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Gravitational model of the three elements theory

The study of this article suggests an explanation for today gravitational mysteries. This explanation is based on a modification of Newton’s law. This modification is conducted from an Euclidean vision of relativity. This modification is thoroughly explained in [12].
The following issues are addressed: Pioneer anomaly after and before the location of Saturn, Earth flyby anomalies, a predicted Saturn flyby anomaly of Pioneer11 probe trajectory, the “missing asteroids” mystery in solar system, the advance or precession of the perihelion of Mercury and Saturn, PPN formalism, Tully-Ficher relation, sideral gravity and the disparity of the measurements of the gravitational constant.

1. INTRODUCTION

The aim of this article is to study some gravitational mysteries with the help of the gravitational model of the three elements theory. This gravitational model is described in [12].
The first analysed mystery is the Pioneer anomaly. For this mystery, an explanation is given which yields the theoretical value of $7.25\ 10^{-10}\ m/s^2$ in place of the measured value $8.74\ 10^{-10}\ m/s^2$, as the added acceleration value toward the sun. But this modification doesn’t give the exact curve for the measured anomaly. In order to retrieve the exact curve, the “second modification of Newton’s law” (explained in [12]), must be used. The second modification comes from the presence of the Kuiper belt. Its mass, when taken into account, yields the exact measured curve of the Pioneer anomaly.

Here is the set of mysteries which are addressed by this article:

- The Pioneer anomaly (Saturn and beyond).
- The Pioneer anomaly before Saturn.
- Earth flyby anomalies.
- The Saturn flyby by Pioneer 11.
- The Perihelion advance or precession of Mercury and Saturn.
- Sideral gravity.
- The disparity of the gravitational constant measurements.
- PPN formalism.
- Tully-Ficher relation.
- Solving the sign issue for the galaxies speed profiles.

The reader is strongly invited to read [12] before reading this document. Otherwise, the next chapter will remind shortly the content of [12]. The conscienscious reader is advised to read at first chapter

<<12. GENERAL RELATIVITY EQUATIONS>>. This chapter will give a mathematical understanding of what is the “first correction of Newton’s law”, and the “second correction of Newton’s law”, terms which are oftenly used in this document.

There exists also a short version of this document (document [14], 5 pages). Chapters 2 and 3 are studying the Pioneer anomaly. The following chapters (chapters 5, 6, 7, and 8) are studying the space-time deformation contributions and their mode of propagation. They can be skiped for a first lecture. The other mysteries are studied starting from chapter 9.

2. THE GRAVITATIONAL MODEL (REMINDER)

This model is thoroughly explained in [12]. Let's summarize it. It is based on the inner mechanisms of restricted and general relativity. Each of the Lorentz transformation details, space-time deformation by energy, following geodesics, and Einstein equivalence principles has been used in order to construct this gravitational model.
Moreover, the attempt is to explain Lorentz transformations with the help of those general relativity principles. During this attempt, some inconsistencies are found. The resolution of these errors lead to stating that matter is composed of indivisible particles, called "luminous points". Those "points" are always travelling in space at the speed of light. They generates around them a radical space-time deformation. These deformations are propagated at the speed of light in every space directions, and are combined together with the help of a non-associative and non-linear operator, called the "relativistic operator". This operator is only applied once, at any point of space and at any time. Therefore, the shape of space-time at any space-time point is determined by the propagated space-time deformations coming from the luminous points in the universe, along the relativistic cone centered on this point. Therefore, local gravitation law depends strongly on energy distribution among the universe.
By construction, this mechanism retrieves Lorentz transformation details. But moreover, it allows to retrieve Newton's law for long distances and for a constant and homogeneous distribution of energy in the universe. The key point is that this Newton's law is only retrieved for long distances, and for this special energy distribution. For short and intermediate distances, Newton's law is no longer retrieved. That's what is called "first modification of Newton's law". And for a non constant distribution of energy, Newton's law is even more modified. This one is called the "second modification of Newton's law".

3. USING THE FIRST MODIFICATION OF NEWTON'S LAW

The "first modification of Newton's law" which is described in [12] uses a fitting of Newton's law for long distances.

In fact, this fitting must not be done for long distances. It must be done for a distance from the sun where the heliocentric gravitational constant is known to be perfect. This value is around the sun to Saturn distance. This can be seen on the measured curve of the Pioneer anomaly. The figure 1 below shows this curve, which is extracted from [1].

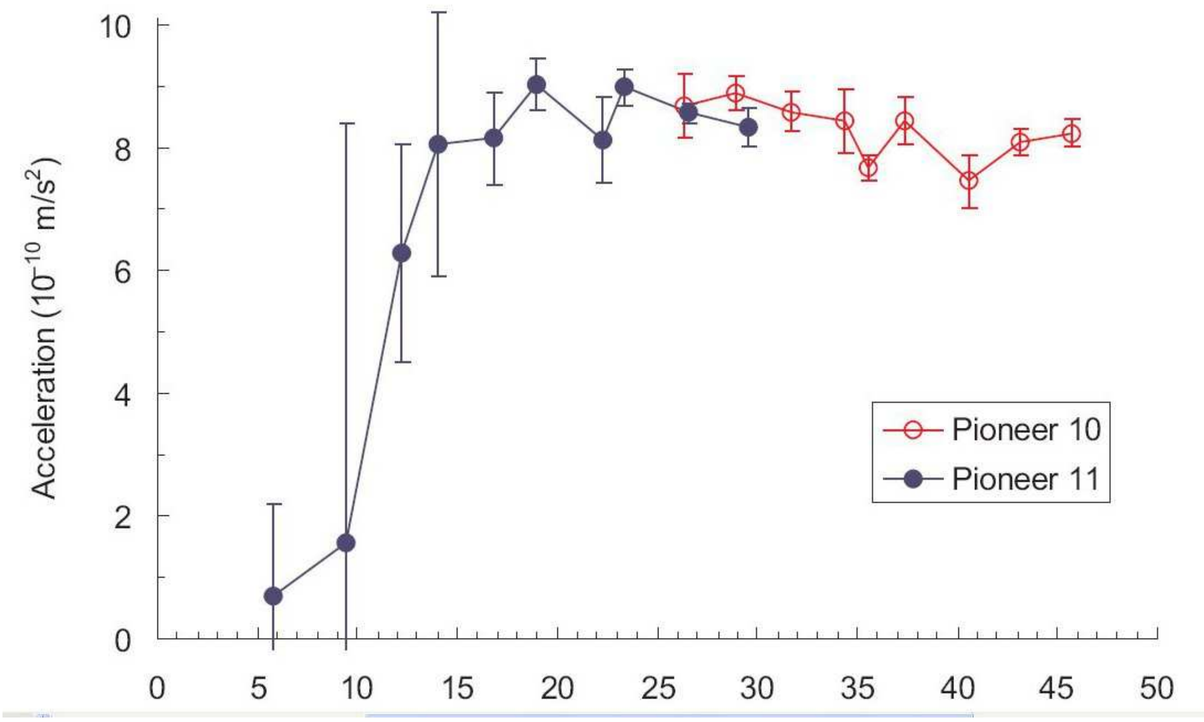


Figure 1: Measured curve of the Pioneer anomaly. This figure has been extracted from [1]. *X*-coordinate is in *AU*.

On this figure, we can see that the anomaly almost completely vanishes around 10 Astronomical Unit from the sun. This value is the distance of Saturn from the sun ($x_s = 9.55\ AU$).
Therefore, it is for this exact distance x_s from the sun that the correction of Newton's law has no effects.
Hence, the following calculations consists simply in stating that the corrected Newton's law is equal to Newton's law for this x_s distance.

$$a \simeq -\frac{G'M}{x_s^2}\left(1-3\sqrt{\frac{2R'}{x_s}}\right) \simeq -\frac{GM}{x_s^2} \quad \textbf{(1)}$$

a is the real acceleration toward the sun: $a = \frac{F}{m}$

m is the mass of the Pioneer probe.
M is the mass of the sun.

F is the sun attracting force.
G is the gravitational constant (therefore, inside the solar system).
G' is the gravitational constant value outside of the solar system, valid for long distances.
R' is equal to <<G'M/c²>>.
x_s is the sun to Saturn distance. We will use: $x_s = 1429\ 10^9\ m \simeq 9.55\ AU$.

The approximated equation (1) has been calculated in [12]. Now the heliocentric gravitational constant, G_h , will be used, and its "outside of solar system value", G_h': $G_h = GM$ and $G_h' = G'M$. Hence we get $R' = G'M / c^2 = G_h' / c^2$ and:

$$a \simeq -\frac{G_h'}{x_s^2}\left(1-\frac{3}{c}\sqrt{\frac{2G_h'}{x_s}}\right) \simeq -\frac{G_h}{x_s^2} \qquad \textbf{(2)}$$

Let us write: $y = \sqrt{G_h'}$. Then (2) becomes:

$$\frac{3}{c}\sqrt{\frac{2}{x_s}}y^3 - y^2 + G_h \simeq 0 \qquad \textbf{(3)}$$

This equation is resolved in appendix 1. It yields the value of *y*, then G_h'. This G_h' value is close to G_h value. The next step is to calculate an approximation value for the Pioneer anomaly, Δ*a*. We have simply:

$$\Delta a \simeq \frac{G_h'}{x^2}\left(1-\frac{3}{c}\sqrt{\frac{2G_h'}{x}}\right) - \frac{G_h}{x^2}$$

Now for each possible value of x. It has been written that this anomaly is equal to the approximated value of the corrected Newton's law, minus the classical Newton's law. After a little calculation, this yields:

$$\boxed{\Delta a \simeq \frac{A-\frac{B}{\sqrt{x}}}{x^2}} \qquad \textbf{(4)}$$

With $A = G_h' - G_h$ and $B = 3\frac{G_h'}{c}\sqrt{2G_h'}$. The curve of Δ*a* has a "bell shape". Its maximum $\Delta a_{\max}$ can be calculated in the following way.

$$\frac{d(\Delta a)}{dx} = \left(\frac{5B}{2} - 2A\sqrt{x}\right)x^{-\frac{7}{2}} = 0$$ After calculation. Hence:

$$\frac{5B}{2} - 2A\sqrt{x} = 0 \quad \text{and} \quad x_l = \frac{25B^2}{16A^2}$$

x_l is the *x*-coordinate of the maximum. The maximum of Δ*a* is then $\Delta a_{\max} = \left(A - \frac{B}{\sqrt{x_l}}\right) / x_l^2$

Numerical application:

$$G_h = 0.13271243\ 10^{21} m/s^2$$

$$G_h' = 0.13273052\ 10^{21} m/s^2$$

$$A = 0.1810\ 10^{17} m^3/s^2$$

(G_h' is calculated in appendix 1).

$$B = 0.2163\ 10^{23} m^{\frac{7}{2}}/s^2$$

$$x_l = 0.2229\ 10^{13} m$$

and finally

$$\boxed{\Delta a_{\max} = 7.25\ 10^{-10} m/s^2} \qquad \textbf{(5)}$$

which is very close to the Pioneer anomaly value : $8.74\ 10^{-10} m/s^2$.
This is 17% of precision.

This theoretical value is very close to the measured one, and it has been calculated without any fitting.

Now let's plot the whole curve.

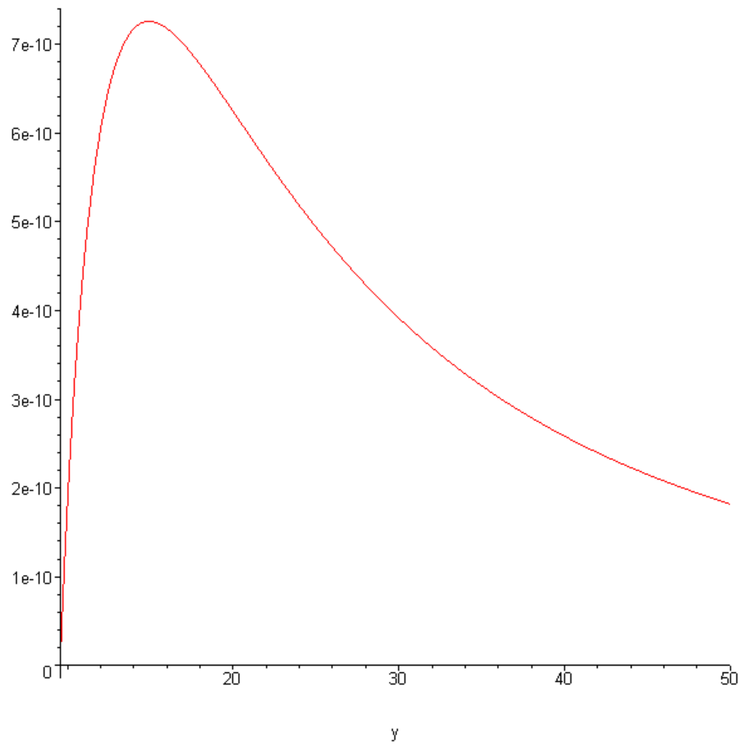


Figure 2: First theoretical curve of the Pioneer anomaly. *X*-coordinate is in *AU*, and *y*-coordinate is in *m/s²*.

This curve has been plotted with the MAPLE program in appendix 2.
The curve is very good for *x*-coordinate between 10 and 20 *AU*, which are roughly the distances of the locations between Saturn and Uranus. But for *x* greater than 20 *AU*, Uranus location, the curve is decreasing too much, as compared with the measured curve (Figure 1).
That's because we didn't took into account the existence of the Kuiper's belt. This is a belt of asteroids which are located beyond Neptune, on the ecliptic plan.

But as it has been shown in [12], chapter “Star speed mystery”, the presence of such a varying mass distribution has an important impact on the correction of Newton’s law.
Hence, it is important to take into account the mass distribution of the Kuiper’s belt in the calculation on the correction of Newton’s law. This will be done in the next chapter.

4. TAKING INTO ACCOUNT THE KUIPER'S BELT

The next chapter will explain why the Kuiper's belt must be taken into account as symmetric contributions in the calculation of the relativistic operator, and why the planets of the solar system must not.
But let's see immediately the consequences of this statement.

The first step is approximating that the effect of the Kuiper's belt is mainly coming from the addition of symmetric contributions to the relativistic operator. This approximation is explained in appendix 4.
The second step is to modeling simulate the space-time deformation symmetric contributions coming from the Kuiper's belt, in order to see if this retrieves the measured curve of the Pioneer anomaly.
The following modeling of L_k, the Kuiper's belt symmetric contribution, will be used.
In fact only $L_k / \left(L_u + L_g\right)$ is needed, since $L_u + L_g$, the galactic and extra-galactic contributions value, will be simplified in the calculation of the relativistic operator.

<u>*Warning*</u> *: from now on, in this Kuiper study, we will note L_g in place of $L_u + L_g$. A next version of this document will correct this. But this false notation doesn't change the final result. Therefore,* $L_k / \left(L_u + L_g\right)$ will be written $L_k onL_g$.

$$L_k onL_g \quad = \quad L_k onL_{g\max} \frac{2}{\pi} \arctan\left(\frac{2}{\pi}\frac{x_{Kuiper}}{x}\right) \qquad \textbf{(5')}$$

$L_k onL_g$ is the value of the L_k / L_g quotient.
$L_k onL_{g\max}$ is the value of $L_k onL_g$ when located on the sun, which is the maximal possible value. The following value will be used:

$$L_k onL_{g\max} = 0.0015 \qquad \textbf{(5'')}$$

x_{Kuiper} is not the maximum distance of the Kuiper's belt to the sun (48 AU), but $x_{Kuiper} = 114\ AU$.

Of course $L_k onL_{g\max}$ and x_{Kuiper} values has been fitted in order to retrieve the best possible curve.
Let us see what is the curve of $L_k onL_g$.

