The Dark Matter Problem
General Relativistic Galactic Rotation Curves in a Friedman Dust Universe with Einstein's Lambda

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1 Abstract

In this paper, the general relativistic replacement for the Newtonian inverse square law of gravitation is obtained from the Friedman Cosmology equations. This version of the inverse square law is shown to contain information about the amount of dark energy mass contained in a specific region through a mass term M_Λ^- dependent on Einstein's Lambda and, importantly for this paper, it also contains information about the amount of dark matter mass in the same region through a term M_P^+. This work derives from the Dust Universe Model which gives a complete cosmological description of the movement and evolution of the astrophysical space substratum which as usual is represented by a spatially uniform or constant mass density distribution at zero pressure. Thus definite spatial regions of the substratum can only be regarded as holding regions for un clumped mass, as primitive galaxies might be described. Consequently, to describe actual galaxies that have condensed from such a region, the more general solution of Einstien's Field eqtions involving the pressure term is needed to explain clumping and the resultant galactic form. The general relativist version of the inverse square law is written in a form applicable to the case of bound circular orbiting about a spherically symmetric central gravitational spatially distributed source force. Thus the behaviour of masses cycling within or outside the source region can be analysed. The formula for the galactic rotation curves for stars rotating within or outside the source region is obtained. A very simple galactic model is used consisting of just two components, the halo and the bulge with all visible orbiting stars, The conclusion is that the pressure term from general relativity and in the consequent Friedman equations is adequate to explain the constancy of the function of rotational velocity as a function of orbital distance from the centre of gravity starting at the massive core of the galaxy. A simple and parameter adaptable computer program using Mathematica has been constructed to display diagrams of galactic rotation curves. This program is available for downloading.

Keywords: Cosmology, Dust Universe, Dark Energy, Dark matter
Galactic Rotation Curves, Friedman Equations
General Relativity, Pressure, Inverse Square Law

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2 Introduction

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [33]. All of this work and its applications has its origin in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7],[25],[3]). The applications can be found in ([23],[24],[33],[37],[35][41]). Other useful sources of information are ([17],[3],[31],[27],[30],[29]) with the measurement essentials coming from references ([1],[2],[11],[38]). Further references will be mentioned as necessary. In the following pages, I shall introduce a general relativistic substructure into the dust universe model that can be used to describe galactic rotation phenomena and show how these relatively stable massive systems, having condensed from the local substratum expansion, can exhibit the constant with respect to radial distance rotation curves that have recently been observed. The dust universe model is left completely intact by this addition but local redistributions of the uniform mass within the expanding substratum mass platforms of that model can now rearrange and, in particular, can clump to form mass distributions separated from neighbouring platforms. The total mass involved in any such redistribution will remain constant. This is achieved by using a general relativistic generalisation of Newton's inverse square law of gravitation at the local galactic level. Thus the *dark matter problem* can be resolved using standard general relativity theory.

3 Relativity Generalised Inverse Square Law

This generalisation is most simply represented by an equation that can be obtained from the Friedman equation for the acceleration field due to density and is

$$\frac{\ddot{r}(t)}{r(t)} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho(t) + \frac{3P(t)}{c^2}\right). \tag{3.1}$$

This equation includes a contribution from the Lambda term, Λc^2. This is a very important equation in relation to the acceleration due to gravity at radius r and how that depends on the mass density term $\rho(t)$. Clearly, the pressure term $3P/c^2$ adds to the mass density to produce an effective or physical mass density $3P/c^2 + \rho$. In the dust universe model, the pressure term is taken to be zero at all times t so that in the dust universe model case the equation above can be written as

$$\frac{\ddot{r}(t)}{r(t)} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\rho(t). \tag{3.2}$$

If we define a time dependent mass density, $\rho_P(t)$, that includes the pressure term as

$$\rho_P(t) = \left(\rho(t) + \frac{3P(t)}{c^2}\right), \tag{3.3}$$

the original equation (3.1) can be represented as

$$\frac{\ddot{r}(t)}{r(t)} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\rho_P(t) = \frac{\ddot{r}_P(t)}{r_P(t)}. \tag{3.4}$$

