Light Deflection by a Gravitating Body a Hidden Deception in General Relativity Theory

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In this paper it is shown that the calculation of the light deflection in the field of a gravitating body does not follow the well know Fermat principle from optics. An ad-hoc formula is typically used for calculations to obtain an agreement with observations, which does not have any correspondence to similar formulas anywhere in physics. The root cause of the problem is traced to the Schwarzschild metric, which does not describe the reality correctly. When a new metric is used the standard Fermat principle can be generalized and used leading to the results agreeing with observations and experiments.

1. Introduction

The confirmation of the light deflection from a straight line by a gravitating body during the Solar eclipse was one of the first major confirmations of General Relativity Theory (GRT) and a milestone that initiated the general acceptance of the theory. Since then several new measurements [1] have been performed, for example with the Cassini space probe, with a much greater precision. It would thus seem that there is no problem and the theory was therefore proclaimed proven. This paper will go over the basic calculations that are behind this phenomenon and investigate the assumptions used in the derivations of the formulas.

2. Derivation of the Light Deflection Formula

The formula for the light deflection angle is derived from the Schwarzschild metric, which is in turn derived as a unique solution of the famous Einstein field equations when the mass energy tensor $T_{jk}=0$ is set to zero:

$$G_{jk}=R_{jk}-\frac{1}{2}R_{(c)}g_{jk}=-\frac{8\pi\kappa}{c^4}T_{jk},\quad T^{j}_{k|j}=0 \tag{1}$$

The resulting Schwarzschild metric solution for a centrally gravitating body of mass M is as follows:

$$ds^2=g_{tt}(cdt)^2-g_{tt}^{-1}dr^2-r^2\left(d\vartheta^2+\sin^2\vartheta d\varphi^2\right) \tag{2}$$

where the metric coefficient g_{tt} equals:

$$g_{tt}=1-\frac{R_s}{r} \tag{3}$$

and where the Schwarzschild radius R_s is defined as:

$$R_s=\frac{2\ \kappa M}{c^2} \tag{4}$$

with κ being the gravitational constant and c the vacuum speed of light. The path of the photons is found by setting the metric line element ds in Eq. (2) to zero. For simplicity and without any loss of generality it is also considered that the photon motion in the space-time of the centrally gravitating body can be limited to the equatorial plane, $\vartheta=\pi/2$. Finally, to find the particular photon trajectory on the remaining 2D surface it is necessary to add another condition that would determine it. There are several ways published in the literature how the light beam deflection can be calculated in GRT, but most of them are omitting to mention the simple fact that they are all mathematically equivalent to the following [2]:

$$\delta\int_r cdt=0 \tag{5}$$

By substituting for the time variable from Eq. (2) when $ds=0$, and $\vartheta=\pi/2$, the variational integral becomes:

$$\delta\int_r \frac{1}{\sqrt{1-R_s/r}}\sqrt{\frac{1}{1-R_s/r}+r^2\left(\frac{d\varphi}{dr}\right)^2}dr=0 \tag{6}$$

The first integral of Euler-Lagrange (EL) equation corresponding to this variational problem is as follows:

$$\frac{r^2}{\sqrt{1-R_s/r}}\left(\frac{d\varphi}{dr}\right)=\alpha\sqrt{\frac{1}{1-R_s/r}+r^2\left(\frac{d\varphi}{dr}\right)^2} \tag{7}$$

where α is an arbitrary constant of integration. After some rearrangements and after determining the value of the integration constant from the condition that at the perihelion r_p the derivative of the radius coordinate with respect to angle is zero Eq. (7) becomes:

$$\frac{d\varphi}{dr}=\frac{r_p}{r\sqrt{r^2(1-R_s/r_p)-r_p^2(1-R_s/r)}} \tag{8}$$

This equation is further rearranged using the following common substitutions: $x=r_p/r$ and $a=R_s/r_p$ resulting in:

$$-\frac{d\varphi}{dx}=\frac{1}{\sqrt{1-x^2}}\frac{1}{\sqrt{1-a\frac{1-x^3}{1-x^2}}} \tag{9}$$

where the variable x ranges from zero to unity: $0\le x\le 1$. This expression is further simplified by assuming that the parameter a is very small compared to unity. This leads to the result:

$$-\frac{d\varphi}{dx}=\frac{1}{\sqrt{1-x^2}}+\frac{a}{2}\frac{1}{\sqrt{1-x^2}}\frac{1-x^3}{1-x^2} \tag{10}$$