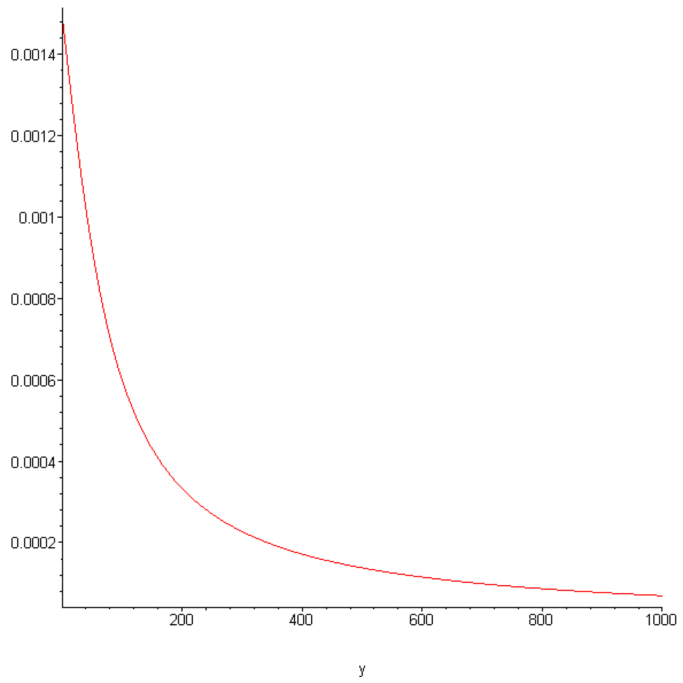


Figure 3: Modeling of the symmetric contribution coming from the Kuiper's belt.

The MAPLE program having plotted this curve is the following one.

```
AU := 1.4959787 * 10**11:
xKuiper := 114*AU:
Lk_on_Lg_max := 0.0015:
Lk_on_Lg := Lk_on_Lg_max * invfunc[tan](( 2/Pi) * xKuiper / x) / (Pi/2):
x := AU * y:
plot(Lk_on_Lg, y=1..1000 );
```

We can check that the value of $L_k onL_g$ is equal to $L_k onL_{g\,\max}$ for $x=0$, and that $L_k onL_g$ is decreasing.

Now let us see immediately the theoretical curve for the Pioneer anomaly, when inserting this space-time deformation symmetric contribution in the calculation of the relativistic operator.

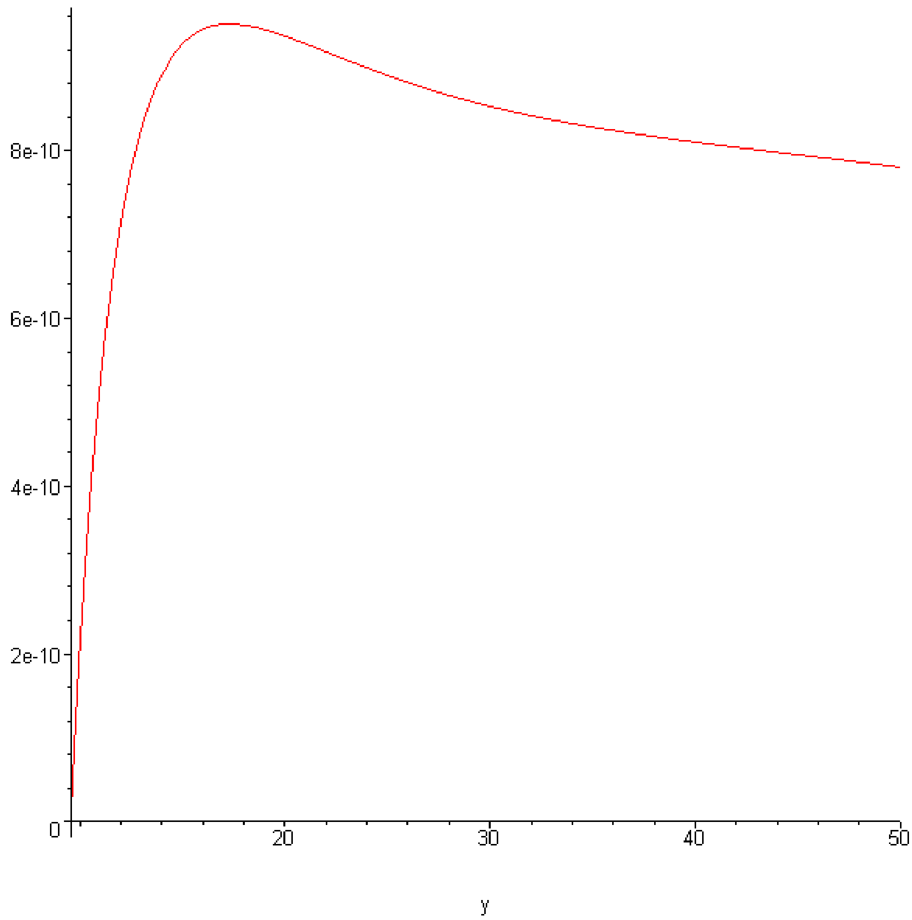


Figure 4: Second theoretical curve of the Pioneer anomaly, taking into account the Kuiper's belt with a modeling of its symmetric contributions on the relativistic operator. *X*-coordinate is in *AU*, and *y*-coordinate is in *m/s²*.

Now this theoretical curve is perfect. It is perfectly fitting the measured curve of the figure 1. Noticeably, the maximum value is $8.7\ 10^{-10} m/s^2$. It is very close to the measured value of $8.74\ 10^{-10} m/s^2$ (0,5% of precision). But, at the same time, the shape of the curve is exactly the same.

The MAPLE program which plotted this curve is in the appendix 3. In this program, the $L_k onL_g$ value of equation (5') has been used in such a way.

$$L_1' = 1 + L_k onL_g + \left(1 + L_{k0} onL_g\right)\sqrt{\frac{8R'}{x}}$$

$$L_2' = 1 + L_k onL_g$$

$$\cos(\alpha) = oper(L_1', L_2')$$ This is the relativistic operator.

α is the slope angle of the local space line.

$L_{k0} onL_g$ is the value of $L_k onL_g$ for $x = x_s$, the location of Saturn.

The equations of L_1' and L_2' above have been obtained by dividing by L_g the following equations:

$$L_1 = L_g + L_k + \left(L_g + L_{k0}\right)\sqrt{\frac{8R'}{x}}$$

$$L_2 = L_g + L_k$$

Of course, we have $L_1' = L_1 / L_g$ and $L_2' = L_2 / L_g$. Those last values are calculated in the following way.

- The symmetric contribution is equal to $L_2 = L_g + L_k$.
- The asymmetric contribution is equal to $\left(L_g + L_{k0}\right)\sqrt{\frac{8R'}{x}}$.

Indeed, the symmetric contribution, locally to Saturn, is equal to $L_g + L_{k0}$, and only the *local* symmetric contribution must be taken into account when calculating the relativistic operator. This has been showed in [12], in the study of the second correction of Newton's law. It is for this location of Saturn, and because $R = MG / c^2$, that the gravitational force is equal to the Newton's equation value, using the G constant.

Of course, the $L_k onL_{g\max}$ and x_{Kuiper} parameters has been fitted in order to obtain this perfect curve.
Now we have to show that this $L_k onL_g$ contribution modeling is very close to the real one.
For this, we must first understand why the Kuiper's belt must be taken into account in the calculation of the relativistic operator. This will be done in the next chapter.
After that, we will actually calculates the real symmetrical contribution coming from the Kuiper's belt. This will be followed by the calculations of the L_k / L_g quotient.
At the end, we will try to analyse the slight differences between this real contribution and the modeled one above, for the Kuiper's belt.

5. WHICH OPERATORS BETWEEN THE CONTRIBUTIONS

This chapter will try to explain why the Kuiper's belt space-time deformations must be taken into account in the calculation of the relativistic operator.

A first important remark must be done at first, about the first modification of Newton's law, which has been used in chapter 2. In this correction, we assume that the distribution of matter inside the solar system is very simple: there is the sun, and the whole space outside of the sun is filled with a constant and homogeneous distribution of matter.
Of course, this is not true. This matter density is not constant. A better approximation consists in supposing that this matter density is only depending of the distance from the sun. This better modelling leads to what was called the "second correction of Newton's law in [12]. In this document, the studied case was the case of the galaxy, not the case of the solar system. But the physical mechanism is exactly the same.
In the solar system, there are many objects. First of all, there are the planets. We will see below that the planets must *not* be taken into account as symmetric contributions in the calculation of the relativistic operator. There are also, the Kuiper's belt, and even some more objects which are located beyond the Kuiper's belt: the scattered disk, Eris, etc…

Now let's analyse the case of the planets in the solar system.
The planets of the solar system are not taken into account as symmetric contributions in the calculation of the relativistic operator. That's because they are almost never aligned.

For example, if Saturn, Neptune, and the Pioneer11 probe are aligned on the same straight line, then the space-time deformations propagated from those two planets must be taken into account as symmetric contributions when calculating the relativistic operator. This yields a final attracting force which is different from the sum of the two classical corresponding Newton's forces.
This case is nothing more than the case of an eclipse. At this time, the total gravitational force is difficult to measure, because this would be done by measuring the Pioneer11's trajectory, but this is not possible because of the extremely short instant of time of this eclipse.

But, on the contrary, if the two planets are not aligned with Pioneer11 on the same straight line, then we must calculate 2 values for the relativistic operator. We must calculate, one relativistic operator value for the Saturn to Pioneer11 straight line, and another one for the Neptune to Pioneer11 line. Those 2 values yields 2 different gravitational forces, in the respective directions of those planets. Thereafter, those two force vectors are summed together to form the total gravitational force vector resulting from the attractions of those planets. This last case is far enough the most frequent case. It is calculated with the classical additional rule of force vectors.

On the contrary, with the Kuiper's belt, it is quite usual that a Kuiper's belt asteroid is just on the line between Pioneer probe and the sun. Therefore, the Kuiper's belt must be taken into account as symmetric contributions in the calculation of the relativistic operator.

6. CALCULATION OF G

Before studying the attenuation rule and the calculation of the symmetrical contributions (following chapters), the calculation of *G* must be reminded.
The equation of the *G* gravitational constant has been calculated in [12], appendix 4. Let's calculate it in a slightly different way. The equation of the asymmetric contribution L_p coming from a luminous point can be written:

$L_p = L_u \sqrt{\frac{8R_p}{x}} = 2\sqrt{2}\,\frac{L_u}{c}\sqrt{\frac{m_p G}{x}} = 2\sqrt{2}\,\frac{L_u}{c^2}\sqrt{\frac{e_p G}{x}}$ With $R_p = \frac{m_p G}{c^2}$, and $e_p = m_p c^2$ being the energy of a luminous point. Now the symmetric contribution L_u is a combination of L_p contributions along some fixed width solid angle:

$$L_u = \sum_p L_p$$

This sum is to be done along a fixed width solid angle.

The width of this solid angle is still to be determinated. Now combining the two equations above, we get:

$$L_u = L_u \sum_p \sqrt{\frac{8R_p}{x}}$$

It results the value of *G* as a function of the L_p contributions calculated along this fixed width solid angle.

$$\sum_p \sqrt{\frac{8R_p}{x}} = 1, \qquad \sum_p \frac{2\sqrt{2}}{c^2}\sqrt{\frac{e_p G}{x}} = 1,$$

and finally

$$G = \frac{c^4}{\left(\sum_p \sqrt{\frac{8e_p}{x}}\right)^2}$$

This is the same result as the one calculated in appendix 4 of [12].

Now the sum $\sum_p \sqrt{\frac{8e_p}{x}}$ located on the denominator of this equation of *G* is answering the following questions.

1) what is the width of this solid angle ?
2) If this width is not small, for example, if it is equal to 2π, in which case the solid angle is covering the half celestial sphere, an issue of coherence arises. It must be ensured that it can be calculated the attracting forces along each space direction, giving the same result. What is this exact formula ?

3) This sum must converge. Hence the maximum distance (the length of the cone) is finite. What is this length ?
4) If this distance, the width of this solid angle, and the "width of the luminous point" (width on which the luminous point deforms space-time and this deformation is propagating to us) leads to the possibility to encounter only partially a luminous point with this solid angle, then the only way to be coherent with the vectorial linearity of gravitational forces, is to postulates that the contributions are combined together with a square root of the sum of the squares law (equation (8)).
5) But if this is not the case, then the same question arises again because it must be unsured that two luminous points, closed to each other, giving separated space-time deformations, will gives a final global force which is the same, either calculated with only one relativistic operator calculated for the two contributions, leading to only one force, or calculated with two relativistic operator, one for each luminous point, and thereafter calculating the global force vector as being the sum of the two vectorial forces.

The only way to answer to question 4) and 5) is to use equation (8) for the the combinations of the asymmetric contributions. This equation (8) must be applied in order to get the final asymmetric contribution which is used later on in the calculation of the relativistic operator.
In this way, it is easy to answer to questions 1) and 2), postulating simply that this width is "small". Here "small" means that there is no need to calculate any incidence angle or whatever could be needed if this width wasn't small enough. Of course this is a practical postulate. But it appears tricky to solve the question 2). Moreover, even if the solution to question 2) could be done, it would be probably a complicated rule (in order to get rid of the non linearity of the relativistic operator). Hence it would be surprising that Newton's law linearity could be explained by a complicated rule. This is for the moment enough for answering those questions 1) 2) 4) and 5).

As a consequence of this, now using equation (8) in place of a simple addition of the contributions like above, we get a different equation for G:

$$\boxed{G = \frac{c^4}{8\sum_p \frac{e_p}{x}}}$$

Let's remark that this equation (8) will also be argued later on with the conservation of the space-time vacuum conservation principle.

For answering question 3), the present document proves the existence of a maximum r_{glast} value inside the galaxy, which results from an occultation mechanism inside the galaxy. It could exist the same mechanism outside the galaxy, for extra-galactic contributions. It could exists an r_{elast} value, which represents the distance beyond which an extragalactic luminous point contribution cannot be noticed. (Let's remind that "contribution" means "space-time deformation propagation contribution"). Let's remark that the answer to question 3) can be given also, differently, simply stating that the universe age is finite.
Anyhow, a way to evaluate r_{elast} will be to compare the equations of the model with experimental data coming from sideral gravity and the disparity of the measurements of G.

7. ATTENUATION RULE FOR THE SPACE-TIME DEFORMATIONS

This is explained in [15]. Let's remind it shortly.

The values which are operating in the relativistic operator are named L_1 and L_2 in [15].
They represents the height, along time axis, of the space-time vacuum which is propagated with the space-time deformation. In fact L_1 and L_2 are the resulting values for the final height of vacuum, after taking into account every space-time propagated vacuum from the corresponding luminous points. Those luminous points are located on the relativistic cone centered on the point where we want to calculate the relativistic operator. Those propagations are coming from the left for L_1 value, (luminous points located on the left) and from the right for L_2 value (luminous points located on the right).
Now in this document, those height of vacuum values will be named simply "space-time deformations", "space-time deformation values", "space-time contributions", or simply "contributions".

The space-time deformations are propagating in space with a $1/\sqrt{r}$ rule. (r is the distance from the emitting point in space and the point in space where we want to calculate the relativistic operator). That's for 2 reasons:

- this rule is retrieving roughly Newton's law for long distances,
- it is justified from a purely theoretical point of view.

This last reason is a geometrical argument. The theoretical assumption which justifies this $1/\sqrt{r}$ rule is the conservation of vacuum during space-time deformation. This vacuum is the well known vacuum created in space-time by a Lorentz transform. It is quantified by the position of the O' point along the time axis in a Lorentz transform. This point has been detailed in [12]. This vacuum is also the key concept which allows relativity to explain the asymmetry of the twin roles in the twin paradox.
As it is described in [15], because of this assumption, the calculation is now the following one.
Let's remind it. We have to study a luminous point .

