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# Introduction to the Flyby Anomaly :
# the Gyrotational Acceleration of Orbiting Satellites

***described by using gravitomagnetism.***

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### Abstract

In a former paper[1], much attention has been given to spinning objects whereof the gyrotational acceleration has been calculated for particles in the spinning sphere and at its surface. The purpose of this paper is to calculate the gyrotational acceleration of orbiting satellites in an orbital plane under an angle with the Earth's equator. It is found that strong influence is possible, depending from the orbit's inclination.


*Keywords*: flyby anomaly – gravitation – gyrotation – prograde – retrograde – orbit.
*Method*: Analytical.

## 1. The Maxwell Analogy for gravitation: equations and symbols.

The Maxwell Analogy for gravitation is the closest theory to the General Relativity of Einstein, while the universe remains Euclid and is not curved. The double aspect of the gravitational field is expressed by the Newtonian gravitation field, supplemented with the *gravitomagnetic* field that I call *gyrotation*. This latter field has been proposed by Oliver Heaviside at the end of the 19th century. The so-called Gyro-gravitation Theory (= gravitomagnetism), which is this very same theory, but including a new physical definition for 'the observer'[1], is suitable to explain celestial mechanics for steady and quasi-steady systems. The retardation of gravitation due to its finite velocity is not taken in account and this does not affect the results noticeably.
For the basics of the theory, I refer the reader to my paper: "*Analytic Description of Cosmic Phenomena Using the Heaviside Field*"[1]. The most relevant parts are summarized in the next paragraphs.
The gyro-gravitation laws can be expressed in equations (1.1) up to (1.6) below.
The electric charge is then substituted by mass, the magnetic field by gyrotation, and the respective constants are also substituted. The gravitation acceleration is written as $\boldsymbol{g}$ , the so-called *gyrotation field* as $\boldsymbol{\Omega}$, and the universal gravitation constant out of $G^{-1} = 4\pi\,\zeta$, where $G$ is the universal gravitation constant. We use the sign $\Leftarrow$ instead of = because the right-hand side of the equations causes the left-hand side. This sign $\Leftarrow$ will be used when we want insist on the induction property in the equation. $F$ is the resulting force, $v$ the relative velocity of the mass $m$ with density $\rho$ in the gravitational field. And $\boldsymbol{j}$ is the mass flow through a fictitious surface. Bold fonts represent vectors.

$$\boldsymbol{F} \Leftarrow m\,(\,\boldsymbol{g} + \boldsymbol{v} \times \boldsymbol{\Omega}\,) \quad (1.1) \qquad div\,\boldsymbol{j} \Leftarrow -\,\partial\rho\,/\,\partial t \quad (1.4)$$

$$\nabla\,.\,\boldsymbol{g} \Leftarrow \rho\,/\,\zeta \quad (1.2) \qquad div\;\boldsymbol{\Omega} \equiv \nabla.\;\boldsymbol{\Omega} = 0 \quad (1.5)$$

$$c^2\,\nabla\times\boldsymbol{\Omega} \Leftarrow \boldsymbol{j}\,/\,\zeta + \partial\boldsymbol{g}\,/\partial t \quad (1.3) \qquad \nabla\times\boldsymbol{g} \Leftarrow -\,\partial\boldsymbol{\Omega}\,/\,\partial t \quad (1.6)$$

It is possible to speak of gyro-gravitation waves with transmission speed $c$.

$$c^2 = 1\,/\,(\,\zeta\,\tau\,) \quad (1.7) \qquad \text{wherein} \qquad \tau = 4\pi\,G/c^2.$$

## 2. Calculation of the gyrotation of a spinning sphere.

For a spinning sphere with rotation velocity $\omega$, the result for gyrotation outside the sphere is given by the vector equation (2.1) . In fig. 2.1, one equipotential line of the gyrotation vector $\boldsymbol{\Omega}$ has been traced for a spinning sphere with radius $R$ , a moment of inertia $I$ and a spinning velocity vector $\boldsymbol{\omega}$ at a distance vector $\boldsymbol{r}$ from the sphere's centre.