Thus the alternatively expressed original equation (3.4) that includes pressure is indistinguishable from the dust universe form of equation (3.2) that does not involve pressure, except for the subscript P on the density function. It follows that the solution for $r(t)$ that is involved in the dust universe model is the same as that for the more general non dust models with the plain $\rho(t)$ replaced by $\rho_P(t)$. To avoid confusion when using the more general case, I shall use the subscripted form of the equation given by the second equality in equation (3.4) above. It is possible to be rather more precise about the meaning of this formula and also show that it is indeed a generalisation of Newton's inverse square law as follows. In its original form as derived from the Friedman equations the non Λ part of this formula represents the radial acceleration just outside a sphere of radius $r(t)$ due to the gravity of a spatially uniform distribution of positively gravitating mass centred on a sphere of radius $r(t)$. The Λ part of this formula represents the radial acceleration just outside a sphere of radius $r(t)$ due to the gravity of a spatially uniform distribution of density $\rho^{\dagger}_{\Lambda}$ of negatively gravitating mass centred on the same sphere. If we use the volume, $V_P(t)$, of the sphere of radius $r_P(t)$, the formula for $\rho^{\dagger}_{\Lambda}$ and the formulas for total positively gravitating mass M^{+}_{P} and total negatively gravitating mass M^{-}_{P} within the volume at time t together with the definitions of the gravitational coupling constants for positively G_+ and negatively gravitating material G_- respectively given below

$$\begin{aligned}
V_P(t) &= 4\pi r_P^3(t)/3 && (3.5)\\
\rho^{\dagger}_{\Lambda} &= \frac{\Lambda c^2}{4\pi G} && (3.6)\\
M^{+}_{P} &= \rho_P(t) V_P(t) && (3.7)\\
M^{-}_{P}(t) &= \rho^{\dagger}_{\Lambda} V_P(t) && (3.8)\\
G_+ &= +G && (3.9)\\
G_- &= -G && (3.10)
\end{aligned}$$

these definitions can be used to express (3.4) in the form

$$\ddot{r}_P(t) = -\frac{G_- M^{-}_{P}(t)}{r_P^2(t)} - \frac{G_+ M^{+}_{P}}{r_P^2(t)}. \qquad (3.11)$$

Expression (3.11) is the general relativity generalisation for Newton's inverse square law of gravitation that is implied by Einstein's field equations with Λ. This generalises Newton's inverse square law in three respects. Firstly, the radius variable $r_P(t)$ can depend on time. Secondly a contribution of negatively gravitating material is taken into account through the time dependent mass term $M^{-}_{P}(t)$ and thirdly the additional positively gravitational mass due to pressure is taken into account through the non time dependent mass term M_P^+. The derivation above depends on both types of mass density not depending on space variation. However, once this formula is obtained that restriction can be removed because the well known result from Newtonian gravitation theory that says that the mass distribution within the volume with radius $r(t)$ at some fixed time t can be redistributed in any way, with the formula remaining valid, provided its numerical value remains constant and its centre of mass remains at the centre of the sphere. Reverting back to the original form of the formula (3.4), we have

$$\frac{\ddot{r}(t)}{r(t)} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho(t) + \frac{3P(t)}{c^2}\right) \tag{3.12}$$

and this can be claimed to be the same thing as equation(3.11), the general relativistic generalisation of Newton's law of gravitation. This formula can certainly be used in the context of studying well know problems in classical gravitation theory to see what differences the general relativity structure from which it emerged brings to the classical solutions. The objective of this paper is to do just that in the case of the dark matter problem of galactic dynamics. As shown above the formula (3.12) can be applied to the case of the gravitational field generated by the fixed amount of mass M_P^+ contained within the time variable volume $V_P(t)$. However, using the purely spherical symmetric case for all functions for simplicity of presentation, it can also be seen to give the general relativistic generalisation of Newton's law that would apply to the more familiar case of the gravitational effect of a fixed amount of mass within a fixed volume when the radial component of velocity $v_2(t)$ is zero and the transverse accelerations $\alpha_2(t)$ is zero, just by dropping the time dependence of $r(t)$ and assuming that $\rho(t) \to \rho(r, r_i)$ and $P(t) \to P(r, r_i')$ and assuming the density and pressure are constant for radii r up to radii r_i and r_i' respectively and zero at greater radii. This last change is certainly a simplifying device to avoid mathematical complications. In effect this is making use of a very simplified model for the mass distribution in a galaxy. One sphere of uniform mass density $\rho(r, r_i)$, extending from $r = 0$ to represent the central bulge and its orbiting stars of radius r_i and a second concentric uniform mass density $\rho(r, r_i')$ sphere extending from $r = 0$ of radius r_i' to represent the halo with $r_i' > r_i$. In more detail these two overlapping densities are defined as,