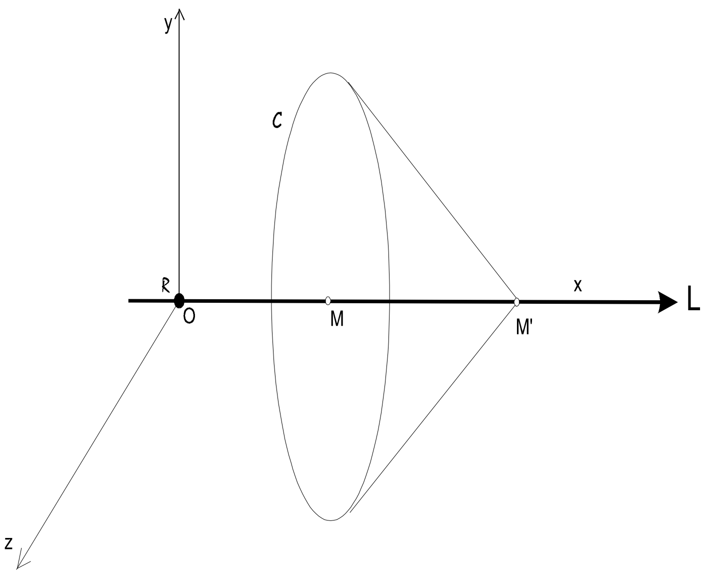


Figure 5: Location in space, of the propagated space-time deformation, generated by a straight line luminous point trajectory.

We study a luminous point, whom trajectory is the Ox axis. The luminous point is travelling along the Ox axis, x increasing.
Let's note, at $t = t_0$, the time at which the luminous point is located on Ox axis on x_0 coordinate. At $t = t_0 + dt$, the luminous point is located at $x_0 + dx$ on Ox axis.

Let's analyse the propagation, in space, of the space-time deformation generated by this luminous point trajectory segment [x_0, $x_0 + dx$].
This space-time deformation generated between t_0 and $t_0 + dt$ instants along Ox axis, is propagating with the c speed in space. This propagation is done in the OyOz plan, which is parallel to the Ox trajectory of the luminous point.

Thereafter, at $t > t_0$, this deformation has been propagated along a circle with a ray equal to $r = c\ (t - t_0)$. This circle is located on the plan which is parallel to the OyOz plan, and which intersect the Ox axis at x_0 coordinate.

Now let's note $c\ dt$ the height of the vacuum which has been propagated at the t instant. (With any space-time deformation propagation, there is also the propagation of a space-time vacuum. This is described in [15]).
Let's note *dr* the length of this vacuum along each circle ray.
This space-time vacuum is located along the circle, and has a thickness equal to $c\ dt = dr$ along each ray of this circle. This is also a direct application of the postulate 7: the vacuum generated by a luminous point always gets an isosceles triangle shape.

The space-time propagated vacuum volume is proportional to $c\ dt\ dr\ dl\ L.$ $(L = 2\pi r)$.
That's because this is the volume of a 4D torus, with a cubic section:

- $c\ dt$ is the height of this torus along time axis.
- dr is the length of the torus along each ray of the circle above.

- $dl = \sqrt{2}\, dx$ is the length of the torus along the envelope of the space-time propagations coming from the luminous point trajectory (the cone of the preceding figure).

Hence we get :

$dV = k\, c\, dt\, dr\, dl\, L$ k, is a constant. Hence we get, since dr = cdt:

$dV = k\left(c\, dt\right)^2 \sqrt{2}\, dx\, 2\pi r$ **(5''')**

$dV = k'\left(c\, dt\right)^2 r$ k' is a constant.

dV stays constant during the propagation (postulate 7, see [15]). Let's call dV_0 this constant value. We get:

$$c\, dt = \sqrt{\frac{dV_0}{k' r}}$$

$$c\, dt = \sqrt{\frac{k''}{r}}$$ k'' is a constant.

The 1/√r law is retrieved.

<u>Remark</u> *: in fact, the propagation schema is more complicated than the one depicted by Figure 5. The envelope of the circles is a not a regular but a deformed cone because the fundamental trajectory of the luminous point is not a straight line but a circle. The center of this circle is the gravity center of the attracting mass which has to be studied. The result of this modification is that the Newton law must be identified directly* $-\frac{mMG}{x^2} = mc^2 \frac{d \tan(\alpha)}{dx} \frac{\tan(\alpha)}{\left(1-\tan^2(\alpha)\right)^{3/2}}$.

Otherwise the identification should be $-k\frac{mMG}{x} = mc^2 \frac{d \tan(\alpha)}{dx} \frac{\tan(\alpha)}{\left(1-\tan^2(\alpha)\right)^{3/2}}$*, k being a constant. Indeed, with the Figure 5 description, the C circle encounter fewer and fewer space points as its ray is increasing. In other words, there is no complete space diffusion of the propagation as time goes by. Hence the Newton's law 1/x² must be multiplied by k x before identification with the model.*

Therefore there is an issue when trying to explained this 1/√x law with the space-time vacuum conservation. Indeed, now we have $dl = \alpha \sqrt{2}\, x\, dx$ *(*α *being a constant angle value) in place of* $dl = \sqrt{2}\, dx$*. It has to be postulated or explained why this diffusion along the Ox axis doesn't decrease the c dt contribution value when propagating. This work is in progress.*

8. CALCULATION OF THE SYMMETRICAL CONTRIBUTIONS

This issue is a general one. It is one of the greatest issues of the dark matter explanation. In the dark matter explanation, this issue has been discarded by an indirect calculation. But here with Pioneer anomaly problem, we cannot avoid the real calculation of those symmetric contributions.

The aim is to find the operator which combines two asymmetric contributions. A first remark is that the additive operator doesn't retrieve the additivity of the gravitational forces. (A crossed term is added….). In order to retrieve the addition of gravitational vector forces the "square root of the sum of the squares" operator must be used for combining the contributions.
Indeed, let's take the simple case of two parallel gravitational forces, coming from two distinct attractive objects, whom masses are M_1 and M_2, and let's write $R_1 = M_1 G / c^2 , R_2 = M_2 G / c^2$.
The relativistic term $\left(1-\tan^2(\alpha)\right)^{-3/2}$ will be neglected for each gravitational force:

$$F = mc^2 \frac{d \tan(\alpha)}{dx} \frac{\tan(\alpha)}{\left(1-\tan^2(\alpha)\right)^{3/2}} \simeq mc^2 \tan(\alpha) \frac{d \tan(\alpha)}{dx} = \frac{mc^2}{4} e \frac{de}{dx}$$

e being the ratio of asymmetric over symmetric contribution as usual: $e = \frac{L_{asym}}{L_{sym}}$

Using the postulated operator we get $e = \sqrt{e_1^2 + e_2^2}$. Hence:

$$F = \frac{mc^2}{4} \sqrt{e_1^2 + e_2^2} \frac{d\sqrt{e_1^2 + e_2^2}}{dx}$$

With, as usual: $e_1 = \sqrt{\frac{8R_1}{x_1 - x}}$ and $e_2 = \sqrt{\frac{8R_2}{\left|x - x_2\right|}}$

This yields immediately:

$$F = \frac{mc^2 R_1}{\left(x - x_1\right)^2} + \varepsilon \frac{mc^2 R_2}{\left(x - x_2\right)^2}$$

With ε being 1 or -1 depending of the location (respectively right or left) of the second object, supposing the first object located on the right. Finally:

$$F = F_1 + F_2$$

Which is the addition of the gravitational forces.

In the chapter studying the measurements of *G*, the analysis of sideral gravity tends to show that the operator combining a star contribution and the galaxy-and-extragalactic contribution, is an <u>addition</u>. Otherwise, using the "square root of the sum of the squares" law, the experimental sideral gravity ratio of 0.054% is not retrieved. (In this sideral gravity analysis, it is supposed that extragalactic contribution is equal (or same order) to galaxy contribution. This is a result from the study of the velocities of the galaxies inside their groups).
This addition of the contributions predicts a violation of gravitational forces additivity at galaxy scales. It gives the idea of an experimentation for checking this violation at earth scale (refers to appendix 16). This is a crucial information since in this present document, the addition is always used for combination of contributions. (Each calculations should be re-executed using the "square root of the sum of the squares" law; this will be done hopefully in a next version of this document).

Therefore, the next step is to check wether or not this "square root of the sum of the squares" law might be explained by the gravitational model of the three elements theory. Of course the used principle will be the conservation of space-time vacuum.

Before entering into the real issue, we must understand how the space-time deformations are propagated and combined between themselves.
Once again, the "conservation of space-time vacuum" principle is dictating the combination of those contributions.
Let's take the example of two luminous points. We want to calculate the intensity of the resulting space-time deformation generated by those points on some location in space.
As described in [12], the intensity of the space-time propagated deformation is in fact the height of space-time vacuum which is propagated with the space-time deformation. All we have to do is quantifying this height of vacuum, assuming that its volume stays constant.

We have, on some M point in space:

$$dV_1 = 2\pi\sqrt{2}\, k\, r_1\, dx_1 \left(c\, dt_1\right)^2 \qquad \text{for the first luminous point,}$$

$$dV_2 = 2\pi\sqrt{2}\, k\, r_2\, dx_2 \left(c\, dt_2\right)^2 \qquad \text{for the second luminous point.}$$

This is equation (5'''), applied for each luminous point. The lengths of the trajectory segments are supposed equal to *dx* for the two cases ($dx = dx_1 + dx_2$). Indeed, we considerer only one time interval on M point for calculating the relativistic operator.
The space-time vacuum conservation law is dictating the calculation rule. Hence, we have, on some M point in space:

$$dV = dV_1 + dV_2 \qquad \text{Total space-time vacuum.}$$

$$= 2\pi\sqrt{2}\, k\, r_1\, dx_1 \left(c\, dt_1\right)^2 + 2\pi\sqrt{2}\, k\, r_2\, dx_2 \left(c\, dt_2\right)^2$$

Let's now assume that the two luminous point trajectories are close together and parallel, and that the M point is quite far away from those trajectories.
Hence we have: $r \simeq r1 \simeq r2$, and:

$$dV \simeq 2\pi\sqrt{2}\, k\, r\, dx \left[\left(c\, dt_1\right)^2 + \left(c\, dt_2\right)^2\right] \qquad \textbf{(6)}$$

Now, since the two trajectories are close to each other and parallel, we have also:

$$dV = 2\pi\sqrt{2}\, k\, r\, dx \left(c\, dt\right)^2 \qquad \textbf{(7)}$$

Where $r = r_1 = r_2$, and $c\, dt$ is the resulting height of the resulting space-time propagation on M point.
Combining (6) and (7), we get:

$$2\pi\sqrt{2}\, k\, r\, dx \left(c\, dt\right)^2 = 2\pi\sqrt{2}\, k\, r\, dx \left[\left(c\, dt_1\right)^2 + \left(c\, dt_2\right)^2\right]$$

$$\boxed{c\, dt = \sqrt{\left(c\, dt_1\right)^2 + \left(c\, dt_2\right)^2}} \qquad \textbf{(8)}$$

Now we would have to generalize this calculation above without assuming that the luminous point trajectories are parallel, and the M point far away. But this will not be done.
This general rule will be used for now on, for calculating the final space-time height of vacuum generated by two luminous points.

Now, we are ready for the calculation of the contributions of many luminous points.
We have to estimate the following values.

- L_k, the space-time deformation contribution coming from the Kuiper's belt. More precisely, it is the part or the symmetric contributions coming on the M point. The M point is the location in space where we want to calculate the relativistic operator (the location of the Pioneer probe).

- L_g, the space-time deformation contribution coming from the galaxy (outside of the solar system).

Let's take the example of a sphere filled with a constant homogeneous repartition of matter.
It must be assumed, as usual, that the universe is not empty (…). Otherwise, the relativistic operator vanishes and the physical result is stupid: $\alpha = \pi/2$. That is the physical principle of the three element theory: the space-time shape at any point is always dictated by the whole universe repartition of matter. The usual linear macroscopic aspect of space-time is due to the isotropy and the homogeneity of the farthest region of the universe.
Therefore, we will assume that, out of the sphere, the space is filled with a uniform repartition of matter.

ρ is the matter density inside the sphere.
R is the ray of the sphere.
O point is the middle of the sphere.

We want to calculate *L*, the space-time resulting deformation height on O point. Only the space-time deformations coming from the sphere are calculated in *L*. The contributions coming from out of the sphere are not taken into account. We have:

$$L^2 = \sum_p \left(\sqrt{\frac{8R_p}{x_p}} \right)^2 \qquad \textbf{(9)}$$

We have applied the equation (8) rule: the square of the final contribution is equal to the sum of the squares of the initial contributions.

$$L^2 = \sum_p \frac{8R_p}{x_p}$$

where the sum is done along a small solid angle from the O point and for each p luminous point in this solid angle.

x_p is the distance of the generic luminous point from the O point, $R_p = m_p G / c^2$, with $m_p = e_p / c^2$ and e_p is the luminous point energy.

$$L^2 = \sum_{x_p} n_p \frac{8R_p}{x_p} \qquad \textbf{(9')}$$

n_p is the number of luminous points which are located in the solid angle, at the x_p distance from the O point. We have applied the equation (8) rule: this number n_p is located inside the square root of the (9) equation, not outside. Let's calculate the value of n_p. We have:

$$n_p = \frac{L^2}{d^2}$$ *L* is the width of the *α* solid angle cone at x_p location.

$$= \frac{\alpha^2 x_p^2}{d^2}$$ Because *α ≅ tan(α) = L/xp* (*α* is small).

Hence, equation (9') becomes:

$$L^2 = \sum_{x_p} \frac{\alpha^2 x_p^2}{d^2} \frac{8M_p G}{c^2 x_p}$$

$$L^2 = \frac{8\alpha^2 e_p G}{d^2 c^4} \sum_{x_p} x_p \qquad \textbf{(10)}$$

By other mathematical classical means, we have :

$$\sum_{x_p=0}^{R} x_p \simeq \frac{1}{d} \int_0^R x\,dx$$ This approximation is accurate here.

$$= \frac{R^2}{2d}$$

Hence, (10) becomes:

$$L = \frac{2\alpha R \sqrt{e_p G}}{c^2 d^{\frac{3}{2}}} \qquad \textbf{(10')}$$

Now will be used also: $\rho = m_p / d^3$, where ρ is the matter density inside the sphere. Hence:

$$d = \left(\frac{e_p}{c^2 \rho}\right)^{\frac{1}{3}} \qquad \textbf{(10'')}$$

Putting (10'') inside (10'), we get:

$$\boxed{L = 2\alpha \frac{R}{c}\sqrt{G\rho}} \qquad \textbf{(11)}$$

Now we can apply this equation to each cases: L_k and L_g values. In fact we only need the L_k / L_g quotient value, since one of the two values L_k and L_g will be simplified in the quotient calculation of the relativistic operator.
The Kuiper's belt is not located on a sphere, but on a belt. If we note r_k , and $r_k^{'}$ respectively the maximum and minimum ray of this belt, then the integral above is calculated differently. After redoing this integral calculation, the result is that we have to replace *R* by $\sqrt{r_k{}^2 - r_k{}'^2}$. This gives the following result.

$$L \quad = \quad \frac{2\alpha}{c}\sqrt{G\rho\left(r_k{}^2 - r_k{}'^2\right)} \qquad \textbf{(12)}$$

We get, from (11) and (12), applied to the Kuiper's belt and the galaxy:

$$\boxed{\frac{L_k}{L_g} \quad = \quad \frac{\sqrt{r_k{}^2 - r_k{}'^2}}{r_g}\sqrt{\frac{\rho_k}{\rho_g}}} \qquad \textbf{(13)}$$

r_k is the maximum ray of Kuiper's belt (as seen from the sun).
$r_k^{'}$ is the minimum ray of Kuiper's belt.
r_g is the ray of the galaxy.
ρ_g is the matter density of the galaxy.
ρ_k is the matter density of the Kuiper's belt.