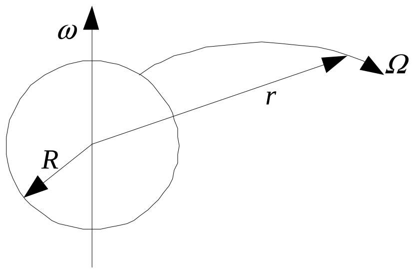


Fig. 2.1 : A spinning sphere with radius $R$ and rotation velocity $\boldsymbol{\omega}$ is generating a rotary gravitation field (or “gyrotation” field) $\boldsymbol{\Omega}$ (whereof one equipotential is shown) at a distance $\boldsymbol{r}$ from the sphere's centre.

$$\Omega_{ext} \Leftarrow \frac{\boldsymbol{G}\,\boldsymbol{I}}{\boldsymbol{2}\,\boldsymbol{r}^3\,\boldsymbol{c}^2}\left(\omega - \frac{\boldsymbol{3}\,\boldsymbol{r}(\omega \bullet \boldsymbol{r})}{\boldsymbol{r}^2}\right) \qquad \text{wherein for a sphere :} \qquad \boldsymbol{I} = \frac{\boldsymbol{2}}{\boldsymbol{5}}\,\boldsymbol{m}\,\boldsymbol{R}^2 \qquad (2.1.a)\ (2.1.b)$$

The value of the gyrotation can be found at each place in the universe, and is decreasing with the third power of the distance $r$ . The factor $\boldsymbol{\omega} \bullet \boldsymbol{r}$ represents the scalar vector-multiplication, and this value is zero at the equatorial level.

In fig. 2.2 , the definition of the angles $\alpha$ and $i$ is shown. The orbital plane of the asteroid is defined by the orbital inclination $i$ in relation to the axis $X$ . The exact location of the asteroid inside the orbit is defined by the angle $\alpha$ . The equipotential line of the gyrotation $\Omega$ through the asteroid has been shown as well. It is clear that the gyrotation of the Earth is axis-symmetric about the Z-axis.

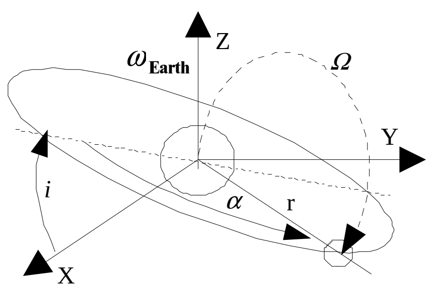


Fig. 2.2 : Definition of the angles $\alpha$ and $i$ . The orbital plane is defined by the orbital inclination $i$ in relation to the axis $X$ . The location of the satellite inside the orbit is defined by the angle $\alpha$ . The equipotential line of the gyrotation $\boldsymbol{\Omega}$ through the satellite has been shown as well.

Now, we need to write the equation (2.1) in full for each of the components, in the case of the spinning Earth.

$$\left(\Omega_x, \Omega_y, \Omega_z\right) = -\frac{\boldsymbol{G}\,\boldsymbol{I}}{\boldsymbol{2}\,\boldsymbol{r}^3\boldsymbol{c}^2}\left(\left(\boldsymbol{0},\boldsymbol{0},\omega_z\right) - \frac{\boldsymbol{3}}{\boldsymbol{r}^2}\left(\boldsymbol{r}_x,\boldsymbol{r}_y,\boldsymbol{r}_z\right)\left(\omega\,\boldsymbol{r}\,\mathbf{cos}(\pi/\boldsymbol{2} - \boldsymbol{i}\,\mathbf{cos}\,\alpha)\right)\right) \qquad (2.2)$$

wherein
$$\left(\boldsymbol{r}_x, \boldsymbol{r}_y, \boldsymbol{r}_z\right) = \boldsymbol{r}\left(\mathbf{cos}\,\alpha\,\mathbf{cos}\,\boldsymbol{i},\ \mathbf{sin}\,\alpha\,\mathbf{cos}\,\boldsymbol{i},\ \mathbf{cos}\,\alpha\,\mathbf{sin}\,\boldsymbol{i}\right) \qquad (2.3)$$

The equations (2.2) and (2.3) constitute the detailed vector formula of the equation (2.1). Remark that $\omega_Z = \omega = \omega_{Earth}$ . In the next chapters we will analyse the torque which is exerted by the gyrotational part of the gyro-gravitation.