$$\rho(r, r_i) = 0, \; r < 0 \tag{3.13}$$
$$\rho(r, r_i) = \rho_{r_i} = a\ constant, \; r \leq r_i \tag{3.14}$$
$$\rho(r, r_i) = 0, \; r > r_i \tag{3.15}$$
$$\rho(r, r_i') = 0, \; r < 0 \tag{3.16}$$
$$\rho(r, r_i') = \rho_{r_i'} = a\ constant, \; r \leq r_i' \tag{3.17}$$
$$\rho(r, r_i') = 0, \; r > r_i'. \tag{3.18}$$

The current thinking on this problem is that most if not all of the missing mass is within the halo. In the case of radial velocity zero, we get

$$\frac{\ddot{r}(t)}{r(t)} \rightarrow -\frac{v_2^2}{r^2} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho(r, r_i) + \frac{3P(r, r_i')}{c^2}\right) \tag{3.19}$$
$$v_1(t) = \dot{r}(t) = 0 \Rightarrow r(t) = r = \ a\ constant \tag{3.20}$$
$$v_2(t) = r\dot{\theta}(t) = r\omega \Rightarrow v_2(t) \rightarrow v_2(r) \tag{3.21}$$
$$\alpha_1(t) = \ddot{r} - r\dot{\theta}^2(t) = -r\dot{\theta}^2(t) \tag{3.22}$$
$$\alpha_2(t) = r\ddot{\theta}(t) + 2\dot{r} = r\ddot{\theta}(t) = \frac{\partial(r^2\dot{\theta}(t))}{r\partial t} = 0 \tag{3.23}$$
$$\Rightarrow r^2\dot{\theta}(t) = l = \ a\ constant \tag{3.24}$$
$$\Rightarrow \dot{\theta}(t) = \omega = \ a\ constant. \tag{3.25}$$

Writing equation (3.19) out again in terms of the masses involved inside the spherical volume of radius r we have

$$\frac{v_2^2(r)}{r^2} = \frac{4\pi G}{3}\left(\rho(r, r_i) + \frac{3P(r, r_i')}{c^2}\right) - \frac{\Lambda c^2}{3} \tag{3.26}$$
$$v_2^2(r) = \frac{GV(r)}{r}\left(\rho(r, r_i) + \frac{3P(r, r_i')}{c^2}\right) - \frac{GV(r)\rho_\Lambda^\dagger}{r}. \tag{3.27}$$

Let us now define three quantities of mass for the positively gravitating density term $\rho(r, r_i)$ the pressure term $3P(r_i, r_i')/c^2$ and the dark energy term $\rho_\Lambda^\dagger$ within the spherical volume $V(r)$ as follows

$$\begin{aligned} M^+(r,r_i) &= V(r)\rho(r,r_i) && (3.28)\\ &= M^+(r_i,r_i),\ r \geq r_i && (3.29)\\ M_P(r,r_i') &= V(r)\frac{3P(r,r_i')}{c^2} && (3.30)\\ &= M_P(r_i',r_i'),\ r \geq r_i' && (3.31)\\ M_\Lambda(r) &= V(r)\rho_\Lambda^\dagger. && (3.32) \end{aligned}$$

At step (3.31) r is outside both distributions of mass. Using this notation, we can write equation (3.27) as follows and compare it with the classical velocity circulation equation at the line (3.36) below,

$$\begin{aligned} v_2^2(r) &= \frac{G}{r}\left(M^+(r,r_i) + M_P(r,r_i')\right) - \frac{GM_\Lambda(r)}{r} && (3.33)\\ &= \frac{GM^*(r)}{r} && (3.34)\\ M^*(r) &= M^+(r,r_i) + M_P(r,r_i') - M_\Lambda(r) && (3.35)\\ v_2^2(r) &= \frac{GM(r)}{r}. && (3.36) \end{aligned}$$