Finally, the equation of the symmetric contribution coming from the Kuiper's belt can be written:

$$L_k \quad = \quad \frac{2\alpha}{c}\sqrt{G\rho_k\left[\left(r_k - x\right)^2 - \left(r_k{}' - x\right)^2\right]} \qquad \textbf{(14)}$$

$$\frac{L_k}{L_g} \quad = \quad \sqrt{\frac{\rho_k}{\rho_g}}\frac{\sqrt{\left(r_k - x\right)^2 - \left(r_k{}' - x\right)^2}}{r_g} \qquad \textbf{(15)}$$

Equation (15) has been obtained by dividing each term of equation (14) by L_g , which has been calculated using (11). This L_k / L_g value is a function of *x*, the distance of the Pioneer probe from the sun. $\sqrt{r_k{}^2 - r_k{}'^2}$ has been replaced in (12) by $\sqrt{\left(r_k - x\right)^2 - \left(r_k{}' - x\right)^2}$, because we suppose now *x* different from 0.
The problem with equation (15) above is that this equation is only correct for *x* weaker than $\left(r_k + r_k{}'\right)/2$. In order to get a correct equation for any *x* value, for the Kuiper belt symmetric contribution, the idea is the following.

- using equation (5'),
- calibrating this equation (5'), in order to have the same value for *x = 0*, and the same tangent to this curve for *x = 0*, as with the equation (15) above.

Equation (15) is representing the contribution coming from the Kuiper belt, but only from the *right*. But in fact equation (15) represents the value of the *symmetric* contribution coming from the Kuiper belt. Indeed, the contribution coming from the left is always greater than the one coming from the right. This is shown in appendix 4.
Therefore, the remaining contribution, which is the asymmetric contribution, is coming from the left. But this asymmetric contribution has an impact on the relativistic operator which is only the classical Newton gravity attraction of the Kuiper belt. This has been shown in [12].

Now we get, after a limited development of (15), of order 1 around $x=0$:

$$\boxed{L_k onL_g = L_k onL_{g\max} - k\,x} \quad \textbf{(16)}$$

With:

$$\boxed{L_k onL_{g\max} = \sqrt{\frac{\rho_k}{\rho_g}}\,\frac{\sqrt{r_k{}^2 - r_k{}'^2}}{r_g}} \quad \textbf{(17)}$$

And k value is such as $L_k onL_g = 0$ for $x = x_{Kuiper} = r_k + r_k' = 78\ AU$.
Now we can compare equation (16) with equation (5'). First of all, the equation (16) is true only for $0 < x < x_{Kuiper}$. For $x > x_{Kuiper}$, this equation must be replaced by $L_k onL_g = 0$. But the whole final curve has a strong derivative discontinuity for $x = x_{Kuiper}$.
Around $x = 0$, the equation (5') and (16) are close together, by construction. The tangent of the curve of equation (5') at $x = 0$ is the straight line passing by ($x = 0$, $y = L_k onL_{g\max}$), and ($x = x_{Kuiper}$, $y = 0$). This line is equation (16) for $0 < x < x_{Kuiper}$.
This x_{Kuiper} value is equal to 78 *AU*. In the program of appendix 3, it has been fitted to 114 *AU*, in order to get the perfect Pioneer anomaly curve. The difference between 114 *AU* and 78 *AU* might come from the presence of other objects beyond the Kuiper's belt. For example, there are the scattered disc objects.

Actually, the numerical value given by equation (17) is not the value of equation (5'') which allows to retrieve the Pioneer anomaly best curve.
For retrieving the value of the equation (5''), we have to replace:

- r_g, the galactic ray, by r_{glast}, where $r_{glast} = 0.9\ 10^{-2}\ r_g$.
- ρ_g, the matter density of the whole galaxy, by ρ_0, the matter density of the galaxy *near the solar system*.

Now with those new values, we get:

$$L_k onL_{g\max} = \sqrt{\frac{\rho_k}{\rho_0}}\,\frac{\sqrt{r_k{}^2 - r_k{}'^2}}{r_{glast}} \quad \textbf{(18)}$$

And the numerical application of (18) is yielding **0,0015**, which is the value of equation (5''). Used values for this calculation: $r_k' = 30\ AU$, $r_k = 48\ AU$, $r_{glast} = 0.9\ 10^{-2}\ r_g$, $r_g = 15.4\ kpc$,

$\rho_k = 0.89\ 10^{-14}\ kg / m^3$, $\rho_0 = 0.71\ 10^{-20}\ kg / m^3$. Those are the values used by the program of appendix 3.
Please refer to appendix 10 for some more details about the calculations of the Kuiper belt symmetric contribution.

The replacement above of r_g by r_{glast} can be explained by a simple physical behaviour.
This behaviour is the occulting of the farthest contributions of the galaxy by the local gas and objects. This mechanism is very similar to the existence of a simple "fog", which is hiding the lights which are located behind it. With this mechanism, the galaxy objects which are situated beyond this "fog" must not be taken into account.
Hence, the r_g distance must be replaced by r_{glast}, the distance at which the "fog" is beginning to hide the further "lights". It is also the distance of the farthest object which has to be taken into account for the space-time propagated deformation contributions when calculating the relativistic operator.
That's why we must replace also ρ_g by ρ_0.

Let's notice that this "hiding" effect is probably the explanation of the opposite sign which has been used in [13], in the MAPLE program: << F := + c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) >> has been executed, in place of << F := - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) >>. (Remark: in [12], the correct sign was used).

We have also to check that the r_{glast} value is less than the thickness of the galaxy, in order to confirm that the sphere surrounding the sun with a ray equal to r_{glast} is completely inside the galaxy. Of course, this is mandatory, because otherwise the model would predict a *G* value (for the gravitational constant) strongly isotropic, (depending strongly on the direction of the gravitational force with respect to our galaxy). This would be immediately ruled out by experiment data.
The numerical values for this checking is the following. We have $r_{glast} = 450\ LY$, compared to the thickness of the galaxy equal to $gal_{thick} = 700\ LY$ in the vicinity of the sun. Therefore we have $r_{glast} < gal_{thick}$. It must be noticed also that r_{glast} is (weaker but) quite close to gal_{thick}. This might be explained by some dynamical mechanism for the galaxy objects. Indeed, when gal_{thick} becomes weaker than r_{glast}, then the local *G* value inside the galaxy becomes do increase, hence the stars mean speed becomes to increase, their ellipsoidal trajectories becomes to expand, and therefore gal_{thick} tends to increase. Finally it exists a regulation mechanism as soon as gal_{thick} tends first to decrease, which might be the case because the galaxy's objects tends to gather themselves as close as possible to the galaxy plane. The permanent mode of this regulation leads to a gal_{thick} value quite close to r_{glast}.

As a conclusion, let's analyse now the fitting of the values which has been done in order to get this theoretical Pioneer anomaly perfect curve. This fitting can be done by modifying r_{glast} and x_{Kuiper} values directly, in the program of appendix 3. When modifying those parameters successively, one may find the perfect curve given by the program of appendix 3. But, if only r_{glast} parameter is fitted, and if x_{Kuiper} is left constant, it is not possible, in the *general* case, to obtain this perfect curve. Also, if r_{glast} is left *constant* to its *fitted* value, then fitting x_{Kuiper} will finally yield an x_{Kuiper} fitted value *close to the theoretical one*.

In other words, r_{glast} has been strongly fitted, but x_{Kuiper} has not neen modified so much from its theoretical value. Indeed, the theoretical value of x_{Kuiper} is 78 *AU* (the sum of 30 and 48 *AU*), and the fitted value of x_{Kuiper} is 114 *AU*. Of course, with respect to the galactic range of magnitudes, 114 *AU* is close to 78 *AU*.
Therefore, for the fitting there were two parameters to be fitted. But the fitting has been done only on one parameter, the other one was not fitted but calculated.
This can be seen as another validation of the model of this document.

9. CONCLUSION ABOUT THE SYMMETRIC CONTRIBUTIONS

9.1. WHAT WE LEARN FROM THE ANALYSIS OF THE PIONEER ANOMALY

Let’s summarize the final state of the symmetric contributions of space-time propagated deformations, used in the calculation of the relativistic operator. This calculation has been used by the program in appendix 3 and plotted the theoretical curve of figure 4.

First of all, the main symmetric contributions comes from the extragalactic objects (L_u) and the galactic objects (L_g). We must add to them the contributions coming from the Kuiper’s belt, L_k .

In the calculation of the relativistic operator, one may only focus only on 2 of the 3 values L_u , L_g , L_k , because the relativistic operator is a quotient and one of those 3 values will be simplified in this quotient. Hence, we have to calculate only, for example, L_u / L_g , and L_k / L_g .

In fact, as we will see below, we can set $L_u = 0$, hence $L_u / L_g = 0$. This supposition will simplify the calculations, without changing the final result for this study. Of course, in reality L_u is not null, even inside the galaxy, and this will be reminded further, when necessary. In fact stating $L_u = 0$ means that in the calculations we suppose L_u constant contribution as part of L_g constant contribution, or in other words: L_g is replaced by $L_g + L_u$ and we put $L_u = 0$.
Also, the OORT cloud contribution is symmetric. But it is is very weak in front of the galactic contribution, L_g . This is shown in appendix 5.

The equation of L_k / L_g is equation (5’). In this equation, the value of $L_k onL_{g\max}$ is given by equation (17), with the classical values for the Kuiper’s belt ($r_k , r_k '$ and ρ_k).
But if we use r_g the galactic ray, and ρ_g , the mean matter density of the galaxy, in equation (5’), then the numerical result is not the one we are waiting for, and which is the value of equation (5’’). Hence we must use different values for those parameters.
If we use $r_{glast} = 0.9\ 10^{-2}\ r_g$ in place of r_g , and $\rho_g = \rho_0$, the matter density in the vicinity of the sun, then we retrieve the correct value of equation (5’’). Of course, this r_{glast} value has been fitted in order to retrieve this final value.

In physical terms, this means that the symmetric contributions might be stopped quickly during their propagations through the galaxy. This “stopping propagation mechanism” gives an explanation for the following issues.

- the replacement r_g of by r_{glast} as seen above for fitting coherently the Kuiper contributions..
- the sign of the force used in [13]. This has to be evaluated in a next version of [13]. In this document, an encouraging attempt is done in the chapter untitled <<GALAXIES SPEED PROFILES: SOLVING THE SIGN ISSUE>>.

- It might prevent to get an infinite value for the extra-galactic contributions in the equation of G in [12]. It is written "might" because another possible explanation of this finite sum is to postulate a universe finite age.

- It prevents to get a strongly isotropic value for G. This is confirmed by $r_{glast} < gal_{thick}$, with gal_{thick} being the thickness of the galaxy in the vicinity of the sun. Nevertheless r_{glast} is quite close to gal_{thick} and this might be explained by some regulation mechanism.

- The r_{glast} fitted value allows also to retrieve the order of magnitude for the sideral gravity issue (chapter untitled << MEASUREMENTS OF G>>).

- The r_{glast} value allows also to retrieve the order of magnitude for the disparities of the measurements of G (chapter untitled << MEASUREMENTS OF G>>).

The used value for x_{Kuiper} in equation (5') is not $r_k + r_k' = 78\ AU$, the theoretical value, but $x_{Kuiper} = 114\ AU$. This value has been fitted in order to retrieve the best possible curve.
Therefore, the shape of the curve of equation (5') is not decreasing quickly enough for modelling correctly the Kuiper's belt which ends at 48 *AU*.
In physical terms, those remarks means that there are some other objects (or gas), beyond the Kuiper's belt, which must be taken into account. Noticeably, the miscellaneous objects are candidates. A next version of this document will try to take them into account.

9.2. COMBINATION BETWEEN THE CONTRIBUTIONS

Now let's try to summarize the different kinds of combination between the contributions.
Once those combinations executed, L_1 and L_2 values will result, and it will be possible to calculate the value of the relativistic operator.
What is the combination of two distinct contributions arriving on the same space point ? The operator used to combine those contributions will depend on the type of the contribution: are they generated by one luminous point, or by numerous luminous points?

A) ONE LUMINOUS POINT

First of all, there is the case of a small number of isolated luminous points, among a uniform and homogeneous distribution of matter.
In this case, there is globally an Euclidean space-time metric, because of the uniform space distribution of matter. In some M point in space, the different contributions coming from those luminous points must be taken into account, in the calculation of the relativistic operator (when they are aligned on the same straight line with M). The operator used in this case is the square root of the sum of the squares (equation (8)).
Those kind of contributions are stopped quickly when propagating inside a galaxy. They vanishes after propagating around $r_{glast} = 450$ light year inside the galaxy. This vanishing mechanism seems to come from an occulting effect provoked by the gas of the galaxy.

B) NUMEROUS LUMINOUS POINTS

The next case is the case of a very important number of luminous points.

For example, the case of extragalactic contributions, L_u, generated by the extragalactic energy, is one of them.
In this case, those contributions are took into account using a "+" operator. For example, this is the case of the calculation of the approximation of Newton's law, in [12]. Let's remind it. We used, in [12] the following equation.

$$L_1 \quad = \quad L_{1u} + L_{1m} \quad = \quad L_u + g\left(x\right)$$

L_{1u} is the contribution coming from the extragalactic contributions, for example.

L_{1m} is the contribution coming from a unique luminous point, received on the same point. as above for L_{1u}.

<u>Remark</u> : we may considerer also L_{1u} as being the contribution coming everywhere except the considered luminous point: from the solar system, except the considered luminous point, from the local galaxy energy situated before the famous r_{glast} distance, and coming also from the extra-galactic energy. In this example the receiving point in space is located inside the solar system.

The "+" operator between L_{1u} and L_{1m} is used here because L_{1u} is a contribution coming from a very important number of luminous points.

Those kind of contributions are not stopped when propagating inside a galaxy.

C) TRYING PHYSICAL EXPLANATIONS

First of all, the galactic center generates a global contribution which is not vanished during its propagation in the galaxy. That's because it is generated from numerous luminous points. Of course this is mandatory because the gravitational effect of the galactic center is proven by experimental data.

The explanation of those different behaviours (single or numerous luminous points) must be driven by the analysis of the geometry of the space-time propagated vacuum, as usual.

The operator used in the first type of contributions (isolated luminous points) has been explained in this document. Equation (8) has been calculated.

For the second type of contributions, (numerous luminous points), we have to imagine the shape of the space-time vacuum which is resulting from the numerous space-time propagated vacuums, on some fixed M point.
At each *t* instant, on the fixed M space point, there are numerous propagated space-time vacuums coming to.
The combination of these vacuums on M, obeys to the rule of the conservation of the whole space-time 4D volume of those vacuums. Hence, the final shape of this resulting vacuum, is locally a narrow slide of space-time. The effect of this slide is the addition of a fixed value to the relativistic operator.

Now, this fixed onset of time is a constant function in space. Hence the frequency of this function is very low. This makes a big difference with the first case in which the frequency was very high, for the shape of the different vacuums on M, either along the time or any space direction.

This very low frequency might explain the "absorption mechanism" which has been noticed. Hence, the galactic gas might behave like a low pass filter on those space-time propagations. Of course, this filter is vanishing high frequency space-time deformations only after propagations of them through some quite long distance. That's why we get this r_{glast} maximum value for the emitting distances of the contributions for the relativistic operator. Therefore this low pass filter behaviour of the galaxy gas might explains the maximum r_{glast} value for the contributions coming from the luminous points located *inside* our galaxy.
But the contributions coming from energy located *outside* the galaxy, are generated by numerous luminous points (of course!). Hence they generate a low frequency global shape, which is not stopped inside its propagation inside the galaxy. But also, for those contributions, there exists also a maximum distance beyond which no luminous point contribution can be received. Indeed, this is the only way to get a finite denominator for the equation of *G*.

9.3. THE REAL RG_LAST VALUE

The calculation which has been done in this document supposes that the contributions coming from energy located outside the galaxy are equal to 0. This has been supposed because it makes simpler calculations, and it doesn't change the final result. But of course, this is not true. In fact, the r_{glast} value which has been used is only a maximum possible value for r_{glast}. The real calculation of r_{glast} must take into account the extragalactic contributions. Let's call them L_u. We get the following.

$$r_{glast\_real} = r_{glast} \frac{L_g}{L_g + L_u} \qquad \textbf{(18')}$$

Where r_{glast} is the value which has been used in this document, supposing that $L_u = 0$.
r_{glast_real} is the real value for r_{glast}, supposing that $L_u \neq 0$.
L_g is the contribution coming from galactic energy. It is proportional to r_{glast_real} value as proven by equation (11), where one must replace R by r_{glast_real}. Similarly, the galactic contribution L_{g0}, supposing that $L_u = 0$, is proportional to r_{glast}.
This equation (18') comes from the following calculation. *SymmetricContributions* = L_2 = $L_u + L_g + L_k = L_{g0} + L_k$ => $L_u + L_g = L_{g0}$ => $r_{glast\_real} / r_{glast} = L_g / L_{g0} = L_g / \left(L_u + L_g\right)$ => equation (18').