The trajectory deviation from a straight line is then obtained by integrating Eq. (10) (perihelion to infinity), multiplying the result by two, and subtracting π. This leads to the final result:

$$\Delta\varphi = a\int_0^1\left(1+\frac{x^2}{1+x}\right)\frac{dx}{\sqrt{1-x^2}} = 2a = 2\frac{R_s}{r_p} \tag{11}$$

This result agrees with observations and the Cassini space probe measurement, so this is a good reason for celebration and for proclamation that GRT is correct. But wait a minute, this is all nice, but where did Eq. (5) come from? Shouldn't this equation be derived from some other more general principle to agree with the rest of the physics? It is not reasonable to just postulate an equation and claim that this is it when it agrees with observations. We have the rest of the physics to make the principle to agree with.

3. Derivation and Application of the Relativistic Fermat Principle

The optics engineering is well developed into a whole industry with a sophisticated science behind it and many optical instruments are fabricated every day to satisfy the various demands of customers and scientists. The lenses and prisms are designed and calculated based on a simple concept that the photons propagate along the lines satisfying the Fermat principle:

$$\delta\int_L n\ dl = 0, \tag{12}$$

where for the index of refraction n holds: $n = c/v$ and where v is the velocity of light in the glass medium. It is well known that the gravitational field affects the speed of light, so one would expect that the Fermat principle should also apply there and could be used to calculate the light trajectory by simply considering that the space around the gravitating body has an index of refraction different from unity. Let's consider that light propagates close to the radial direction for most of the travelled distance in these trajectory deflection tests. The radial light speed from Eq. (2) is then:

$$c_r = cg_{tt} \tag{13}$$

It is important to use the physical radius in Eq. (12), since the coordinate radius is also affected by gravity and this must be taken into consideration. For the physical length increment dl in the radial direction ($dl = \sqrt{g_{tt}^{-1}dr}$) follows also from Eq. (2) that:

$$\sqrt{g_{tt}}\,cdt = dl \tag{14}$$

By substituting these equations into the Fermat principle the following result is obtained:

$$\delta\int_L \frac{cdt}{\sqrt{g_{tt}}} = 0 \tag{15}$$

This is the correct relativistic equivalent of the Fermat principle from optics that the photon path should follow in a gravitational field. From the fact that the integral depends only on the radial coordinate it is reasonable to extrapolate this principle to any direction of light propagation, similarly as it was considered in the GRT case, even though the apparent speed of light may not be isotropic in this space-time. This variational integral should then be used instead of the integral in Eq. (5) to calculate the photon trajectory. The calculations are simple to repeat starting from the integral:

$$\delta\int_r \frac{1}{1-R_s/r}\sqrt{\frac{1}{1-R_s/r}+r^2\left(\frac{d\varphi}{dr}\right)^2}\,dr = 0 \tag{16}$$

with the result:

$$\frac{d\varphi}{dr} = \frac{r_p\sqrt{1-R_s/r}}{r\sqrt{r^2\left(1-R_s/r_p\right)^2 - r_p^2\left(1-R_s/r\right)^2}} \tag{17}$$

finally leading to the expression for the light beam deflection:

$$\Delta\varphi = 2a\int_0^1\left(1-\frac{x}{2}+\frac{x^2}{1+x}\right)\frac{dx}{\sqrt{1-x^2}} = 3a = 3\frac{R_s}{r_p} \tag{18}$$

Unfortunately, this result does not agree with observations and experiments. How is this possible? What is wrong? The answer is not too difficult to find. The problem is the Schwarzschild metric and the Einstein field equations from which the Schwarzschild metric is derived. To prove this point let's use another metric derived elsewhere [3], which does not predict the existence of Black Holes and which is not derived from Einstein field equations. The metric is:

$$ds^2 = e^{-R_s/\rho}(cdt)^2 - e^{R_s/\rho}dr^2 - \rho^2 e^{-R_s/\rho}\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \tag{19}$$

where the variable ρ is found from the differential equation:

$$d\rho = e^{R_s/2\rho}dr \tag{20}$$

The variational integral according to the Fermat principle in Eq. (15) is then as follows:

$$\delta\int_\rho e^{R_s/2\rho}\sqrt{e^{R_s/\rho}+\rho^2\left(\frac{d\varphi}{d\rho}\right)^2}\,d\rho = 0 \tag{21}$$