Note that $M^*(r)$ is only constant for values of r such that $r > r_i'$ when the $M_\Lambda^\dagger$ is excluded. In the classical case, $M(r)$ means the total amount of mass within a spherical volume of radius r about the radial origin and by construction $M^*(r)$ means the same thing. The minus sign in front of the Λ mass does not mean that this mass is negative, rather it goes with the G at (3.34) to give the negatively gravitational coupling constant $G_- = -G$ involved with negatively gravitating mass. The expression $M^*(r)$ is a mathematical convenience. The classical version of the formula for boundary velocity in terms of enclosed gravitating mass, (3.36), has been used to find the amount of mass $M(r)$ within a spherical region of radius r in terms of its transverse outer edge velocity $v_2(r)$, (3.37), and also to give the boundary transverse velocity at the edge of an enclosed amount of mass, $M(r)$, (3.38),

$$\begin{aligned} M(r) &= \frac{v_2^2(r)r}{G} && (3.37)\\ v_2(r) &= \left(\frac{GM(r)}{r}\right)^{1/2}. && (3.38) \end{aligned}$$

These formulae are applicable to planetary systems and were thought to apply to galactic systems composed of stars in rotational motion. The formula (3.38) involves $M(r)$, the mass within a sphere of radius r and gives the velocity at the surface of that sphere but it says nothing about the way the mass is arranged in that sphere except that the centre of mass has to be at the centre. Thus $M(r) = M(r_i) =$ a constant for all $r > r_i$. Or alternatively expressed from (3.38), $v_2(r)$ is inversely proportional to $r^{1/2}$ for $r > r_i$. We can use this to write down a result to be used later. In the case when the radial variable r is outside the region where the density fuction is non zero the mass function can be written as

$$M(r, r_i) = M(r_i, r_i) = \textit{ a constant}. \tag{3.39}$$

Let us now consider the case for $r \leq r'_i$. That is when the variable r is within the region in which the density function $\rho(r'_i)$ is constant and *not zero*. In that situation we can write

$$v_2^2(r) = \frac{GM(r, r'_i)}{r} = \frac{GV(r)\rho(r, r'_i)}{r} = \frac{4\pi G r^2 \rho(r'_i, r'_i)}{3} \tag{3.40}$$

and because $\rho(r, r'_i)$ is constant and equal to $\rho(r'_i, r'_i)$ for $r \leq r'_i$, we can now claim that $v_2(r)$ is directly proportional to r. We can use this to write down a second result to be used later. In the case when the radial variable r is inside the region where the density fuction is non zero the mass function can be written as

$$M(r, r'_i) = V(r)\rho(r, r'_i). \tag{3.41}$$

$$v_2^2(r) = \frac{GM(r, r'_i)}{r} = \frac{GV(r)\rho(r, r'_i)}{r} = \frac{4\pi G r^2 \rho_{r'_i}}{3} \tag{3.42}$$

where $M(r, r'_i)$ is the total amount of pressure related mass within the sphere of radius r and because $\rho(r, r'_i) = \rho_{r'_i}$, a constant by (3.14).

The two results (3.38) and (3.42) are the well know classical results from Newtonian theory the first giving the case when gravitational effects outside the source distribution are considered and the second case when gravitational effects within the source gravitational distribution are considered. The first case is clearly a good account of the observed speeds of planets in the solar system where the sun has the dominant effect on the planets. The second case is the usually assumed form for how gravity would affect the motion of a free particle within a cavity inside the earth's interior or any other planet for that matter. The dark matter problem has arisen from astronomical observational measurements that indicate that the transverse velocities for stars at the edges of galaxies obey neither of the two cases above but rather obey an approximately constant relation between velocity $v_2(r)$ and the radial distance from the centre r. That is to say the measured galactic velocities squared are greater than the theoretical value suggested by the tailing off first case above and less than the quadratic increasing second case above might imply. Clearly the formula (3.38) implies that if rotation velocities at some distance are to be higher then more mass within the region is required. Thus if theory is to adapt to observation the extra mass within the region has to be arranged in some special way. This is the *dark matter problem* of how much extra mass there has to be and where this missing mass needs to be located within the galactic volume so that it leads to constant velocity against distance rotation curves for cycling stars. There is nowadays some consensus that the missing or dark matter part of galactic structure is about four or five times the normal mass part. There seems to be quit a lot of variability over this estimate, it partly depend on how much dark energy the universe is assumed to contain at any time. In the dust universe model, there is 75% dark energy mass and 25% normally gravitating matter present now. This seems to me to favour four for the ratio of dark matter to ordinary matter in the positively gravitating sector, M_P^+. These figures come from reference [1] The analysis above of the rotation curves structure from (3.36) onwards has used the classical Newtonian theory and as we have seen that does not seem to explain why the galactic rotation curves are flat. However, we have above the more elaborate general relativistic generalisation of Newtonian inverse square law gravitation theory that will be applied to the study of galactic structure in the next section.

