Mathematical Inconsistency in Relativity's Original Paper of 1905

Cameron Rebigsol
11124 NE Halsey, PMB 577, Portland, OR 97220
e-mail: crebigsol@aquasoil.net

A criticism has been found quite popular in scientific study that the acceptance of relativity is based on faith. Tragic to relativity, this criticism is found to be evidenced by the most fundamental equation from relativity's original paper (herein after referred to as The Paper), published in 1905 [1]. Believing in relativity, two observers moving with respect to each other may not feel troublesome to accept that they do not have the same time readings regarding the same sequence of events. What must trouble them is to accept whose time reading should have dilated to match the time reading from the other observer. Of course, to someone who has followed relativity well with an unshakable faith, it may be only a simple mathematical matter to find out. However, can relativity really make things so simple for the two observers? With the same principle that relativity allures them to accept the concept of time dilation, relativity also agitates arguments that are equally legitimate between them to disagree with each other. When they finally settle some physical quantities that both can commonly accepted for time comparison, they found relativity has only left them with 1=0. Faiths from each of them toward relativity only end up with confronting each other, as well as confronting with acceptable mathematical rules.

1. Introduction

In The Paper, relativity's mathematical foundation about mechanical movement: is as follows:

$$½\left[\tau(0,\ 0,\ 0,\ t)\ +\ \tau\left(0,\ 0,\ 0,\ t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right]$$
$$=\tau\left(x',\ 0,\ 0,\ t+\frac{x'}{c-v}\right) \tag{1}$$

This function expresses the mathematical relationship of the coordinates, both spatial and temporal, between two inertial frames. These two frames in The Paper are referred to as **K**(x, y, z, t) and k(ξ, η, ζ, τ). For convenience, The Paper refers **K**(x, y, z, t) as a "stationary" system most of the time while k(ξ, η, ζ, τ) as the moving system. With this coordinate system established this function is vitally linked to relativity's mathematical validity. Only if this function can withstand the verification done by accepted mathematical rules can it serve to support all relativity's claims. But can it? It is the purpose of this paper to answer that question.

2. Dependence on Opinions

In The Paper, we find the following statement:

(Quotation one, noted by this author**,** in §2, [1]**)** *Let a ray of light depart from A at the time* t_A*, let it be reflected at B at the time* t_B*, and reach A again at the time* t'_A*. Taking into consideration the principle of the constancy of the velocity of light we find that*

$$t_B-t_A=\frac{r_{AB}}{c-v} \tag{2}$$

and

$$t'_A-t_B=\frac{r_{AB}}{c+v} \tag{3}$$

where r_{AB} *denotes the length of the moving rod—measured in the stationary system.*

Given that r_{AB} *denotes the length of the moving rod*, and if r_{AB} has a rest length L, according to the idea of length contraction from relativity, they must have the following relationship:

$$r_{AB}=L\sqrt{1-\left(v/c\right)^2} \tag{4}$$

Relativity further reasons:

(Quotation two, noted by this author**,** in §3, [1]**)** *If we place* $x'=x-vt$*, it is clear that a point at rest in the system k must have a system of values* x'*, y, z, independent of time. We first define* τ *as a function of* x'*, y, z, and t. To do this we have to express in equation that* τ *is nothing else than the summary of the data of clocks at rest in system k…*

From the origin of system k let a ray be emitted at the time τ_0 *along the X-axis to* x'*, and at the time* τ_1 *be reflected thence to the origin of the co-ordinates, arriving there at the time* τ_2*; we then must have* $½(\tau_0+\tau_2)=\tau_1$*, or, by inserting the argument of the function* τ *and applying the principle of the constancy of the velocity of light in the stationary system:-*

Eq. (1) is displayed right after this quotation. Retaining exactly the same mathematical content, we can slightly rewrite (1) to read as

$$\tau(0,\ 0,\ 0,\ t)\ +\ \tau\left(0,\ 0,\ 0,\ t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)$$
$$=2\tau\left(x',\ 0,\ 0,\ t+\frac{x'}{c-v}\right) \tag{5}$$

With a factor of 2, the right side of (5) is a function expression for the total time that the ray of light needs to complete the round trip journey of two equal distances: from emission to reflection to being intercepted at the emission point again. Clearly, such time duration can only be directly registered by the observer in the frame of k(ξ, η, ζ, τ) with his own clock. Thus, if r_{AB} is placed on the ξ axis, and if r_{AB} is assigned a finite value of rest length L, quotations one and two must lead to a replacement in the right side of (5) to read as $\frac{2L}{c}$.