The main issue here is that it is difficult to extract $L_g / \left(L_u + L_g\right)$, (or L_g / L_u), from experimental data. In appendix 9, it has been supposed that L_g / L_u value is between 1/8 and ½.
(In appendix 9 it has been written $L = r / x$, where L is the relative contribution coming from the galaxy energy, and received in the galaxy in a location at which x is the distance from the galactic center. Hence we get $L_g / L_u = r / x_s$, where x_s is the distance between the sun and the galactic

center. r is the distance from the galactic center where $L_g / L_u = 1$. It has been used $x_s = 8\,kpc$, and r located between $1\,kpc$ and $4\,kpc$, this gives L_g / L_u between 1/8 and ½).
This last ranging value for r has to be refined. For this, a comparison may be done with the measured matter density profile of the galaxy. Of course it is very rough. Moreover the whole appendix 9 calculation has to be refined. This would yield a precise L_g / L_u value resulting from experimental data. The important point is that it could invalidate or validate the above reasoning (low pass filter, r_{glast} value, etc…).

Nevertheless, the galaxy study has to be completely rewritten taking into account the correct sign in the equation of the gravitational force. After the correction of this strong error, the coherence with the study of the solar system will be possible. This will be done in another version of this document, after writing another version of [13].

10. FLYBY ANOMALIES

The theoretical explanation of the Pioneer anomaly above is probably able to explain also the “flyby” anomalies.

In this document we will focus on 2 kind of flyby anomalies:

- Earth flyby anomalies
- Saturn flyby anomaly.

10.1. EARTH FLYBY ANOMALIES

Earth flyby anomalies are explained in [5]. This document reports anomalies for 8 NASA probe trajectories when traveling in the vicinity of the Earth.

We uses exactly the same theoretical explanation as above for the Pioneer anomaly, except that the sun is replaced by the Earth, and the anomaly is seen for x lesser or greater than x_0, rather than only for x greater than x_0.

x is now the distance between the probe and the Earth.
x_0 is the distance at which the geocentric gravitational constant is known to be perfect, (that is, probably the distance at which it has been calculated). In the program of appendix 6, we have used the value of 16200 *km* for x_0. This value has been fitted in order to retrieve exactly the measured anomaly of the Galilleo I probe.

The program in appendix 6 is calculating the theoretical perigee velocity of 6 probes.
Its results is the following.

Probe	Measured anomaly	Theoretical anomaly
Galilleo I	2,56	2,53
Galilleo II	- 2	-11
NEAR	7,21	- 6,6
Cassini	- 1,7	4,5
Rosetta I	0,67	22
Messenger	0,008	29

Table 6: Comparison between the real and the theoretical flyby anomalies for 6 probes. Unit is *mm/s*.

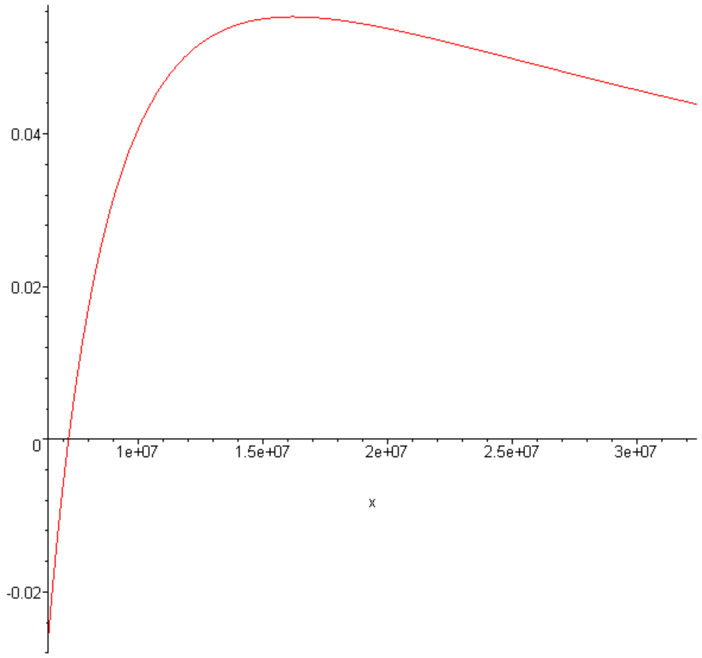


Figure 7: Theoretical perigee velocity increase curve for the Messenger Earth flyby. *Y*-coordinate is in *m/s*, and *x*-coordinate is in meter.

The curve above is showing the value of the anomaly when *x* is varying. *x* is the distance between the probe and Saturn at the perigee, expressed in meters.
For plotting the curve above, the following approximated equation has been used.

$$D_v \simeq \sqrt{2}\,\frac{G_t^{\frac{3}{2}}}{c v_n x}\left(\frac{3}{\sqrt{x_0}} - \frac{2}{\sqrt{x}} - \frac{9 G_t^{\frac{3}{2}}}{c v_n^2 x_0 x}\right) \qquad \textbf{(19)}$$

Where G_t stands for the geocentric gravitational constant, v_n stands for the Newtonian velocity at perihely.
This approximated equation has been calculated using the approximated equation (3), transforming this equation (3) in a second degree polynomial equation, (using G_t' in place of G_h'), calculating the resulting approximated equation for G_t', using this equation of G_t' in the classical potential energy equation. This equation is validated in the program of appendix 6.

This equation is interesting because it explains the bell shape of figure 7. Also, it shows that the anomaly is barely equal to the measured value. This seems to be an invalidation of our theoretical explanation.
However, those calculations are much too simpler, because they suppose that the Earth referential frame is inertial. Of course, this is not true. The calculations must be done at least in the Schwarzchild metric of the sun, and using the motion of the Earth, the exact trajectory of the probe, and the symmetric contributions of surrounding matter (asteroids, planets, etc …).

Meanwhile, the order of magnitude is retrieved. It can be seen on table 6 that the theoretical calculated values share exactly the same order of magnitude as the measured values, for those six probes flyby anomalies. Therefore, here there is no contradiction between the three elements gravitational model and gravitation.

Of course, Rosetta I and Messenger yields bad results in table 6. But this might be explained by the second correction of Newton's law because those probes are probably travelling along the ecliptic plane.

10.2. SATURN FLYBY ANOMALY

As well as for the next chapter, a probe passing by Saturn should experiment a flyby anomaly, for exactly the same theoretical reasons. Therefore, our Newton's law modification is calculating a flyby anomaly for the Pioneer 11 probe when traveling in the vicinity of Saturn.

The characteristics of this anomaly for Pioneer 11 probe trajectory, is the following.

- anomalous decrease of the probe before the Saturn encounter,
- anomalous increase of the probe after the Saturn encounter,

Indeed, the Pioneer probe is going away from the sun, and the first modification of Newton's law is calculating a weaker Saturn gravitational force than Newton's one, when flying near by Saturn.
The sun's attraction is weakened (by Saturn gravitation), less than what Newton's equation is calculating, before the Saturn encounter. Also, after the encounter, the sum of the sun's attraction and the Saturn's attraction is weaker than Newton's prediction.

The program of appendix 7 is calculating this flyby Saturn anomaly for Pioneer 11. It yields the following result.

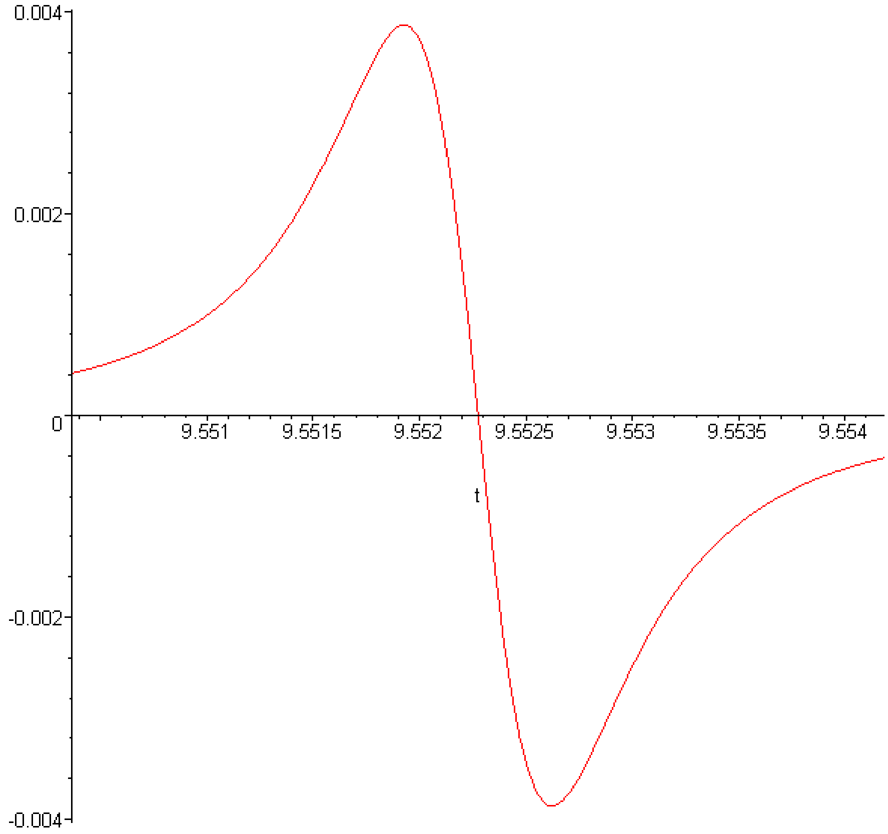


Figure 8: Theoretical anomalous added acceleration toward the sun when encountering Saturn. *Y*-coordinate is in *m/s²*, x-coordinate is in *AU.*

The program of appendix 7, yielding this figure, has been using the following values.

- $l_s = 20000\,km$, is the nearest distance from Pioneer 11 to Saturn, of its trajectory.
- $x_{0s} = 2\,AU$, is the distance from Saturn at which the Saturn gravitational constant, GM_s, is known to be perfect (M_s stands for Saturn mass).

The values of the figure above are very strong, as compared to the Pioneer anomaly values, but the distances range when this anomaly occurs is very short. This range of this flyby anomaly is roughly [9,550 *AU*, 9.555 *AU*].
We must check that the global Pioneer anomaly curve is still correct for this Pioneer 11 travel, after taking into account this Saturn flyby anomaly. In fact, this global curve shape depends strongly on the x_{0s} value.
For getting the Pioneer anomaly curve of the figure below, x_{0s} value has been fitted in order to get still this correct global Pioneer anomaly curve. This fitted value is the one above, and is used in the program in appendix 7. This program is yielding figures 8 and 9.

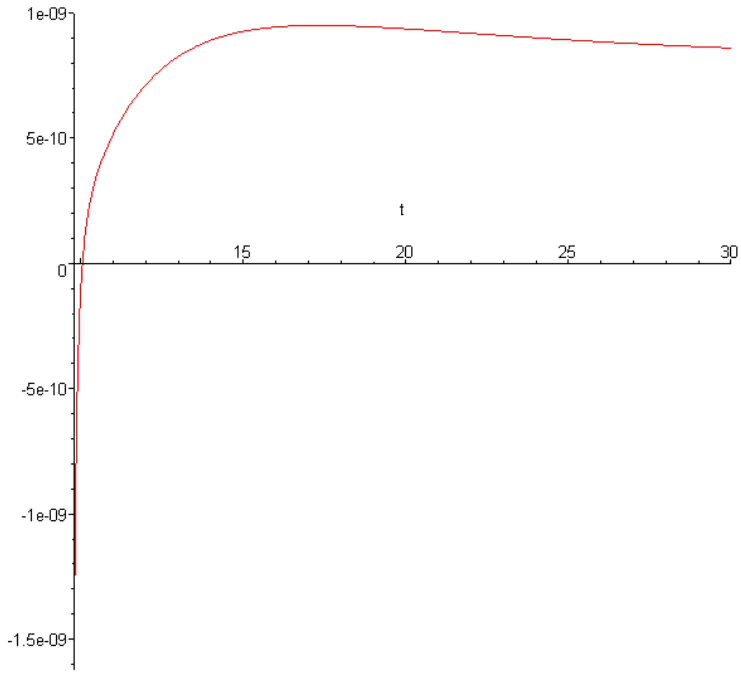


Figure 9: Theoretical anomalous added acceleration toward the sun when encountering Saturn. *Y*-coordinate is in *m/s²*, x-coordinate is in *AU*.

11. PERIHELION ADVANCE

The MAPLE program of appendix 8 is calculating the perihelion advance in the context of our study. It yields values which are very close to the General Relativity values :

Planet	**GR value**	**3elt value**
Mercury	42,7848	GRvalue - 0,0014
Saturn	1,66291	GRvalue + 0,00056

Table 10: Change in the perihelion advance or precession by the three elements theory. Units are arc-second by century.

Those modifications of GR values are coming from the first modification of Newton's law described in [12]. They do not explain the anomaly of the precession of the Saturn perihelion explained in [2], which is of -0.006 arc-second by century.
But, since the calculated values are very close to the GR values, it can be concluded that the three elements theory is compatible here with GR. Anyhow, those corrections are probably insignificants in front of the precisions of actual measurements.
Once again, like for the Earth flyby anomalies, the second correction of Newton's law must be calculated in this case. This might retrieve the anomalous advance of the perihelion of Saturn described in [2].

12. MEASUREMENTS OF G

The aim of this chapter is to explain the disparities in the measured values of *G*, the gravitational constant.

The issue is well defined. For nearly three centuries, *G* has been measured, without getting globally a better precision than 0.7%. The reason of this globally poor precision is the fact that many measurements values are contradicting each others, taking into account their precision interval. The details of this issue is, for example, well described in [6].
Let’s remark, first, that our model prediction is a *G* value depending of the surrounding matter distribution.
This is not a new idea, it has been suggested in [8], [9], and [10]. Moreover, it has been proven by experimental data in [7], that *G* value varies with orientation. This behaviour is also a consequence of this theoretical idea.

Calculations shows that what is called the <<first correction of Newton’s law>>, in [12], is not able to explain the issue. Indeed, the order of magnitude of the theoretical error is below the experimental one. Let’s check this quickly with some little calculation.
We use the right end of equation (2), which can be written:

$$G = G'\left(1-\frac{3}{c}\sqrt{\frac{2MG'}{x}}\right) \qquad \textbf{(20)}$$

G' is the value of G for long distances (between the attracting masses).
M is the mass of the attracting object used in the experimentation.
For the G measurement of [3], we use this equation (20) with $M_1 = 900\ g$, which is the mass of the attracting ball, and $x_1 = 0.02\ m$, which is the approximated distance between the balance and the attracting ball.
For the G measurement of [4], we use equation (20) with $M = M_2 = 10.5\ kg$, which is the mass of the attracting ball, and $x = x_2 = 0.0001\ m$, which is the minimum distance between the balance and the attracting ball. I cannot have the exact value of this distance x_2 in [4], therefore i use a minimum estimated value. This x_2 value is the best case in order to get a maximum difference between the two values of *G*. Therefore we get:

$$G_1 = G'\left(1-\frac{3}{c}\sqrt{\frac{2M_1G'}{x_1}}\right) \qquad G_2 = G'\left(1-\frac{3}{c}\sqrt{\frac{2M_2G'}{x_2}}\right)$$

$$G_1 - G_2 = 3G'\frac{\sqrt{2G}}{c}\left(\sqrt{\frac{M_2}{x_2}}-\sqrt{\frac{M_1}{x_1}}\right)$$

For the numerical application, *G’* value is $G' \simeq G = 6.6742867\ 10^{-11}\ N\ m^2/kg^2$, and we get:

$$\frac{G_1 - G_2}{G} = 3{,}7\ \ 10^{-11} \qquad \textbf{(21)}$$

This ratio must be compared with the measured values of G in [3] and [4], which are $G_{1m} = 6.7174\ 10^{-11}\ N\ m^2 / kg^2$, and $G_{2m} = 6.6740\ 10^{-11}\ N\ m^2 / kg^2$:

$$\frac{G_{1m} - G_{2m}}{G_{1m}} = 6{,}5\ 10^{-3} \qquad \textbf{(22)}$$

The measured value of equation (22) is much greater than the theoretical value of equation (21). Therefore, the first modification of Newton's law is not able to explain this issue.