4 General Relativistic Rotation Curves

Let us now consider what the general relativistic generalisation of Newton's inverse square law can contribute to the problem of the missing mass and the rotation curves. In the general case for any r we have,

$$
\begin{aligned}
v_2^2(r) &= \frac{G}{r}\left(M^+(r,r_i) + M_P(r,r_i')\right) - \frac{GM_\Lambda(r)}{r} &(4.1)\\
M_P(r,r_i') &= V(r)\frac{3P(r_i')}{c^2} &(4.2)\\
M_\Lambda(r) &= V(r)\rho_\Lambda^\dagger. &(4.3)
\end{aligned}
$$

Equation (4.1) is the general relativity rotation curve for transverse velocity in terms of distance from the origin r. The two following masses are the pressure induced mass and the dark energy mass within the spheres of radius r. These are clearly additional masses within the region concerned that help determine the form of the function $v_2(r)$. Pressures in cosmology are used in conjunction with an *equation* of state that expresses the pressure in term of a related density and a dimensionless function denoted by ω that can depend on space and time parameters as below but with no time dependence and only radial spatial distance involved for this problem.

$$
\begin{aligned}
P(r,r_i') &= c^2\rho(r,r_i')\omega(r,r_i') &(4.4)\\
M_{GR}^+(r) &= M^+(r,r_i) + M_P(r,r_i') = V(r)\rho(r,r_i) + V(r)\frac{3P(r,r_i')}{c^2} &(4.5)\\
&= V(r)\rho(r,r_i) + V(r)3\rho(r,r_i')\omega(r,r_i'). &(4.6)
\end{aligned}
$$

It is not obvious what the function $\omega(r,r_i')$ should be except that we know that it represents positively gravitating material. In the case of negatively gravitating material associated with the Λ term, we do know it has the value -1. Thus a first reasonable shot at the value for the ω above is the value $+1$ which gives for equation (4.6)

$$
M_{GR}^+ = V(r)(\rho(r,r_i) + 3\rho(r,r_i')). \qquad (4.7)
$$

At equation (4.7) we see just how much, $3\rho(r,r_i'))$, additional mass the pressure term from general relativity adds to the classical density term $\rho(r,r_i)$. Further, r_i' is an adjustable parameter so that it can be chosen to give any ratio between these two terms. For example, if we decide four to one is the correct value, $\rho(r,r_i')$ could be chosen so that

$$
\rho(r,r_i') = \rho(r,r_i) \qquad (4.8)
$$

with $r_i' = r_i$. That is to say, the mass distribution and the density distribution coincide in value and terminal radius. The general relativity rotation curve for this case, without taking into account the Λ term would be

$$v_2^2(r) = \frac{4\pi r^2 G}{3}\left(\rho(r,r_i) + 3\rho(r,r_i')\right) \tag{4.9}$$