The total time required for the round trip of light should be the same on the left side of (5) as the right side. If the reading shown by the right side of (5) has been assigned to the observer in k(ξ, η, ζ, τ), the left side of (5) is then the only place to relate the function with the time reading made by the observer on the x axis, or **K**(x, y, z, t).

To the observer in the frame of **K**(x, y, z, t), with his own clock, to see the ray of light completing the same journey as ~~what~~ the observer on k(ξ, η, ζ, τ) sees, (2) and (3) must lead him to register a time duration as

$$\frac{r_{AB}}{c-v}+\frac{r_{AB}}{c+v}=\frac{L\sqrt{1-(v/c)^2}}{c-v}+\frac{L\sqrt{1-(v/c)^2}}{c+v} \qquad (6)$$

Can the left side of (5) be replaced by either side of (6), similarly to what was done to the right side of (5)? If it can, (5) will then be equivalently replaced with

$$\frac{L\sqrt{1-(v/c)^2}}{c-v}+\frac{L\sqrt{1-(v/c)^2}}{c+v}=\frac{2L}{c} \qquad (7)$$

However, this replacement will not meet the approval of the observer in the **K**(x, y, z, t) system. Relativity has taught him that there is a time dilation factor involved. Therefore, from his point of view, according to the time his own clock has registered, the only equation he should be able to accept is

$$\left(\frac{L\sqrt{1-(v/c)^2}}{c-v}+\frac{L\sqrt{1-(v/c)^2}}{c+v}\right)\sqrt{1-(v/c)^2}=\frac{2L}{c} \qquad (8)$$

Should the observer in the k(ξ, η, ζ, τ) system agree with him on this equation? With equal legitimacy, and with the time duration of $\frac{2L}{c}$, and also guided by relativity's time dilation concept, the observer staying with k(ξ, η, ζ, τ) must reject (8) but only accept an equation written as

$$\frac{L\sqrt{1-(v/c)^2}}{c-v}+\frac{L\sqrt{1-(v/c)^2}}{c+v}=\frac{2L}{c}\sqrt{1-(v/c)^2} \qquad (9)$$

Regardless of how each of them understands the concept of time dilation, both (8) and (9) are brought about by the same events: light emission, light reflection, and light interception. Although (8) holds equal on both sides in more detailed calculations, (9) must lead to $v=0$. In other words, what (8) and (9) together have demonstrated is that function (1) can lead to different results, *depending on different people's opinions*. Shouldn't this cause us to wonder what relativity is doing to the validity of mathematical rules? This further tells us that, the mathematical construct based on a hypothetically invariable speed of light with respect to any inertial frame, results in skepticism that equation (1) has a consistent set of solutions. Equation (1) needs to be considered in more detail to see whether or not it can be supported by sound material evidence that is not clouded by opinions or faith.

3. Considerations

Consideration 1. Equation (8) and (9) come together to tell us that relativity has bridged quantities on both sides of (1) with an equality sign while inevitably clouding this equation with uncertainty ~~by~~ due to the concept of time dilation. Honesty and clarity are thus compromised if the 1905 relativity paper must be viewed as setting up a continental divide in theoretical science.

Consideration 2. In quotation two, relativity presents ~~to us~~ the expression $x'=x-vt$. Customarily, $x'=x-vt$ is an expression that is found in ~~the~~ classic Newtonian Physics to link the spatial coordinates between two moving frames. As we know, challenging the accuracy of Newtonian Physics is what the "modern" relativity aims to do. Therefore, x' in $x'=x-vt$ cannot be expected to be a point from the ξ axis in k. Instead, point x' and point x are meant to be from the same axis in The Paper; this is plainly expressed in the quotation.