But the << Second correction of Newton's law>> of [12] gives much better results.
Let's compare first with data coming from [7]. In this document, the amplitude of the variation of the G value is more than 0.054% as compared to its absolute value.
Here the calculation based on the "Second correction of Newton's law" is the following.

Step	Action
1	Calculating the number of stars which has an impact on the final value of G during the measurement. Those stars are located on a sphere surrounding the Earth, and the ray of this sphere is equal to r_{glast}. Let's remind that r_{glast} is the greatest distance for an object in the galaxy, in order to have any gravitational effect on Earth. r_{glast} has been fitted to 450 light year (138 parsec), in the program of appendix 3.
2	Calculating α, the mean solid angle for each star in this sphere, as seen from the Earth.
3	Calculating β, the solid angle done by the "attracting lines" between the attracted masses used in the measurement of G. Those "attracting lines" are the straight lines, in space, driven by the gravitational forces vectors between two attracting objects. One must pay attention to the fact that there exists a gravitational force between each couple of points. Each such couple of points is composed by one given point on an attracting object, and the other one being on the other attracting object.
4	Calculating the ratio α/β, which has the same order of magnitude as the mean relative variation of the symmetric contributions.
5	Multiplying it by 2, because of the square power over the contribution in the equation of G. The equation of G is available in [12]. The result is roughtly the mean relative variation of G.

Table 11: Calculation procedure for a simple estimation of the theoretical relative error of G measurements, when changing the attracting lines direction with respect to the system of fixed stars.

Hence, a good estimation of the order of magnitude of the theoretical relative amplitude is equal to 2 α/β.
This value is calculated in the program of appendix 9. The result of this program is the following.
The prediction of the gravitational model of the three elements theory for this value is a mean value of 0.013%, and a real value greater than this mean value. Therefore, the order of magnitude of this model prediction is close to experimental data: **0.054%**.

This is without taking into account the contributions coming from extragalactic energy. In fact, the study of the galaxies speed profiles shows that those extragalactic contributions might be taken into account. But the value of those contributions for the Milky-way are difficult to get from experimental data. If we suppose that those extragalactic contributions are equal to eight time the value of galactic contributions, then we obtain a relative amplitude equal to **1 %**.

If it is supposed that those extragalactic contributions are equal to twice the value of galactic contributions, then we obtain a relative amplitude equal to 0.12 %.
If it is supposed that those extragalactic contributions are null, then we obtain a relative amplitude equal to **0.013 %**. Those results are given in appendix 9.

Remark: extragalactic contribution can be estimated on return with the measured variation of G (0.054 %.). There is two way for this. The first one is to simply calculate Lu. We have, inverting the equations used in table 11 and appendix 9:

$$L_u \quad = \quad L_g \left(\sqrt{\frac{SolidAngleBal\,.\,NbStarsLoc}{8\pi}\left(\frac{r_{glast}}{r_{gloc}}\right)^3 \frac{\Delta G}{G}} - 1 \right) \quad = \quad 1.01\ L_g \ \simeq \ L_g \quad \textit{and}$$

$$R_{extragal} \quad = \quad r_{glast}\sqrt{1+\left(\frac{L_u}{L_g}\right)^2 \frac{\rho_g}{\rho_{universe}}} \quad = \quad 680000\,kpc \ = \ 0.05\ R_{universe}.$$

This last result shows that the farthest distance beyond which no contribution are ever received, has the order of magnitude of the "ray" of the universe (taking in account the precision of these calculations). It tends to shows that there is no occulting mechanism during the propagation of the contributions coming from the farthest region of the universe. This is consistent with the existence of a cosmic microwave background radiation. On the other hand, there is an occulting mechanism inside the galaxy, which is the case also for some light frequencies as we know. This value $L_u \simeq L_g$ *is consistent with the calculation done at the beginning of [13], which uses the velocity of the galaxies inside their groups. The second way to evaluate it is the following. Extragalactic contribution could be much lower (near null) along an attractive line which is contained in the galactic plan. That's if the occulting effect of the galaxy* ***is acting*** *for those extragalactic contributions. But, on the contrary, along an attractive line which is not contained in the galactic plan, extragalactic contributions are much greater. The difference between those two cases is constraint by the relative measured variation of G. The angle of the cone of the attractive lines of the balance used during those measurements is equal to roughtly 12°. This angle is weaker than the angle from which the width of the galaxy is seen from the sun. Probably, the direction of those attracting lines during the day period is crossing over the galaxy (to be confirmed). Therefore, the variation of G is probably completely affected by this mechanism. The equation gives then dG/G = 2 dLu/(Lu+Lg) then 0.054 % = 2 Lu/(Lu+Lg) and Lu/Lg = ½ 0.054 % = 2.7 10-4. This value has to be checked with the other calculations (galaxy speed profiles noticeably). This is not compatible with the first result above (*$L_u \simeq L_g$*). This tends to confirm that those extragalactic contributions are* ***not*** *occulted by the galaxy dust.*

Now let's try a simple calculation for estimation, but without using the stars.
It will be supposed that the stars has no effect in the measurement of *G*. We will calculate the effect of the surrounding mountains, which may be located around the apparatus during the measurement of *G*.
It is assumed that exactly the same experimentation is executed, but in two completely different places. And the important difference between them is the distribution of matter in the surrounding neighbourhood during those experimentations.

- experimentation 1) is done at the very top of a hill on the floor of a desert, and this floor is completely plane outside the hill on which we are located.

- experimentation 2) is done in the middle of a valley which is surrounded by mountains.

The interesting thing is that the measured value of *G* will be completely different between those two cases, even if exactly the same experimentation apparatus and measurement procedure is applied. Indeed, the presence of the surrounding mountains in the second measurement has an important effect on the final measured value of *G*.

Let's remind equations from [12]:

$$e1 = \frac{L_0 \sqrt{\frac{8R}{x}}}{L_0} = \sqrt{\frac{8R}{x}} \quad \textbf{(23)}$$

$$e2 = \frac{L_0 \sqrt{\frac{8R}{x}}}{L_0 + L_m} \quad \textbf{(24)}$$

Where L_0 stands for the result of the symmetric contributions in the case of the experimentation 1), which are those of the solar system, L_{sol}, the galaxy, L_g, and the extra-galactic objects, L_u.
L_m is the added contribution in the case of experimentation 2), which is coming from the nearby mountains.
For the two cases, we still get the following equations, coming from [12], with e being either e_1 or e_2, depending of the experimentation:

$$\cos(\alpha) = oper(L1, L2) = \frac{\sqrt{1 + e}}{1 + \frac{e}{2}} \quad \textbf{(25)}$$

$$\tan(\alpha) \simeq \frac{e}{2} \quad \textbf{(26)}$$

$$F = mc^2 \frac{d \tan(\alpha)}{dx} \frac{\tan(\alpha)}{(1 - \tan^2(\alpha))^{3/2}} \quad \textbf{(27)}$$

Therefore, using equations (26) and (27), we get, for experimentation 1):

$$F_1 \simeq \frac{mc^2}{4} \frac{de_1}{dx} e_1 \quad \textbf{(28)}$$

And for experimentation 2):

$$F_2 \simeq \frac{mc^2}{4} \frac{de_2}{dx} e_2 \quad \textbf{(29)}$$

Now, using (28) and (29) we get:

$$\frac{F_1 - F_2}{F_1} \simeq \frac{e_1^2 - e_2^2}{e_1^2}$$

$$\simeq \quad 2\ \frac{L_m}{L_0}$$

For the estimation of L_m / L_0 we will use equation (13), in the case of these contributions:

$$\frac{L_m}{L_0} = \frac{r_m}{r_{glast}}\sqrt{\frac{\rho_m}{\rho_g}} \qquad \textbf{(30)}$$

In fact, this r_{glast} value supposes that extragalactic contributions are equal to 0, and this is not true. But this supposition doesn't change the final result below for $(G_1 - G_2)/G_1$. Indeed we get $L_0 = L_{sol} + L_g + L_u \simeq L_g + L_u = 2\alpha \frac{r_{glast}}{c}\sqrt{G\rho_g}$, which is equation (11), from which r_{glast} has been calculated with the help of Kuiper fitted contributions.
Because of supposed Newton's law, this gives the same value for the corresponding ratio of G:

$$\frac{G_1 - G_2}{G_1} = \frac{F_1 - F_2}{F_1}$$

$$= 2\ \frac{r_m}{r_{glast}}\sqrt{\frac{\rho_m}{\rho_g}}$$

Where :

r_m stands for the maximum distance between the surrounding mountains, and the emplacement of the experimentation 2). We will use $r_m = 4\ km$.
r_{glast} stands for greatest distance between a galactic object and the place of the experimentations. We have used above, for the Pioneer anomaly curve calculation, the value $r_{glast} = 450\ LY$ for this.
ρ_g is the matter density of the galaxy, like above for equation (17). Its value has been set to $0.003\ M_0 / AL^3 = 0.709\ 10^{-20} kg / m^3$, which is the matter density in the galaxy, near the solar system, M_0 being the sun mass.
ρ_m is the mean matter density of the surrounding mountains of experimentation 2). We will use $\rho_m = 1.4\ g / cm^3$, which is weaker than granite matter density ($\rho_{granite} = 2.7\ g / cm^3$).
With those numerical values, the final result is the following.

$$\boxed{\frac{G_1 - G_2}{G_1} = 0{,}9\ 10^{-3}} \qquad \textbf{(31)}$$

The order of magnitude of this theoretical value is the same as the measured one, of equation (22).
This proves that the order of magnitude of the measured difference can be explained by this correction of Newton's law.
The next step would be to get the information of the exact locations where the two experimentations of [3] and [4] took places. It will be needed also the exact values of the distances between the attracting objects, in [3] and [4], in order to calculate precisely the theoretical ratio as above, and to compare this theoretical ratio to the measured one of equation (22).

As an intermediate conclusion, the gravitational model of this study, explained in [12], might explain precisely the great historical disparity between the measurements of *G*.
We have shown that this theoretical value of *G* is depending on the distribution of matter in the surrounding neighbourhood (buildings, hills, mountains, Earth surface, the sea …) of the place where the measurement of *G* is done.
Moreover, this study might give, with the help of our big database of today *G* measurements, a precise value for the *G’* gravitational constant, which is the “classical” gravitational constant *G*, but valid only for very long distances.
Thereafter, with this value of *G’*, it will be possible to predict the exact value of *G* at any distances and in any cases. Noticeably, whatever the distributions of the surrounding matter in the neighbourhood, it would be possible to calculate the value of *G*. This value of *G* will be valid only for the local application of Newton’s law.
If this case, there is no doubt that the precision of the measurement of *G’*, and the following calculations of *G*, will be much better than today, and with no longer disparities.
It must be pointed also that the gravitational model of this document is predicting that the same measurement of *G*, done in two different places, will give completely different values, and that this difference can be actually calculated by this model.

13. GENERAL RELATIVITY EQUATIONS

13.1. EQUATION OF THE GRAVITATIONAL FORCE

The correction of Newton's law is given by the following equation in almost any cases.

$$F_3 = -\frac{mMG'}{x^2} \frac{\left(s + \sqrt{\frac{2R'}{x}}\right)\left(s + 2x\frac{ds}{dx}\right)}{\sqrt{s^2 + s\sqrt{\frac{8R'}{x}}}\left(s^2 + s\sqrt{\frac{8R'}{x}} - \frac{2R'}{x}\right)^{3/2}} \qquad \textbf{(32)}$$

M is the attracting mass, *m* the attracted mass, and *G'* the gravitational constant valid only for long distances. *G'* is roughly equal to *G* plus 0,04% of *G*. We have also $R' = MG'/c^2$.
This is the gravitational force between two pin pointed masses inside a universe whom matter density is varying. In this equation, s is the varying symmetric contribution. In other words, the contributions coming from the universe matter density are supposed to be like below.

$L_s = L_0\, s$ is the symmetric contribution,

$L_a = L_0 \sqrt{\frac{2R'}{x}}$ is the asymmetric contribution. $R' = MG'/c^2$, where *M* is the attracting object generating this asymmetric contribution.

Equation (32) has been calculated using this classical math tip

$$F = mc^2 \frac{d\ \tan(\alpha)}{dx} \frac{\tan(\alpha)}{\left(1-\tan^2(\alpha)\right)^{3/2}} = \frac{mc^2}{2}\frac{d}{dx}\left(\left(\tan(\alpha)\right)^2\right)\frac{1}{\left(1-\tan^2(\alpha)\right)^{3/2}}$$

For $s = 1$, equation (32) becomes:

$$F_3 = -\frac{mMG'}{x^2} \frac{1 + \sqrt{\frac{2R'}{x}}}{\sqrt{1 + \sqrt{\frac{8R'}{x}}}\left(1 + \sqrt{\frac{8R'}{x}} - \frac{2R'}{x}\right)^{3/2}} \qquad \textbf{(33)}$$

It gives what has been called in this document the "first correction of Newton's law".
A limited development at order two, of this last equation, as a function of $\sqrt{\frac{R'}{\mathrm{x}}}$, yields the following

$$F_3 \simeq -\frac{mMG'}{x^2}\left(1 - 3\sqrt{\frac{2R'}{x}} + \frac{19R'}{x}\right) \qquad \textbf{(34)}$$

Therefore this last equation is correct only for *R'* weak in front of *x*. It will be used for the calculation of the new Schwartzchild metric, below.

Now for *s* different from 1, and for *R'* very weak in front of *x*, equation (32) becomes:

$$F_3 \quad = \quad - \frac{mMG'}{x^2} \; \frac{s + 2x\dfrac{ds}{dx}}{s^3}$$

$$F_3 \quad = \quad - \frac{mMG'}{x^2} \; \frac{1 + f + 2x\dfrac{df}{dx}}{(1+f)^3} \qquad \textbf{(35)}$$

It gives what has been called in this document the "second correction of Newton's law".
It has been written $s = 1 + f$ where "1" stands for the (relative) extragalactic symmetric contributions, and *f* for the (relative) galactic symmetric contributions. This Newton's law corrective term (correction term on right hand side of equation (35)) is negative if $df \,/\, dx$ is negative and strong in front of $(1+f)/(2x)$. This explains the negative values for the gravitation force in the case of NGC 3310 and NGC 7541 galaxies, for example. Indeed, as soon as the derivative value $df \,/\, dx$ is negative and strong, as well as *x* quite strong, then the gravitational force becomes negative. Those cases can be found in the matter density distributions of the NGC 3310 and NGC 7541 galaxies, for example.
In the special case of a galaxy with a matter density varying with a 1/x² law, we get the following.

$$\left.\begin{array}{l} \rho = \dfrac{k}{x^2} \quad \Rightarrow \quad f = \dfrac{r}{x} \\ x >> R' \end{array}\right\} \quad \Rightarrow \quad F_3 \;\simeq\; -\, m\, G_h \, \frac{x - r}{(x+r)^3}$$

Which explains the classical bell shape of the classical measured speed profiles. It explains also the "flat" shape of the right hand sides of those speed profiles.
The gravitational force is null near the galactic center, for *x* = *r*. It explains also the solid part of a galaxy with an analytical calculation.