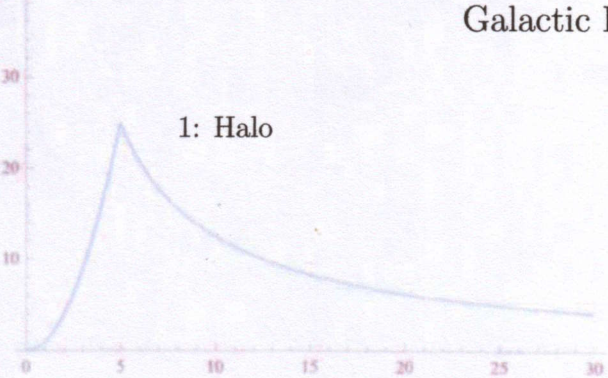
$$\frac{M_P(r_{out},r_i')}{M^+(r_{out},r_i)} = \frac{4\rho(r_{out},r_i')}{\rho(r_{out},r_i)} = 4, \tag{4.10}$$

$$\tag{4.11}$$

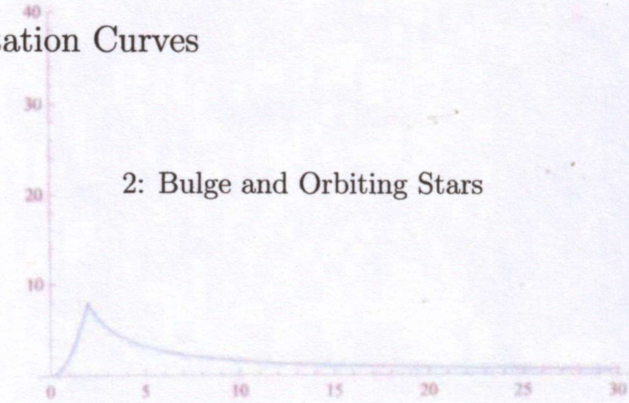
where r_{out} is a radial position outside both mass distributions and at (4.10), we have obtained the radio of halo mass to visible mass to be 4. It is clearly easy to find the values for the quantities concerned, the $\rho(r,r_i)$s and the r_is to find any value that might be determined from experiment to be the correct value. Thus I have constructed a simple computer program using mathematica to display graphically a whole range of rotation curves in the general relativity case that correspond to the astronomically observed rotation curves. The diagrams $1, 2$ and 3 0n page 12 were copied form this program. This program in Mathematica note book language can be downloaded in the file *grcs.nb* from my website at, QMUL Maths.

I have carried through this case for identifying dark matter as originating from the pressure term that appears in Einstein's field equations and consequently also in the Friedman equations with an extremely simple model for a galaxy. The model is unrealistic in a number of ways, two constant spherically constant mass distributions have been used both of which terminate sharply at their radial limits r_i and r_i'. Galaxies are certainly not like that, particularly in the respect that the distribution of stars in the visible part of a galaxy tail off at a very indefinite outer boundary. Also the actual visible distributions observed are not uniformly constant but often have very complicated spiral shapes for example. One consequence of this sharp boundary aspect is that the curves I have calculated have sharp cusps so that the orbits calculated have to refer to stars at the lower boundary and indeed I have taken the constant curves to start at that boundary. I have used this simple model to avoid mathematical complications and to get quickly to a clear conclusion. Certainly the model could be made very much more realistic by giving the whole structure more sections with more character. However, in spite of the simplifications, I think it is established that Einstein's pressure term does account for the missing dark matter and the extra gravitational power that exists within a galaxy and holds it together.