Therefore, x', with its increasing value as time progresses, serves as a "pilot" point to identify the moving state of one of the end points of a moving rod. This moving rod, designated as r_{AB} in quotation one, and considering the context before and after quotation two, is lying on the ξ axis and is supposedly having point A coincide with its origin at $t=\tau=0$. The end point to be identified is therefore the point where light reflects (at time instant t_B); such an end point is point B of r_{AB}. Subsequently, relativity must have point x' and point B coincide forever in its calculation for the identification to work out.

The reasoning embedded in quotation two is further reinforced by a later statement found in the same section: *…where for brevity it is assumed that at the origin of k, τ=0, when t=0.* With this statement, however, difficulty of understanding the value of x' becomes inevitable because of $x'=x-vt$.

It is easy to see from the relationship $x'=x-vt$ that $x'=x$ at $t=\tau=0$. This means that point B, which must permanently coincide with x', takes the coordinates of x on the x axis at $t=\tau=0$. With the concept of length contraction for a moving rod (from quotation one: *r_{AB} denotes the length of the moving rod—measured in the stationary system*), relativity must decide which of the following two diagrams, fig. 1 or fig. 2, viewed by the observer on the x axis, should be chosen to start its calculation:

In fig. 1, point A of r_{AB} coincides with both the origins of k and K. Therefore, with length contraction shown by (4), should point B coincide with

$$x'=x+vt=L+v\cdot 0=L\ ? \qquad \mathbf{(10)}$$

or with

$$x'=x+vt=r_{AB}+v\cdot 0 \qquad (11)$$

then

$$x'=r_{AB}=L\sqrt{1-(v/c)^2}\ ? \qquad (12)$$

Obviously, at $t=\tau=0$, (10) and (12) only create uncertainty for B's location on the x axis for all nonzero speed of r_{AB}.

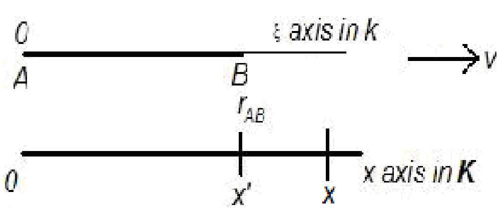

Fig. 1 Uncertainty of B's Location

In fig. 2, point B is said to coincide with $x'=x$ at $t=\tau=0$ because of $x'=x-vt$. Since point A is where the ray of light emits at $t=\tau=0$, (1) must designate the origin of the ξ axis as where the ray of light is emitted. This must also mean that point A and the origin of ξ axis coincide at $t=\tau=0$. However, allowing point B to take the value of x', length contraction will not allow A, which has coincided with $\xi=0$, to coincide with $x=0$. Coinciding with point A, the origin of the ξ axis thus must therefore be seen to have its location shifted on the x axis without any time passing and with no explanation or reason. Such shifting is completed at $t=\tau=0$. So here is the question relativity must answer: How can both origins of the axes both coincide and not coincide at the same instant of time, when $t=\tau=0$?

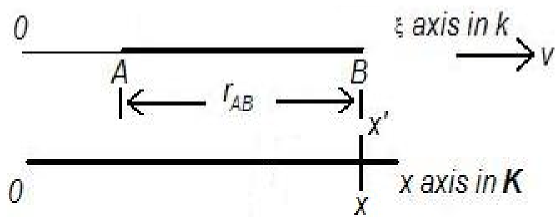

Fig. 2 Uncertainty of A's Location

In this situation, Figures 1 and 2 cannot be reconciled. If we inspect the moving states of the two systems between two instants in time that are designated $(0-1/g)$ and $(0+1/g)$, where g can be any positive number, $x'=x-vt$ must result in some inexplicable speed for some material points of r_{AB}. We really don't know how r_{AB} in fig. 1, for example, will have point B complete its movement from $x=L$ toward point A and stay at a point of $x=L\sqrt{1-(v/c)^2}$ before the instant of time $(0+1/g)$ can be defined.