Firstly, we have to analyse the effects of gyrotation on the satellite. Some of the components of the gyrotation will affect the spin or the motion of the satellite, other components will not affect the satellite's motion.

## 3. The gyrotational accelerations of the satellite

When applying the equation (3.1) for each of the components, we get the forces that works onto the satellite due to gyro-gravitation. Let us write this result as an acceleration only, and omit the gravitational part, because it does not play any role for unusual accelerations of the satellite.

Hence,
$$\left(a_x, a_y, a_z\right)_{\mathrm{sat}} = \left(v_y \Omega_z - v_z \Omega_y, v_z \Omega_x - v_x \Omega_z, v_x \Omega_y - v_y \Omega_x\right) \tag{3.1}$$

Herein,
$$\left(v_x, v_y, v_z\right)_{\mathrm{sat}} = \left(\omega_x, \omega_y, \omega_z\right)_{\mathrm{sat}} \times \left(r_x, r_y, r_z\right) \tag{3.2}$$

with
$$\left(\omega_x, \omega_y, \omega_z\right)_{\mathrm{sat}} = \omega_{\mathrm{sat}} (\sin i, 0, \cos i) \tag{3.3}$$

when using (2.3) and (3.3) in (3.2), we get:

$$\left(v_x, v_y, v_z\right)_{\mathrm{sat}} = \omega_{\mathrm{sat}}\, r \left(-\sin\alpha \cos^2 i, \cos\alpha \left(\cos^2 i - \sin^2 i\right), \sin\alpha \cos i \sin i\right) \tag{3.4}$$

Equation (3.4) can be written as:

$$\left(v_x, v_y, v_z\right)_{\mathrm{sat}} = \omega_{\mathrm{sat}}\, r \left(-\sin\alpha \cos^2 i, \cos\alpha \cos 2i, \tfrac{1}{2} \sin\alpha \sin 2i\right) \tag{3.5}$$

Combining (3.1) with (2.2) , (2.3) and (3.5) gives the gyrotational accelerations of satellites in circular orbits.

We get:

$$a_x = -\frac{G\, I\, \omega \omega_{\mathrm{sat}}}{2 r^2 c^2} \left(\cos\alpha \cos 2i \left(1 - 3 \cos\alpha \sin i \sin(i \cos\alpha)\right) + \frac{3}{2} \sin^2\alpha \sin 2i \cos i \sin(i \cos\alpha)\right)$$

$$a_y = -\frac{G\, I\, \omega \omega_{\mathrm{sat}}}{2 r^2 c^2} \left(-\frac{3}{4} \sin 2\alpha \sin 2i \cos i \sin(i \cos\alpha) + \sin\alpha \cos^2 i \left(1 - 3 \cos\alpha \sin i \sin(i \cos\alpha)\right)\right)$$

$$a_z = -\frac{3 G\, I\, \omega \omega_{\mathrm{sat}}}{2 r^2 c^2} \left(\sin^2\alpha \cos^3 i \sin(i \cos\alpha) + \cos^2\alpha \cos 2i \cos i \sin(i \cos\alpha)\right)$$

(3.6.a.b.c)

## 4. Axial transform: rotation about the Y-axis

It is useful to get the value of the accelerations in the orbit plane. Therefore, the axis will be rotated about the Y-axis and the system ($X$, $Y$, $Z$) will be transformed into the system ($X'$, $Y'$, $Z'$).