Following the same steps as in the derivation from the Schwarzschild metric, the EL equation and from that the differential equation for the trajectory becomes:

$$\frac{d\varphi}{d\rho} = \frac{\rho_p}{\rho\sqrt{\rho^2 e^{-R_s/\rho_p} - \rho_p^2 e^{-R_s/\rho}}} \tag{22}$$

where ρ_p is the physical distance at the perihelion. Again, after the customary substitutions: $x = \rho_p/\rho$ and $a = R_s/\rho_p$ the result is:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{e^{-a} - x^2 e^{-a\,x}}} \tag{23}$$

This formula is simplified assuming again that the parameter a is small compared to unity:

$$-\frac{d\varphi}{dx} = \frac{1}{\sqrt{1-x^2}}\frac{1}{\sqrt{1-a\dfrac{1-x^3}{1-x^2}}} \tag{24}$$

After some rearrangements, integration from zero to unity, multiplying the result by two, and subtracting π the light beam deflection from a straight line is:

$$\Delta\varphi = a\int_0^1\left(1+\frac{x^2}{1+x}\right)\frac{dx}{\sqrt{1-x^2}} = 2a = 2\frac{R_s}{\rho_p} \tag{25}$$

This is the same result as obtained from the standard GRT formula, but using the well known and many times verified Fermat principle. The only difference is that the standard coordinate radius is replaced by the physical radius ρ. This difference could be possibly detected by the technology that is available today or will be detected in a near future.

4. Comparison with Observations

In order to find out what differences can be expected from these formulas it is convenient to evaluate the following ratio:

$$\varepsilon = \frac{\Delta\varphi(Sch)}{\Delta\varphi(new)} = \frac{\rho_p}{r_p} \tag{26}$$

First, numerically computing the physical distance of the trajectory perihelion for the Sun, the ratio becomes $\varepsilon = 1.000029$. It is also possible to make more precise numerical calculations using Eq. (23) without neglecting the higher order terms when assuming that the parameter a is small compared to unity. The result for this case is $\varepsilon = 1.000027$. The classical result thus slightly overestimates the real deflection that should be observed. The recent measurements with the Cassini space probe [1] have reached the value of $\varepsilon - 1 = (2.1 \pm 2.3) \times 10^{-5}$, which is not too far from the accuracy needed to unquestionably establish this difference. The deflection of light rays by the Sun therefore agrees with the GRT formula, provided that the coordinate distance is replaced by the physical distance. The deflection angle is evaluated to be $\Delta\varphi = 1.753\ (401)$ arc seconds, which is a well known value considered today to be the correct number for this phenomenon. However, the GRT computation based on the variational principle in Eq.5 has no corresponding equivalent in physics, and since the Schwarzschild metric is not physical and should not be used, this coincidental agreement with the measurement is not the confirmation of the theory. The variational formula in Eq. (5) can thus be considered only a lucky guess.

5. Conclusion

It seems inconceivable that the problem presented in this paper is not well known to those who work in this field for their entire professional lives. We have two principles according to which to calculate the path of the photons in a gravitational field and two different metrics that describe the geometry. It is now easy to see which of these are describing the reality correctly. For the Schwarzschild metric, with its hallmark prediction of such absurdities as Black Holes, it is necessary to invent a new variational principle, not based on anything similar in physics, to find the photon trajectory and the value of the light trajectory deflection that agrees with observations. This is a clear example of fudging and "bending" the generally accepted laws of physics to force an agreement with observations and measurements. This is not how the science should be done. For the new metric the standard and well established Fermat principle from optics was generalized and the correct formula for the light trajectory deflection derived that gives the result agreeing with observations quite naturally and with a better accuracy. The only reasonable conclusion that can be drawn from this work is that the theoretical physicists are purposely covering up this problem in order to maintain the aura of "greatness and beauty" of supposedly proven General Relativity Theory. The Schwarzschild metric and consequently Einstein field equations do not correspond to reality and should be abandoned. A new theory of gravity should be developed that also agrees with the well established fundamental laws of physics.

References

[1] J. D. Anderson et al., paper presented at 22nd Texas Symposium on Relativistic Astrophysics (December 2004).

[2] H. Stephani, **General Relativity, An Introduction to the Theory of the Gravitational Field** (Cambridge University Press, Cambridge, 1990).

[3] J. Hynecek, "The Galileo Effect and the General Relativity Theory", Physics Essays, v 22, No 4, (2009) p. 551.