Consideration 3. Two physical states, a state of rest and a state of relative movement between two frames, are required to start the derivation of the theory of relativity. This theory also requires a transition from a state of no length contraction to a state in which length contraction is said to occur. It also demands these events to be completed instantaneously. This is ~~so~~ evidenced by the following statement:

> **(Quotation three,** noted by this author**,** in §2, [1]**)** *Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is* ***then*** (boldfaced by this author) *imparted to the rod.*

We can see that l herein mentioned has the same significance of L that we found in (4). Before the time instant that is signified by the word ***then***, which enables only $t=\tau=0$ to be defined but nothing else, the two systems are at rest with respect to each other. After that instant it does not matter how large the positive number g is, when the instant in time is defined as $(0+1/g)$, length contraction, if real, would have to have been completed. Can relativity propose some explanation how, and with what speed, all material points have moved and completed the length contraction at $t=\tau=0$? This consideration ends up with the same question that we have in consideration 2. In other words, with equation, such as $x'=x-vt$, or just plain words, relativity relies on an inexplicable speed, and subsequently a mysterious history of movement, to complete its derivation. What is the counterargument if someone disputes that relativity has built its credibility with discredit?

Consideration 4. The significance of x' that gives the time $\frac{x'}{c-v}$ in the expression $\tau\left(0,\,0,\,0,\,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)$ must mean that light is emitted at the origin of some inertial system -- but which system, k or K? Instead of a clear answer, relativity is ambivalent about pinpointing such an origin. We will come back to this argument later here.

Equation $t'_A - t_B = \frac{r_{AB}}{c+v}$ found in quotation one indicates that the point emitting the light is also the moving point that later intercepts the reflected light. This idea has been repeatedly implied by the behavior of point A of r_{AB} in the text of The Paper. The behavior of point A makes this clear: the expression $\frac{x'}{c-v}$ requires that the origin of the ξ axis is the moving point from which light is emitted. The same origin is also the point intercepting the reflected ray of light later. The origin of the **K** system simply cannot be the moving point, because the x axis is not where r_{AB} lies. Had r_{AB} lay on the x axis, the observer on the x axis would not have been able to employ equations (2) and (3).

On the other hand, given that point B of r_{AB} permanently coincides with x', as analyzed previously, the significance of x' found in $x'=x-vt$ must mean that x' is the point reflecting the light. But, then, $\frac{x'}{c+v}$ must allow no point other than the origin of the x axis to intercept the reflected light. Now, with this new choice of origin of the x axis as an intercepting point, function (1) is found to allowed two origins to intercept the reflected light, although these two origins are supposed to have been moving with nonzero speed in relation to each other. Why should a supposedly accurate function enjoy such freedom of uncertainty? Considering the above, we cannot help but ask: Were the conclusions assumed before the calculations were done? This may be exactly what happened. Look at this statement again from quotation two:

> *From the origin of system k let a ray be emitted at the time τ_0 along the X-axis to x', and at the time τ_1 be reflected thence to the origin of the co-ordinates, arriving there at the time τ_2…*

In this quoted sentence, light is definitely being emitted from the origin of k(ξ, η, ζ, τ), but when the reflected light is intercepted, relativity allows it to be intercepted by *the origin of the co-ordinates*. The plural form of *co-ordinates* must allow all of the origins to be the point intercepting the reflected light. Relativity not only relies on a mysterious time problem to set up its equations, but also masks its lack of precision in its language. When this mask is stripped off, the only way relativity can be true is when $v=0$.

If at $t=\tau=0$, which is signified by the word ***then*** in quotation three, relativity cannot have x' decisively take the value of (10) or (12) for (1), and therefore (1) has a good reason to be rejected. If at the time instant τ_2, the time of light interception in quotation two, relativity cannot pinpoint which origin of the two frames intercepts the reflected light, (1) has another good reason to be rejected. Troubles for (1) do not end here. There is a third discrepancy in (1).

From the point of view of the x observer, the distance that the emitted light travels on the x axis is $\Delta x + v(t_B - t_A)$, where Δx is whatever r_{AB} may be allowed to match on the x axis by relativity {Refer to (10) and (12)]. However, the distance that the reflected light travels along the x axis is [$L\sqrt{1-\left(v/c\right)^2}$ -$v(t'_A-t_B)$], which is certainly different from $\Delta x + v(t_B - t_A)$ for all nonzero v. The right side of (5), or equivalently (1), presents a quantity resulting from the round trip of light, as seen by the ξ axis observer. However, the left side of (5) presents a quantity which results from a journey consisting of two unequal distances, as seen by the observer on the x axis. Here is the contradiction: x', a mathematical element which indicates light must return to the origin of the x axis, is forced to serve on the left side of the equation to mean that the light cannot return to the same origin for all nonzero speeds between two frames. Unless this problem is settled, the equal sign in (1) is invalid. Thus, equation (1) in relativity has been constructed with invalid suppositions.