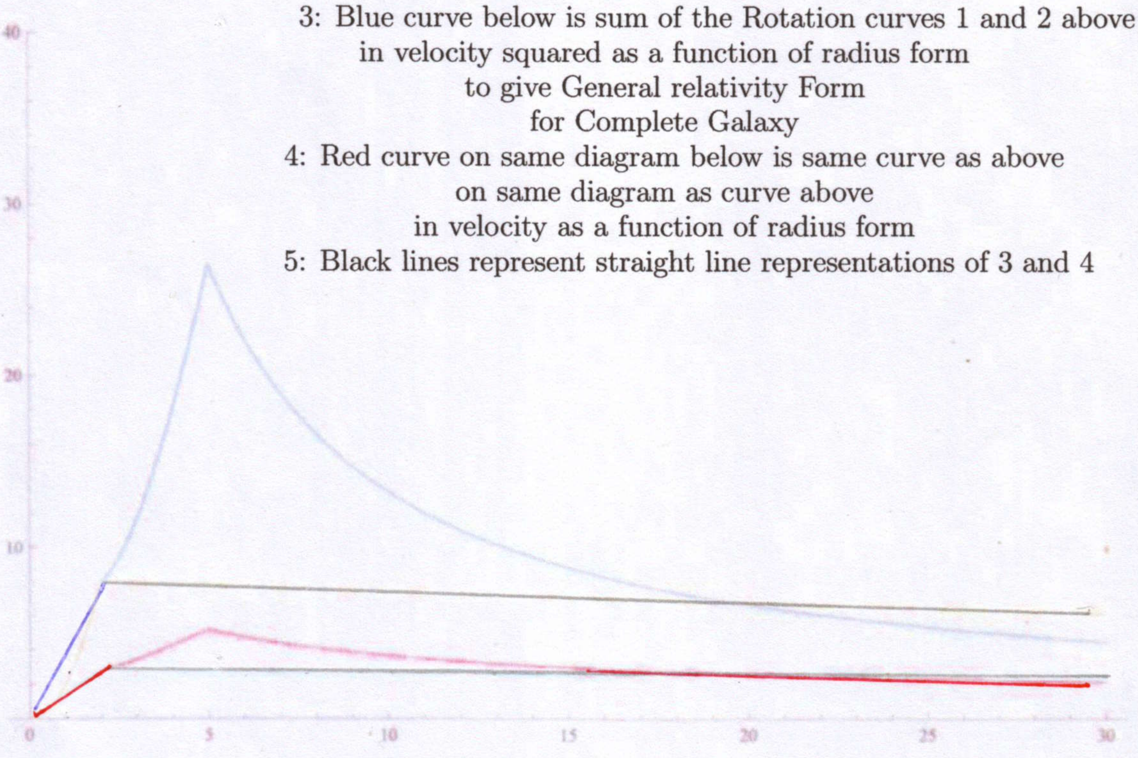
Galactic Rotation Curves



1: Halo



2: Bulge and Orbiting Stars



3: Blue curve below is sum of the Rotation curves 1 and 2 above
in velocity squared as a function of radius form
to give General relativity Form
for Complete Galaxy

4: Red curve on same diagram below is same curve as above
on same diagram as curve above
in velocity as a function of radius form

5: Black lines represent straight line representations of 3 and 4

.

5 Conclusions

The dark matter problem has been around for about a century. Fritz Zwicky[46] was an early astronomer to remark on this problem following his studies of the masses of galaxy clusters. There has been vast numbers of papers written on this subject and many varied attempts to find the solution of this problem which is essentially the problem of explaining and locating what appears to be vast quantities of mass in the universe that appears to exert gravitational attraction but cannot be seen with most of the observation astronomical equipment that exists at present. One dominate attempt at solving this problem has been the suggestion that Newton's theory of gravity needs to be modified particularly in the way it describes gravity at large distances from the gravitational source and this has seemed to imply that Einstein's general theory of relativity would also need to be modified. This connection arises because Newton's theory of gravity is a limiting case of Einstein's Theory. However, there seems to have been little clear recognition of the actual *form and structure* taken by the Einstein general relativity replacement for the Newton inverse square law. This failure I have rectified in this and earlier papers by deriving and displaying the full content of the general relativity inverse square law of gravity (3.11). This formula contains the essential addition of Einstein's *dark energy mass* and its pressure contribution both contained in the mass term, M_Λ^- and also the full positively gravitating mass density contribution which includes the pressure induced part both contained in the mass term, M_P^+. The addition of Einstein's mass *pressure term* is here used to described the so called mysterious *dark matter*, apparently invisible contribution, here identified as the halo, that explains the missing gravitating mass of galaxies or their clusters. I feel that this claim is reinforced by an observation I made in an earlier paper, QMUL Maths, with regard to *dark energy* is also applicable in part to *dark matter* repeated here as follows:- Thus the mystery of the origin of the dark energy density, $\rho_\Lambda = \Lambda c^2/(8\pi G)$ in Einstein's form or in my revised form $\rho_\Lambda^\dagger = 2\rho_\Lambda$, within the universe is completely resolved by this theory. Possibly this is the reason that dark energy is not visible. It could be because *pressures* are not usually visible and the *pressure status* of the dark energy density is its dominant characteristic. However, it seems to me that dark energy with approximately an equivalent density of 5 hydrogen atoms per cubic meter would not be visible anyway.

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