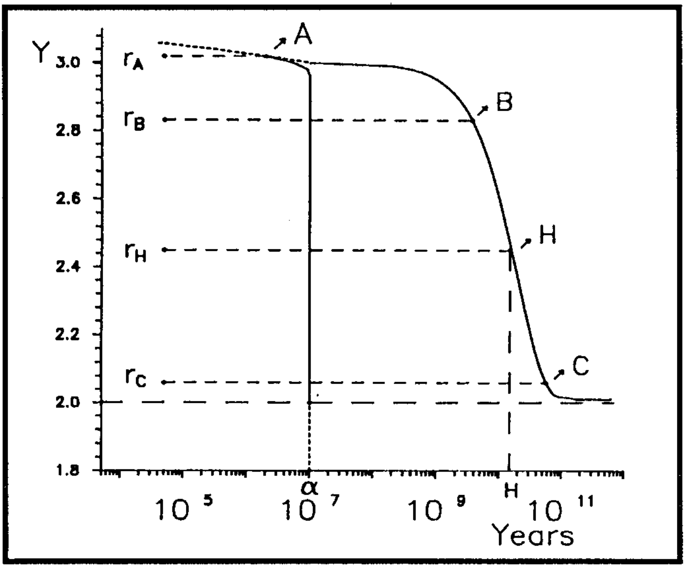

Figure 2
Correlation between Schwarzschild time and proper time for falling mass between r/M=2 and 3. In order to illustrate more clearly the correspondence between times and distances, the vertical scale is expanded non-linearly between 2 and 3. This maintains exactly the same absolute value r=Y at r/M=2 and r/M=3. This is obtained using the equation $Y = 2+(r-2)^W$. Here W is arbitrarily set to 1000, leading to the relation: $r=2+(2-Y)^{0.001}$. We consider here a star formed in 10 million years (proper time). "α"is the time of final catastrophe (proper time) and H is the Hubble time.

Schwarzschild time is 2 billion years old but the observer in proper time will be only about 9.9 million years old. At a distance r_H, the observer in Schwarzschild time is 15 billion years old. This is the Hubble time, *i.e.*, the "age"of the universe. Matter created at the beginning of the universe should be only 10 million years old in proper time when it reaches the distance r_H from the star.

Consequently, if primeval matter started to form a star at the same instant as the Big Bang, that matter cannot have reached the Schwarzschild radius yet. That matter is still at a distance r_H (corresponding to $Y_H = 2.48$) whether it is considered by the outside observer or by the falling observer. However as seen above, these two observers have quite different ages depending on whether we observe matter from faraway or if we accompany the matter in its fall.

No matter can possibly reach the radius r_C (Fig. 2) because matter in a star originated far outside the star, and this matter should have started its fall 55 billion years ago. According to the Big Bang theory, the universe did not exist at that time. Even if only a few seconds of proper time t are required to pass from r_H to r_C, the outside universe (Schwarzschild time) must wait an extra 40 billion years before these few seconds can elapse in proper time. If we accept the Big Bang cosmological model (that is, the Universe is finite in time) and Einstein general relativity, we must conclude that no matter has ever crossed the Schwarzschild radius.

Consequently, Black Holes cannot exist unless the Big Bang theory is erroneous and the universe is infinite. An infinite universe can be easily described if the cosmological redshift is not interpreted by a Doppler effect. It has been demonstrated previously (Marmet 1988) that redshifts can be interpreted naturally by known physical mechanisms without any new hypotheses. Many observations (Marmet 1989a, 1989b, 1990) confirm these calculations.

Finally, the problem described here is related to the so called "twins paradox". In fact, the so-called twins paradox is clearly not a paradox at all when one understands relativity. Relativity explains clearly that the accelerated twin is not as old as the twin at rest when they come back together. That difference of age is perfectly real and verifiable experimentally. For example, many experiments like the one using the measurement of the lifetime of cosmic mesons or with atomic clocks moving around the earth prove the physical reality of the space-time phenomenon that is now generally accepted. Consequently, one must conclude that it is a physical reality that the falling observer cannot cross the Schwarzschild radius before the outer observer reaches an infinite age.

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