Consideration 5. First, x' in (1) obviously increases with the advancement of time, because $\frac{x'}{c+v}$ means that light will be intercepted at the origin from which the value of x' is determined. Second, point B of r_{AB} permanently coincides with x'. Taken together, this must mean that r_{AB} keeps stretching with the increase of time. We can immediately recognize how much confusion relativity causes in formulating its most essential equation, equation (1) -- it even rejects r_{AB} as a rod with finite length. Let's imagine such a picture: Point B travels along the x axis at speed v; meanwhile point A, or equivalently the origin of the ξ axis, is able to travel at the same speed but also able to share the interception of light with the origin of the x axis at the same time and at the same location. Unless relativity is proposing that a geometrical point has length, it has created a contradiction that no one can resolve.

Consideration 6. As pointed out in consideration 1, equation (1) has allowed preconceptions to obscure what is actually happening. Only solid material evidence, that both observers can accept together, can eliminate this obscurity. Does such evidence exist? Yes. In spite of different time readings, both observers must accept that there is a distance, but only one, on the x axis that the origin of ξ can fulfill when it intercepts the reflected light.

Suppose the ray of light emitted from point A, mentioned in quotation one, needs a time lapse of Δt to be intercepted again after its emission, where Δt is registered from a clock on the x axis. At interception, point A coincides with a coordinate point named x_2 on the x axis. Equations (2) and (3) in quotation one easily lead the observer on the x axis to have

$$\Delta t = \frac{|r_{AB}|}{c-v} + \frac{|r_{AB}|}{c+v} \tag{13}$$

$$x_2 = v\Delta t = v\cdot\left(\frac{|r_{AB}|}{c-v} + \frac{|r_{AB}|}{c+v}\right) = v\cdot\left(\frac{L\sqrt{1-(v/c)^2}}{c-v} + \frac{L\sqrt{1-(v/c)^2}}{c+v}\right) \tag{14}$$

The time duration recorded by a clock on the ξ axis for the light to complete a round trip on r_{AB} is

$$\Delta\tau = \frac{2L}{c} \tag{15}$$

Naturally, an observer on the ξ axis will have

$$x_2 = v\cdot\Delta\tau = v\cdot\frac{2L}{c} \tag{16}$$

Coupled with the information of x_2, observers on both frames, stationary or moving, must come to an agreement such that

$$v\cdot\left(\frac{L\sqrt{1-(v/c)^2}}{c-v} + \frac{L\sqrt{1-(v/c)^2}}{c+v}\right) = x_2 = v\cdot\frac{2L}{c} \tag{17}$$

or

$$\frac{L\sqrt{1-(v/c)^2}}{c-v} + \frac{L\sqrt{1-(v/c)^2}}{c+v} = \frac{2L}{c} \tag{18}$$

Equation (18) is an exact copy of (7).

Since c and v are both considered constants in relativity, we can replace c with $c=nv$ in (18), where $n>1$; $n=1$ cannot be accepted because of the consideration of (2). After the replacement of $c=nv$, (7), which is now equation (18), it can gradually evolve as shown in the following:

$$\begin{gathered}\frac{L\sqrt{1-(v/nv)^2}}{nv-v} + \frac{L\sqrt{1-(v/nv)^2}}{nv+v} = \frac{2L}{nv}\\ \sqrt{1-(v/nv)^2}\left(\frac{1}{n-1}+\frac{1}{n+1}\right) = \frac{2}{n}\\ \frac{1}{n}\sqrt{n^2-1}\left(\frac{2n}{n^2-1}\right) = \frac{2}{n}\\ \frac{n}{\sqrt{n^2-1}} = 1\\ n^2 = n^2 - 1\\ 1 = 0\end{gathered} \tag{19}$$

If relativity allows its own most fundamental equation to be trapped with the dramatic result of $1=0$, it has no validity; the validity simply does not exist.