13.2. SCHWARTZCHILD METRIC

The Schwartzchild metric is modified in such a way.

$$ds^2 = g_{tt}c^2dt^2 - g_{rr}dr^2 - r^2d\theta^2$$

1) General relativity

$$M = \frac{mG}{c^2} \qquad g_{tt} = 1 - \frac{2M}{r} \qquad g_{rr} = \left(1 - \frac{2M}{r}\right)^{-1}$$

2) Three elements theory

$$M' = \frac{mG'}{c^2} \qquad g_{tt} = 1 - \frac{2M'}{r}\left(1 - 2\sqrt{2M'}r\right) \qquad g_{rr} = \left(1 - \frac{2M'}{r}\left(1 - 2\sqrt{2M'}r\right)\right)^{-1}$$

In other words, general relativity equation remains the same, but $M = mG/c^2$ is replaced by

$$M' = \frac{mG'}{c^2}\left(1 - 2\frac{\sqrt{2mG'}}{c}r\right).$$

The corrective term, $1 - 2\frac{\sqrt{2mG'}}{c}r$, comes from the first order part of equation (34) (the first correction of Newton’s law).

G' is the gravitational constant which is valid only for very long distances. For calculations of this corrective term, one just redo the calculations of the Schwartzchild metric of general relativity, using the new gravitational force in place of the Newton one:

$$F_n = -\frac{mMG}{r^2}$$ is Newton’s law,

$$F_{3elt} = -\frac{mMG'}{r^2}\left(1 - \frac{3}{c}\sqrt{\frac{2MG'}{r}}\right)$$ is the corresponding new gravitational force.

13.3. PARAMETERIZED POST-NEWTONIAN FORMALISM

The PPN parameters (Parameterized Post-Newtonian formalism) are exactly the same as for relativity. Indeed, the only differences between relativity and the gravitational model of the three elements theory are the following.

- Lorentz equations are true only between inertial reference frames which get “energy attached locally”. “Energy attached locally” to a reference frame (O; ct, x, y, z) means that there is a particle or a group of particles whom inertial point is constantly equal to O.

- Newton’s law is not used, like in relativity, but retrieved. There is a slight difference between Newton’s law and this corrected Newton’s law.

There exists another difference, but which is not impacting the macroscopic scale. In the context of the three elements theory, the luminous point space-time deformations are asymmetric, and this yields an isotropic behaviour of the gravitational force. But this doesn’t impact the macroscopic scale. However, this gives the opportunity to develop a unifying theory.

Anyhow, the compatibility with relativity is such that each post Newtonian parameters are equal to their relativity values.
Each relativity equations remains also the same, except Einstein equation because this one is calculated using Newton’s law. In the gravitational model of the three elements theory, Einstein equation is only an approximation.

14. TULLY-FICHER RELATION

This relation has been proven from experimental data in [11].
For retrieving this relation, equation (32) is used, as well as the classical following equation:

$$v = \sqrt{\frac{-F_3 \; x}{m}} \qquad \textbf{(36)}$$

This last equation supposes a circular trajectory of the stars around the galactic center.
F_3 is replaced in this equation by its expression in (32). The last correction term of (32) has to be set to 1 in this case because we study the speed of the stars in a galaxy: *x* is much greater than *R'*.
Therefore, equation (35) is used in place of equation (32).
The maximum value of *v*, the speed profile, is then searched, with the classical tip of searching where the derivative of this *v* function is equal to 0. The final result of these calculations is the following.

Tully-Ficher relation is retrieved only in the case of ρ following a x^{-2} law, where ρ is the matter density in the galaxy, and *x* the distance from the galactic center.

Of course, this prediction has to be checked from the galaxy data which has been used in [11].

Details of those calculations are given below.

β is the power for matter density ρ inside a galaxy, supposed to be of the form: $\rho = \frac{k}{x^{\beta}}$,

ρ is the matter density inside a galaxy, supposed to be of the form: $\rho = \frac{k}{x^{\beta}}$,

whith k being a constant depending of the galaxy.

M is the (attracting) galactic center mass.

M_g the galaxy's mass.

m mass of the attracted object.

G' gravitational constant valid only for very long distances.

f symmetric contribution coming from the galaxy's objects.

x distance from the galactic center.

v speed of the stars (located at *x* distance from the galactic center).

v_{max} maximum speed of the stars.

$$v = \sqrt{\frac{-F_3 \; x}{m}}$$

$$v = \sqrt{\frac{MG'}{x} \; \frac{1 + f + 2x\frac{df}{dx}}{(1+f)^3}} \qquad \textbf{(37)}$$

$$\frac{dv}{dx} = 0$$ Searching the maximum value of *v*.

$$\Leftrightarrow \qquad x(1+f)(3f'+2xf'') \quad = \quad (1+f+2xf')(1+f+3xf')$$

After calculations. It has been used $f' = \frac{df}{dx}$ and $f'' = \frac{d^2 f}{dx^2}$. Let's note write now: $f = \frac{r}{x^\alpha}$,

therefore $\boxed{\alpha = \frac{\beta}{2}}$. The above equation becomes:

$$\alpha r(2\alpha - 1)(x^\alpha + r) \quad = \quad (x^\alpha + r - 2\alpha r)(x^\alpha + r - 3\alpha r)$$

Now let's write: $y = x^\alpha$ $\quad k = \alpha(2\alpha - 1)$ $\quad a = -1 + 2\alpha$ $\quad b = -1 + 3\alpha$.
With those notations, the last equation becomes:

$$y^2 - (a+b+k)ry + (ab-k)r^2 \quad = \quad 0$$

Therefore,

$$y \quad = \quad \frac{1}{2}\left((a+b+k) \pm \sqrt{(a+b+k)^2 - 4(ab-k)}\right)r$$

$$y \quad = \quad k_0 r$$ k_0 being a constant.

$$x \quad = \quad k_1 r^{\frac{1}{\alpha}}$$ k_1 being a constant.

Now, this *x* value is inserted in equation (37), using $f = \frac{r}{x^\alpha}$, and replacing *r* by its expression as a function of *M*.

14.1. SUPPOSING RGLAST CONSTANT

For this version of the calculations, it will be assumed: $r = k_4 \sqrt{M}$, k_4 being a constant. This means that r_{glast} is supposed to be constant, when changing the galaxies. (Because in this case, *r* is proportional by construction to *L*, which is proportional to the square root of ρ because of equation (11) used when replacing *R* by r_{glast}, and ρ is proportional to *M* everywhere in a galaxy). This assumption means that the r_{glast} "horizon" stays constant whatever *M*, and therefore that dust density stays roughly constant whatever the galaxy. We get finally:

$$v_{\max}^4 \quad = \quad k_2 M^{2\frac{\beta-1}{\beta}}$$

Now it might be written $M = k_7 M_g$ (M_g being the galaxy's mass), and k_7 a constant. Therefore:

$$\boxed{v_{\max}^4 \quad = \quad k_3 M_g^{2\frac{\beta-1}{\beta}}}$$

k_3 being a constant such as:

$$k_3 = G'^2 k_4^{-\frac{2}{\alpha}} k_7^{2\frac{\beta-1}{\beta}} \frac{\left(1+\frac{1-2\alpha}{k_0}\right)^2}{k_0^{\frac{2}{\alpha}}\left(1+\frac{1}{k_0}\right)^6}, \qquad \text{with } k_0 = \alpha^2+2\alpha-1+\alpha\sqrt{\alpha^2+4\alpha-2}$$

Here Tully-Ficher relation is retrieved only for $2\frac{\beta-1}{\beta} = 1$, which means $\boxed{\beta = 2}$

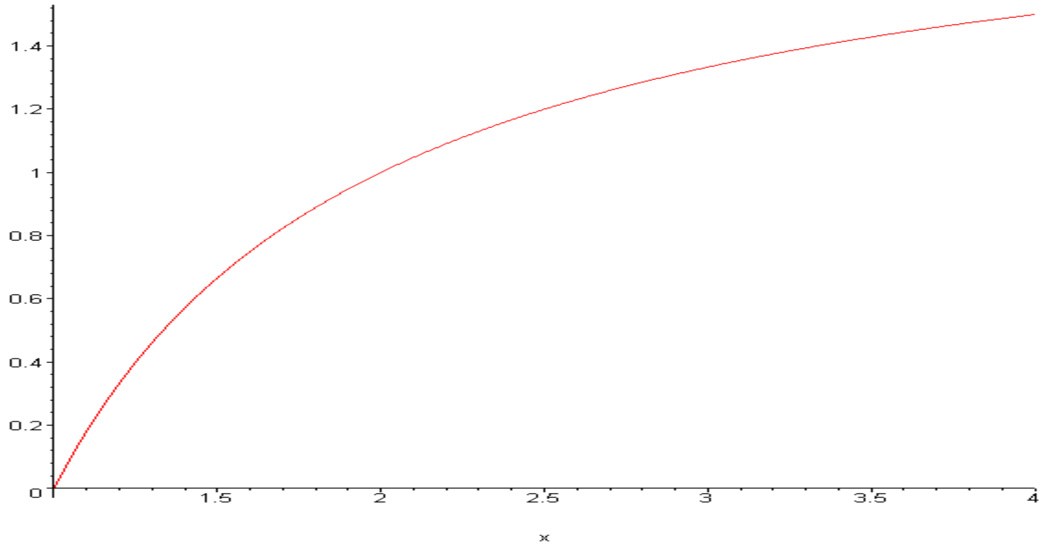


Figure 12: Plotting of the function *y = 2(x-1)/x* for *x* between 1 and 4. The interesting point is (*x*=2, *y*=1).

14.2. SUPPOSING RGLAST NOT CONSTANT

For this version of the calculations, it will be supposed that dust density is allways proportional to galaxy matter density, everywhere, and that the coefficient of this proportionality does not depend of the galaxy in its "Tully-Ficher group".

Therefore equation (11) must be used, replacing *R* by r_{glast}: $L = 2\alpha\frac{r_{glast}}{c}\sqrt{G\rho}$. By oher means, using equation (41), the variation of r_{glast} can be deduced: $e^{-k\, r_{glast}\,\rho^{\frac{2}{3}}} = \eta$, where η is a fixed coefficient without unit, weaker than 1. This yields $r_{glast} = -\frac{\ln(\eta)}{k}\rho^{-\frac{2}{3}}$, and $L = -2\alpha\frac{\ln(\eta)\sqrt{G}}{k\,c}\rho^{\frac{1}{6}}$, therefore *L*, then *r* is proportional to $\rho^{\frac{1}{6}}$. But the density of the dust, ρ, is supposed to be proportional to matter density inside the galaxy. And matter density is globally, when changing the galaxy, proportional to *M* (this last point is coming from the fact that galaxy matter density is supposed to be always of the form $\rho = \frac{k}{x^\beta}$; hence there is only one scale factor for the galaxy masses). Therefore it can be written: $r = k_5 M^{\frac{1}{6}}$, with k_5 being a constant. With this assumption, finally we get:

$$\boxed{v_{max}^4 = k_6 M_g^{2\frac{3\beta-1}{3\beta}}}$$

k_6 being a constant.

Here Tully-Ficher relation is retrieved only with $\boxed{\beta = \frac{2}{3}}$

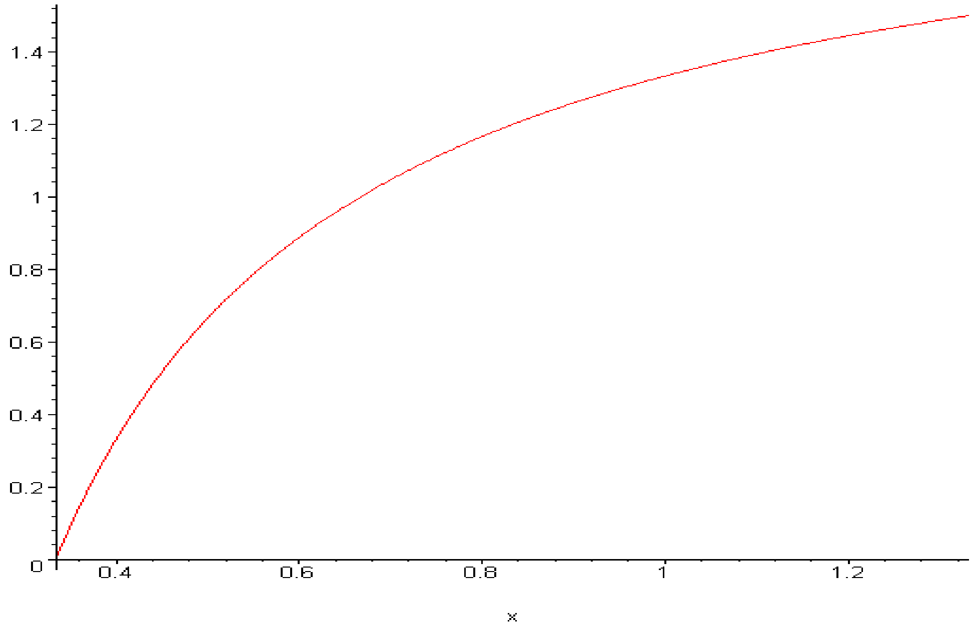


Figure 13: Plotting of the function $y = 2(3x-1)/(3x)$ for x between 1/3 and 4/3. The interesting point is (x=2/3, y=1).

14.3. CONCLUSION

The result depends of the behaviour of the r_{glast} value as a function of matter density. The study of the galaxies speed profiles has confirm ***locally*** the occultation behaviour which explains the existence and the variation of r_{glast} .

With this assumption, the variation of r_{glast} gives the result **β = 2/3.**

The issue with this 2/3 value is that the final ***global*** speed profile is not the measured one.
This is shown by the program of appendix 15. The final curve gets not flat region as it is the case with measured speed profiles.
This might be explained, for example, by a matter density not proportional to galaxy dust, in the galactic center. Indeed, it has been supposed that matter density is proportional to the dust in the

chapter untitled << GALAXIES SPEED PROFILES: SOLVING THE SIGN ISSUE >> This allows to solve the issue of sign, which is a *local* issue. But anyhow this proportionnality cannot be correct globally, otherwise the global theoretical speed profile would be far from the measured one (as mentioned just above). Therefore, it is hoped that matter density is not everywhere in the galaxy, proportional to the dust. Otherwise this would probably mean an invalidation of the model of this document. This has to be checked with experimental data.

On the contrary, if dust density is, for example, roughtly constant through the galaxy, then we get $\boldsymbol{\beta}$ **= 2**. This is completely compatible with the global measured speed profiles. In this case, the local "waves" of the speed profiles are explained by a local proportionality of dust with matter density. If this is true, then our model is explaining each issue correctly. The maximum value of a speed profile is probably more driven by the global shape than the local waves of this speed profile.
But even this case, we get $\beta = 2$ only if the constant dust density is not depending of the galaxy, noticeably if it is not varying with the galaxy's mass. Of course this seems to be uncorrect at first glance.

As a conclusion, these calculations are not enough in order to check if the model is retrieving Tully-Ficher relation.
Anyhow it could be interesting to fit this β value, with experimental data coming from the group of galaxies used in [11].

15. GALAXIES SPEED PROFILES: SOLVING THE SIGN ISSUE

The aim of this chapter is to solve the sign issue in the calculation of the galaxies speed profiles. This issue has been noticed in [13]. It is the following.

- The theoretical speed profiles of the galaxies are not retrieved with the exact theoretical equation (27).

- If another equation is used, then the theoretical speed profiles are closed to experimental ones. This *new* equation is $F \;=\; - \;\; mc^2 \, \frac{d \;\tan(\alpha)}{dx} \, \frac{\tan(\alpha)}{(1-\tan^2(\alpha))^{3/2}}$,
  which is exactly the opposite value of equation (27).

For this reason, and because of the existence of a limit distance, r_{glast}, in the reception of the space-time deformation contributions (showned above during the study of the Pioneer anomaly), an occultation mechanism is suspected to appear in the propagation of the space-time deformations inside the galaxy.
For modelling this occultation behaviour, it is supposed that the dust of the galaxy acts like a amount of particles. It is supposed also that the space-time deformation are propagating along straight lines. Thoses particles stops the propagation of the space-time deformations, when the straight lines representing thoses propagations intersect any of those particles.
Therefore, for any slide of dust, this model leads to an interception ratio which can be calculated as follows.

$$\frac{dN}{N} \quad = \quad - \;\; k \, \rho_s \, dx$$

Where ρ_s is the surfacic density of the dust.
dx is the thickness of the slide.
N is the total number of propagating straight lines intersecting the slide.
dN is the (counted negative) number of propagating lines, among the *N* lines, which are intercepted by the slide of dust particles.
k is a positive constant depending of the cross section of each dust particle.