The transform is given by
$$\left(X', Y', Z'\right) = \left(X \cos i - Z \sin i, Y, X \sin i + Z \cos i\right) \tag{4.1}$$

I am particularly interested in the $Z'$-component of the acceleration, because it represents the tangential (rotational) acceleration upon the orbit, and the place where $\alpha = 0$ is even able to swivel the orbit about the $Y$-axis. The other components only change the shape of the orbit. Applied upon the $Z'$-component of the gyrotational acceleration, where I analyze the case of $\alpha = 0$, I get:

$$a'_z = a_t = -\frac{G\, I\, \omega \omega_{\mathrm{sat}}}{2 r^2 c^2} \left(\sin i \cos 2i \left(1 - 3 \sin^2 i\right) - \frac{3}{4} \sin 4i \cos i\right) \tag{4.2}$$

The result of the tangential gyrotational orbit acceleration for $\alpha = 0$ is given in fig. 4.1, where we clearly see that for prograde orbits, the states of rest are given for an orbital inclination of $i = 0$ and $\pi/4$. For retrograde orbits, they are $i = 0$ and $3\pi/4$.

For inclinations between $i$ = 0 and π/4 (prograde), and for $i$ = 3π/4 and 2π (retrograde), the acceleration tends towards positive values, resulting in a weak rotational drift towards the rotational axis of the Earth.
But for inclinations between $i$ = π/4 and π/2 (prograde), and for $i$ = π/2 and 3π/4 (retrograde), the acceleration will much more strongly tend towards negative values (4 times larger than the former case), resulting in a rotational drift towards the equatorial axis of the Earth. The retrograde orbits are strongly pushed back into prograde orbits.

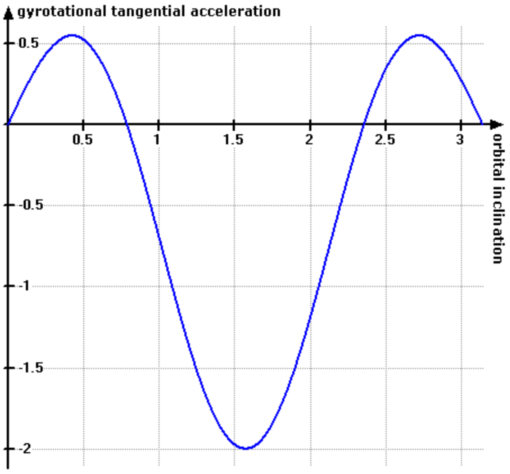


Fig. 4.1. Tangential gyrotational orbit acceleration for $\alpha = 0$ . For prograde orbits, the states of rest are given for an orbital inclination of $i$ = 0 and π/4. For retrograde orbits, they are $i$ = 0 and 3π/4. For inclinations between $i$ = 0 and π/4 (prograde), and for $i$ = 3π/4 and 2π (retrograde), the acceleration tends towards positive values, resulting in a rotational drift towards the rotational axis of the Earth. For inclinations between $i$ = π/4 and π/2 (prograde), and for $i$ = π/2 and 3π/4 (retrograde), the acceleration will much more strongly tend towards negative values, resulting in a rotational drift towards the equatorial axis of the Earth, and retrograde orbits are strongly pushed back into prograde orbits.

## 5. Conclusions.

With the equations (3.6.a.b.c), we found the accelerations of the gyrotational part of gravitomagnetism, that work upon a satellite in a circular orbit about the Earth, whereby the satellite's orbit plane is under an angle with the Earth's equator. Obliviously, none of the accelerations is zero, thus, the satellite will undergo cyclic accelerations due to its orbital speed in the neighbourhood of the spinning Earth.
When analyzing the tangential acceleration for the whole orbit, which I limited to the case for $\alpha = 0$ , it is found that the satellites under an orbit inclination of $i$ = 0, π/4 and 3π/4 are in rest; between $i$ = 0 and π/4 (prograde), and for $i$ = 3π/4 and 2π (retrograde), the acceleration induce a rotational drift towards the rotational axis of the Earth. Between $i$ = π/4 and π/2 (prograde), and $i$ = π/2 and 3π/4 (retrograde), the acceleration results in rotational swivelling towards the equatorial axis of the Earth, and retrograde orbits are pushed back into prograde orbits.
The general conclusion is that in average, satellite orbits that go over the poles, or nearby them, will undergo a strong orbit swivelling towards the Earth's equator, while satellite orbits that go over the Earth's equator, or in an orbit inclination of π/4, will undergo no orbit change.

## 6. References and bibliography.

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