Perhaps someone wants to rescue relativity by arguing that the distance marked by x_2 should have been contracted in the view of the observer on the ξ axis. According to this rescuer, the speed that the ξ observer should have with respect to the x axis is not v, but a new speed v' that reflects the length contraction. Subsequently, he argues, (16) should be rewritten as

$$x_2\sqrt{1-\left(v/c\right)^2} = v'\cdot\Delta\tau = v'\cdot\frac{2L}{c} \tag{20}$$

This argument cannot stand by itself because it involves the argument about whether to use v or v' inside the square root of (20). Such a situation is fully discussed in case 1 in another paper in the NPA proceedings of 2010 [3] and will not be repeated here. If the rescuer feels the argument against relativity shown by equation (19) has no merit unless equation (20) can be resolved, here are few more suggestions.

In the opinion of the x observer, the time dilation factor will bridge his time and the time read by the ξ observer in the following way

$$\Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}=\Delta\tau \qquad (21)$$

Then v' developed from (20) can take the form as

$$v'=\frac{x_2\sqrt{1-\left(\frac{v}{c}\right)^2}}{\Delta\tau}=\frac{x_2\sqrt{1-\left(\frac{v}{c}\right)^2}}{\Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}}=v \qquad (22)$$

In the opinion of the ξ observer, with the reciprocal principle between frames about time dilation, he will have

$$\Delta\tau\sqrt{1-\left(\frac{v}{c}\right)^2}=\Delta t \qquad (23)$$

With (23), v' developed from (20) can take the form as

$$v'=\frac{x_2\sqrt{1-\left(\frac{v}{c}\right)^2}}{\Delta\tau}=\frac{x_2\sqrt{1-\left(\frac{v}{c}\right)^2}}{\frac{\Delta t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}=v\left[1-\left(\frac{v}{c}\right)^2\right] \qquad (24)$$

Now, which value of v', (22) or (24), will the rescuer take to remedy the situation?

Furthermore, regardless whether v or v' to be used for the rescue, one can always find two segments terminated by the two origins: one is x_2 from the x axis, another one is $|\Delta\xi|$ from the ξ axis. Believing length contraction, the ξ observer should have

$$x_2\sqrt{1-\left(\frac{v}{c}\right)^2}=|\Delta\xi| \qquad (25)$$

In the same way, the ξ observer obtains v' [by comparing spatial coordinates from the other axis rather than his own, as shown in (20)], the observer on the x axis would have a speed of

$$v''=\frac{|\Delta\xi|}{\Delta t}=\frac{x_2\sqrt{1-\left(\frac{v}{c}\right)^2}}{\Delta t}=v\sqrt{1-\left(\frac{v}{c}\right)^2} \qquad (26)$$

So the rescuer has a few choices: He either allows the x observer to have two different speeds, which are v and v'', or to have one single speed, which is $v=0$, to reconcile between v and v''. He can also allow the ξ observer to have v or v'. Of course, there is one more choice for him: to leave (18) the way it is without modification; but then, 1=0. Rescuing relativity is not an easy job.

4. Conclusion

Equations (7), (8), and (9), all result from one fundamental function, but each also leads to a different outcome, depending on how equation (1) is interpreted by observers from different frames. The confusion is overwhelming. Even aside from this mathematical confusion, relativity proposes many concepts that are totally incompatible with our daily lives, as well as the theoretical realm of Newtonian Physics. As much as relativity tries to convince us regarding its concepts, it also provides theoretical evidence which negates those very concepts.

If the speed of light must remain constant with respect to any inertial frame, the reason remains mysterious. Could it be that such a mystery is only the result of erroneous physical technique in measurement, either through experiment or observation? It is ~~for~~ certain that such a mystery cannot be resolved by a theory that is pure mathematical exploration while ignoring what we see in material interactions.

References

[1] A. Einstein, "On the Electrodynamics of Moving Bodies", Annalen *der Physik* **17** (1905).

[2] H. H. Ricker III, "The Irksomeness of Einstein's Special Theory of Relativity" (July 8, 2008) http://wbabin.net/papers.htm#Ricker.

[3] Cameron Rebigsol, "Relativity's Length Measurement Inconsistency", *Proceedings of the NPA* (2010).