Bohr Quantum Theory Yields the Relativistic Schroedinger Equation

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Niels Bohr's 1913 approach to Quantum Theory yields the relativistic Schroedinger Equation, suggesting that the physical meaning of the wave function is not probabilistic. The existence and physical meaning of longitudinal waves of E-M Potential are discovered.

1. Introduction

On the circular Bohr's orbit the velocity of an electron is constant. The electron radiates de Broglie's frequency that correspond to its velocity (linear momentum). This radiation "feeds" some standing wave which is possible only if the length of the circle orbit can be divided on the integer number of De-Broglie's wave lengths.

We applied the idea of standing wave to the elliptic orbits and obtained the relativistic Schroedinger Equation. Alongside we developed the physical meaning of the alleged standing wave. It is not an "electromagnetic" wave. It is a longitudinal wave of E-M Potential. We call it "Dummy Wave".

2. Longitudinal Waves of E-M Potential

The connection between E-M Vector Potential Ak (A0, A1, A2, A3) and Electromagnetic field Fik is:

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \quad \text{or} \quad \vec{E} = -\nabla\Phi - \partial_0 \vec{A} \quad \vec{H} = \text{curl}(\vec{A})$$
$$\text{where} \quad \Phi = A_0 \quad \vec{A} = (-A_1, -A_2, -A_3) \tag{1}$$

The minus signs appear due to changing covariant components into contravariant ones.

The Gauge Transformation is an adding a gradient of arbitrary function G to a Vector Potential:

$$A'_k = A_k + \partial_k G \quad \text{or} \quad \Phi' = \Phi + \partial_0 G \quad \vec{A}' = \vec{A} - \nabla G \tag{2}$$

It is obvious that the electromagnetic field will not change with the gauge transformation. It is called "Gauge Invariance". All contemporary physics is based on the assumption that only electromagnetic fields have physical meaning and E-M Potential is only physically meaningful as far as it produces electromagnetic fields. It was supposed that a gauge transformation changes nothing physical. To narrow the choice of gauge transformations it is often used L. Lorenz (not H. Lorentz) "gauge":

$$\partial_k A^k = 0 \quad \text{or} \quad \partial_0 \Phi + \text{div}\,\vec{A} = 0 \tag{3}$$

Another condition on gauge is Coulomb "gauge" : $div\vec{A} = 0$. This "gauge" has no 4-dimensional meaning and cannot be considered for a general theory.

The basic assumption of this paper is that everything in the realm of E-M Potential is physically important and can be represented uniquely by E-M Potential. We also notice that (3) looks like exact copy of the law of conservation of electric charge: $\partial_k j^k = 0$ or $\partial_0 \rho + \text{div}\,\vec{j} = 0$. So, we assert that Lorenz "gauge" (3) is the mandatory condition and represent the conservation of "vacuum polarization" or "vacuum charge".

a) The realm of E-M Potential is physical in spite of the fact that Potential cannot be measured directly. Contrary to the philosophy of "Quantum Theory" we still can describe it (yes, claiming its existence) and obtain the physically measurable consequences from that description.

b) This realm is physical in spite of the fact that not all physical objects of this realm carry energy and momentum. The longitudinal waves of E-M Potential have physical reality but they do not produce electric or magnetic fields and consequently have no energy or momentum. Their physical reality is based upon the conservation of "vacuum polarization" (or vacuum charge) and influence on the boundary conditions for electromagnetic fields therefore yielding the measurable results this way.

c) Obviously we deny "gauge invariance". Different "gauges" represent different physical situations (there is no physical invariance – only invariance with respect E-M field, energy, and momentum).

If we suppose that the vector potential has a gradient form then from (3) we have:

$$A_k = \partial_k G \equiv D_k \quad \text{then} \quad \partial^k \partial_k G = 0 \quad \text{or} \quad \Delta G - \partial_0^2 G = 0 \tag{4}$$

This potential we denote Dk and call "Dummy Potential". The scalar G we call "Dummy Generator". Dummy Potential represent Longitudinal Potential Waves. There is no electromagnetic field or energy that accompany this waves. Dummy Generator always satisfy to the vacuum wave equation no matter the space is vacuum (the density of currents are zero) or not vacuum (the density of currents are not zero). Notice, that Dummy Potential also satisfies the homogeneous wave equation everywhere in space. Dummy Generator could be equal to zero in whole space (because there are no sources) if it were no disruption surfaces in space. The sources of Dummy waves are situated on the surfaces of elementary particles (which are disruption surfaces).

The E-M Vector Potential in general has to satisfy the inhomogeneous Wave Equation:

$$\partial^m \partial_m A^k = 4\pi j^k \quad \text{or} \quad \Delta\Phi - \frac{1}{c^2}\partial_0^2 \Phi = -4\pi\rho$$
$$\Delta\vec{A} - \frac{1}{c^2}\partial_0^2 \vec{A} = -4\pi\vec{j} \tag{5}$$

Inside the elementary particles the current density is not zero. Obviously Dummy Waves alone cannot handle this situation. On the other hand we know that any charges under acceleration radiate in vacuum. But this radiation in some cases can be completely Dummy without any energy requirements, therefore can be continuous. In particular we think that it is exactly what happens in elementary particles and in atoms where electron is orbiting the nucleus not radiating any energy.

3. Niels Bohr's Quantum Theory

Niels Bohr suggested the quantization of angular momentum which in the case of circular orbiting is: mvr=n(h/2π) where n has to be integer. This can be rewritten in a form:

$$2\pi r = n\lambda_d \quad \lambda_d = \frac{h}{p} \quad p = \frac{mv}{\sqrt{1 - v^2 / c^2}} \tag{6}$$

Here λd is De-Broglie's wave length (we consider the relativistic case). It looks like we have here an interaction between orbiting electron generating De-Broglie's frequency and some standing wave. This interaction can only be stable if this standing wave "fits" into the circle orbit. Also it is important that everywhere on the orbit the frequency of a standing wave is equal to electron's De-Broglie's frequency. What is the physical meaning of this standing wave? We suggest that it is the standing wave of scalar Dummy Generator. If so, then we know that it has to satisfy the homogeneous wave equation.

Let us now consider more general case of elliptic orbits. If electron moves in a static centrally-symmetric field with the potential energy $V(r)$ then the relativistic dynamics equation can be integrated without any approximations. It gives the energy integral:

$$\frac{mc^2}{\sqrt{1 - v^2 / c^2}} - mc^2 + eV(r) = \varepsilon = \text{const} \tag{7}$$

which reads: kinetic energy plus potential energy is constant. From here we can find the relativistic linear momentum of electron depending on radius:

$$p^2 = 2m(\varepsilon - eV(r))\left[1 + \frac{\varepsilon - eV(r)}{2mc^2}\right] \tag{8}$$

Using (6) we can find De-Broglie's frequency of the electron depending on its distance from the nucleus.

Now let us suppose that there exist a scalar standing wave of Dummy Generator in the space around the nucleus. Let us propose its mathematical form:

$$G = \Psi(\vec{r})\exp(-i\omega_d(r)t) \tag{9}$$

where Ψ is the amplitude that does not depend on time, but has some phase (complex number). The exponent term is unit by its amplitude and expresses oscillations. What is unusual here is that we allow the frequency of oscillations depend on radius. We think that the interaction between this Dummy Generator "cloud" and the moving electron that De-Broglie frequency of the electron should be equal to the local frequency of Dummy Generator (like in the case of circular orbit). Substituting (9) in homogeneous wave equation we found:

$$e^{-i\omega_d t}\left\{\begin{array}{c}\Delta\Psi + \dfrac{\omega_d^2}{c^2}\Psi - it[2(\nabla\omega_d \cdot \nabla\Psi) \\ + \Delta\omega_d\Psi] - t^2(\nabla\omega_d)^2\Psi\end{array}\right\} = 0 \tag{10}$$

At $t = 0$ the amplitude of Dummy Generator has to satisfy the equation:

$$\Delta\Psi + \frac{2m}{\hbar^2}(\varepsilon - eV(r))\left[1 + \frac{\varepsilon - eV(r)}{2mc^2}\right]\Psi = 0 \tag{11}$$

Here we used (6) and (8). The (11) is a relativistic Schroedinger Equation. The term in square brackets is close to 1 and without it (11) is Schroedinger Equation. The equation (11) is not a wave equation. It has no time dependence. The time dependence was introduced in it later artificially.

What has happened? The wave equation is not fully satisfied by G from (9). It is only satisfied at $t = 0$ and in close vicinity of $t = 0$. But G at different t are different not by amplitude but only by different phases at different locations. The interaction between this Dummy Generator "cloud" and the moving electron at a given time occurs at the particular location where the electron is at a given time. This interaction is a subject to phase shift anyway. We can not request both: that De-Broglie's frequency and phase of the electron and the cloud in particular location were exact match. We managed to satisfy only frequency.

We still do not have a full physical picture of what is happening. The accelerating electron still radiate, but the stable Dummy cloud probably helps to make this radiation energyless. In reality there exist not only standing Dummy waves that correspond to the standing waves of Dummy Generator but also outgoing Dummy waves. If "Dummy cloud" becomes unstable then electron begins to radiate energy.

4. Conclusion

We have proved that Bohr's approach yields the correct Quantum theory. We proved that there exists something physical behind the wave function of Quantum Theory. It is not just a "probability". We still cannot explain why a moving electron generates de Broglie's frequency according to its linear momentum.