We have also $\rho_s = \rho^{\frac{2}{3}}$ where ρ is the (volume) density of the dust. Therefore:

$$\frac{dN}{N} \quad = \quad - \;\; k \;\; \rho^{\frac{2}{3}} \, dx$$

After integrating we get, if ρ stays constant:

$$\ln\left(\frac{N}{N_0}\right) \quad = \quad - \;\; k \;\; \rho^{\frac{2}{3}} \, x$$

Where N_0 is the initial number N before touching the dust.

$$N = N_0 \, e^{-k\rho^{\frac{2}{3}}x} \qquad \textbf{(38)}$$

Now let's go back to equation (9). This equation must be modified, taking into account this decreasing value of the received symmetric contributions. The new equation is the following.

$$L_g^2 = \sum_p \frac{N}{N_0}\left(\sqrt{\frac{8R_p}{x_p}}\right)^2 \qquad \textbf{(39)}$$

$$= \sum_{x_p} 8\, n_p \frac{N}{N_0}\frac{R_p}{x_p} \qquad \textbf{(40)}$$

n_p is the number of luminous points which are located in the solid angle, at the x_p distance from the O point.

$$L_g^2 = \sum_{x_p} 8\,\alpha^2 \frac{{x_p}^2}{d^2}\frac{N}{N_0}\frac{M_p G}{c^2 x_p}$$

$$= 8\frac{\alpha^2}{d^2} e_p \frac{G}{c^4} \sum_{x_p} \frac{N}{N_0} x_p$$

The serie to integral transform is the same as above in this document, with some little add-on:

$$\sum_{x_p=0}^{r_{glast}} \frac{N}{N_0} x_p = \frac{1}{d}\int_0^{r_{glast}} \frac{N}{N_0} x\, dx$$

$$= \frac{1}{d}\int_0^{r_{glast}} x\, e^{-k\rho^{\frac{2}{3}}x}\, dx$$

Let's suppose again that ρ is constant. This is an interesting approximation because the contributions in the integral above are almost vanishing for *x* greater than r_{glast}, which is weak in front of the galaxy scale.

$$= \frac{1}{d\,B}\left[\frac{1}{B} - \left(\frac{1}{B} + r_{glast}\right) e^{-B\, r_{glast}}\right]$$

After some classical calculations, with $B = k\,\rho^{\frac{2}{3}}$. This yields finally for L_g :

$$L_g^2 = 8\,\alpha^2 \frac{G}{k^2 c^2} \rho^{-\frac{1}{3}} \left[1 - \left(1 + k\, r_{glast}\, \rho^{\frac{2}{3}}\right) e^{-k\, r_{glast}\, \rho^{\frac{2}{3}}}\right] \qquad \textbf{(41)}$$

Let's approximate $e^{-k\, r_{glast}\, \rho^{\frac{2}{3}}}$ to 0, which is possible because of the occultation mechanism. Let's just remind that this is possible except for a location at the end of the galaxy. We get immediatly:

$$L_g \simeq 2\sqrt{2}\,\alpha \frac{\sqrt{G}}{k\,c} \rho^{-\frac{1}{6}} \qquad \textbf{(42)}$$

Now there is an opposite sign for the effect of ρ on the final contribution. Hence the sign issue has been solved. But of course it remains to check if those new theoretical speed profiles will be still close to experimental ones.

L_g is the value of the symmetric contribution coming from the galaxy.

Let's note L_u the symmetric contributions coming from outside the galaxy. Equation (40) is used replacing *x* by $L_{end} - x$:

$$L_u^2 = L_{u0}^2\, \alpha^2\, e^{-k'\rho^{\frac{2}{3}}(L_{end}-x)}$$

Equation (40) has been used, replacing (N/N0) by its value in equation (38). The sum has been deleted, since there is only one contribution to be taken into account there. R_p / x_p has been replaced by a constant value, since this extra-galactic contribution have a constant value when encountering the galaxy. It is only slowly decreasing when propagating inside the galaxy because of the absorption mechanism.

L_{end} stands for the ray of the galaxy,
x stands for the distance from the galactic center,
k' is the absoption coefficient for those extra-galacctic contributions. Let's remind that *k'* should be weaker than *k*, because it has been shown (in chapter 8) that the contributions coming from numerous luminous points are less (or not) absorbed by the galaxy dust.
L_{u0} is the value of L_u / α when those contributions are encountering the galaxy.

$$L_u = L_{u0}\, \alpha\, e^{-\frac{k'}{2}\rho^{\frac{2}{3}}(L_{end}-x)}$$

Finally, the symmetric contributions inside a galaxy is the sum of the two above:

$$\begin{aligned} L_s &= L_g + L_u \\ &= 2\sqrt{2}\,\alpha \frac{\sqrt{G}}{k\,c} \rho^{-\frac{1}{6}} + L_{u0}\, \alpha\, e^{-\frac{k'}{2}\rho^{\frac{2}{3}}(L_{end}-x)} \end{aligned}$$

It can be seen, on appendix 11 program resulting curve, that this equation has an irrealistic result for *x* close to 0 (that is for locations near the galactic center). Of course, near the galactic center, the absorption mechanism might behave differently from elsewhere in the galaxy. That's why, in the program of appendix 11, a third term has been added. The role of this last term is to take into account the case near the galactic center. This added term is $L_{gn} = K_0\, x^{-\frac{3}{4}}$, where K_0 is a constant, and "3/4" power has been fitted. Of course, it still remains to be checked that this added term is realistic.

$$L_s = 2\sqrt{2}\,\alpha\frac{\sqrt{G}}{k\,c}\rho^{-\frac{1}{6}} + L_{u0}\,\alpha e^{-\frac{k'}{2}\rho^{\frac{2}{3}}\left(L_{end}-x\right)} + K_0\,x^{-\frac{3}{4}} \qquad \textbf{(43)}$$

The program located in appendix 11 calculates the new speed profiles with this new equation for the symmetric contributions. This program is exactly the same as the program of [13], but modified with this new equation for the gravitational force. The calculation has been done using equation (35), for optimizing computation performances. This equation (35) has been checked, by program, to be equivalent to the exact calculation in this case.
Of course with this equation (43), the sign issue is solved. In the program of appendix 11, the parameters of this equation have been fitted in order to get the best possible curves.
The resulting curves are given at the end of appendix 11. The global shape of the experimental speed profiles is still retrieved , but now without any sign error. The result is worse than the result of the initial program of [13], because the curves are decreasing too much, globally. This is noticeably the case for NGC 7541.

Therefore, this new equation for the symmetric contributions still has to be worked.

16. PIONEER ANOMALY BEFORE SATURN

It has been studied above the Pioneer anomaly for heliocentric distance greater than 10 *AU* (beyond Saturn).
But the theoretical curve of the Pioneer anomaly above predicts a negative anomaly for distances weaker than 10 *AU*. This is the case of two kind of theoretical curves: the “first” and the “second modification of Newton’s law”.
The program of appendix 12 is calculating the theoretical orbital period anomalies for each planet, taking into account the following.

- The main belt of asteroids.
- The Kuiper belt.
- Supposing that G_h' is perfect on Earth.

The results of this program is the right column of table 14. (For the left column, just change one line of appendix 12 program: replace <<`Ls_on_Lg := Lk_on_Lg + Lm_on_Lg:`>> by <<`Ls_on_Lg := Lk_on_Lg:`>> and re-execute the program).

Planet	**Without the main belt.**	**Taking into account the main belt.**
Mercury	-1.28	-1.24
Venus	-0.370	-0.345
Earth	0	0
Mars	0.400	**0.299**
Jupiter	1.31	**1.72**
Saturn	1.56	**1.96**
Uranus	1.76	**2.16**
Neptune	1.85	2.26

Table 14: Theoretical relative errors for the orbital periods of planets, calculated for the heigth planets at once. Units are: $\times 10^{-4}$ .These errors are the periodic orbital value errors, divided by the periodic orbital value itself.

The second column of this table shows an important difference between the four inner planets and the four outer planets. This is due to the presence of the main belt of asteroids, located between Mars and Jupiter. Indeed, the table shows that between Mars and Jupiter, the anomaly is increasing much more with the presence of the main belt than without it.
This behaviour is explained also by the program curves, and by equation (35). Indeed, when passing through the main belt, the value of *f* (the symmetric contributions coming from the main belt of asteroids) is decreasing brutally. This decreasing has two parrallel effects on equation (35). It decreases the denominator, and it increases $1 + 2x\, df/dx$ on the numerator (because *df/dx* is negatice inside the belt, and is vanishing outside the belt). Therefore, the final attraction force is increasing very much when passing through the main belt, from the sun to the outer space.

This mechanism gives an important prediction for the model of this document. It might explains the apparent lack of knowned asteroids in the main belt. Hence, the ephemerides shows that there are missing asteroids in order to explains them correctly.
Here is a possible theoretical explanation of this mystery.

Now the calculation must be done once again, taking into account the four inner planets first, and the four outer planets, after. Indeed, like the ephemerides calculation itself, two different calculations must be done, one for each group of planets.

Planet	**Inner planets only.** **G_h perfect on Earth.**	**Outer planets only.** **Maximum error minimized.**
Mercury	-1.24	
Venus	-0.345	
Earth	0	
Mars	0.299	
Jupiter		1.05
Saturn		1.29
Uranus		1.49
Neptune		1.59

Table 15: Theoretical relative errors for the orbital periods of planets. Units are: $\times 10^{-4}$ .There are two different calculations, one for the four inner planets, on the left, and one for the outer planets, on the right. For the left column, G_h is perfect on Earth. On the right, G_h is perfect when located 10,39 AU away from the sun. With this G_h value, the maximum relative error for the four outer planets is minimized. These errors are the absolute error of periodic orbital time value, divided by the periodic orbital value itself. The left column is exactly the same as the right one of table 14 for the four inner planets, therefore, calculated with the program of appendix 12. The right column has been calculated using the program of appendix 13.

This table shows that the order of magnitude of relative errors is $\mathbf{10^{-4}}$, and the maximum value is for Mercury: $\mathbf{-1.24\times10^{-4}}$.
Now let's calculate the maximum relative error in JPL ephemerides simplified calculations, available in http://ssd.jpl.nasa.gov/?planet_pos.

Planet	**Inner planets**	**Outer planets**
Mercury	1.0×10^{-2}	
Venus	0.467×10^{-2}	
Earth	0.186×10^{-2}	
Mars	0.290	
Jupiter		1.22
Saturn		71.6
Uranus		36.5
Neptune		5.64

Table 16: Maximum cumutaled relative errors of orbital periods of planets, for JPL ephemerides. Units are: $\times 10^{-4}$. The program calculating those values is available in appendix 14. For the details of the signification of those values, please refer to the heading comments of the program which is located in appendix 14.

Now we might compare theoretical values with some experimental data. Let's add the Mercury relative error to the Mars relative error. This gives an estimation of the whole theoretical deviation of table 16.

- For table 15, we get *RelMax0* = $1.24\times10^{-4} + 0.299\times10^{-4}$ = $\mathbf{1.54\times10^{-4}}$
- For table 16, we get *RelMax1* = $1.0\times10^{-6} + 0.290\times10^{-4}$ = $\mathbf{0.30\times10^{-4}}$

And finally:

$$\boxed{\frac{\mathrm{Re}\,l_{\max 0}}{\mathrm{Re}\,l_{\max 1}} = \mathbf{5.1}}$$

This is greater than one.

May be this could be explain by some extra mechanism, such as the effect of the solar wind plasma. Indeed, the solar wind might attenuate the space-time deformation propagations coming from the sun.
Let's remind that a similar mechanism has been studied in this document for the space-time deformations coming from the galaxy's object, which are attenuated by the galaxy's dust. It has been shown that this mechanism would be abble to explain in a coherent manner the sign issue of the galaxies speed profiles, the exact Pioneer anomaly curve, the sideral gravity value and the order of magnitude for the disparities of the measurement of *G*.
Indeed, an attenuation of the sun's contributions would leads to a decreasing value of the gravitational attraction coming from the sun. It would compensate this too strong theoretical anomaly above. But of course, this hypothesis has to be worked.

Remark
*For the four outer planets, the ratio is **below 1.***
To be worked.

17. CONCLUSION

The Pioneer anomaly has a purely theoretical explanation, based on the three elements theory.
The details of this explanation are explained in the chapter untitled << First modification of Newton's law >>, in [12].
The theoretical curve is close to the measured one. First of all, for distances ranging from 10 to 20 *AU* from the sun, the anomaly is perfectly explained, without any fitting modification.
Noticeably, the value of $8.74 \;\; 10^{-10} m / s^2$ is retrieved with a precision of 17%.
For distances greater than 20 *AU*, the theoretical curve is decreasing more than the measured one. But this can be explained by the presence of the Kuiper's belt, and some other objects beyond it. The calculation shows that there might exist an absorption mechanism in the propagation of the space-time deformations generated by the galaxy's objects. This mechanism might elude a calculation issue for the stars speed profiles, in [13].
This mechanism and the "horizon value" of it (r_{glast}) are in accordance with the following.

- the Pioneer fitted anomaly curve. One parameter only was fitted (r_{glast}) where one could except to fit two parameters (r_{glast} and x_{Kuiper}),
- fixing the sign issue for the gravitational force in [13]. This must be completely evaluated in a next version of [13]. But in the actual document here, an encouraging attempt has been done, using this occulting mechanism,
- the width of the galaxy ($r_{glast} < gal_{thick}$),
- sideral gravity,
- the disparity of the measurements of *G*.

In a more general way, the study of the Kuiper's belt symmetric contributions, along with the study of the dark matter mysteries, should enable us to understand fully the mode of propagation of the space-time deformations contributions in the context of the three elements theory.

In this document, it has been shown also that these modifications of Newton's law are predicting the following.

- An increase of the gravitational force toward the sun when passing through the main belt of asteroids in the direction of outer space. This added acceleration might explains the ephemerides "missing asteroids" mystery in solar system.
- An added Saturn flyby anomaly of Pioneer 11 trajectory when encountering Saturn. This is a short distances acceleration value, which must be added to the "classical" Pioneer acceleration anomaly.
- Tully-Ficher relation should be retrieved only for galaxies with a matter distribution following a $1/x^2$ law. (Calculations to be confirmed by experimental data).
- "Sideral gravity", shown in [7], which is theoretically retrieved.

- The very old disparity mystery, for the gravitational constant measurements, with an order of magnitude which is close to the measured one.

- Too strong anomalies for planets orbital periods: theoretical relative errors are fith time the maximum experimental error for Mercury. But this might be explain for example by some attenuation mechanism for the sun's space-time propagated contributions (solar wind).

For the mysteries below, it has been checked that the "first modification of Newton's law" doesn't explain those mysteries. In some way we can conclude that the model of this document is compatible with gravitational measurements here. But the "second modification of Newton's law" must be calculated. In those cases this seems to be complicated to do.

- Earth flyby anomalies.

- Perihelion advance of Saturn.

As a conclusion, we can write that the gravitational model of the three elements theory:

- is compatible with relativity. Noticeably the PPN parameters are equal to those of relativity,

- seems to be compatible with gravitation,

- is encouraging for the explanation of the perihelion advance of Saturn.

- is very encouraging for the explanation of the Earth flyby anomalies,

- gives an explanation for the Pioneer anomaly, the "dark matter" mysteries, sideral gravity, the disparities of the measurement of *G*, and the ephemerides "missing asteroid" mystery.

- But gives too strong anomalies for the ephemerides of Mercury, Venus, and Earth.

As a more global conclusion, the gravitational model of the three elements theory seems to be validated. As such, this is a validation of the three elements theory itself.

A next step will be to solve definitively the sign issue for the galaxies speed profiles mystery. It has been shown by calculations and computing that this sign issue is solved quite completely by the occulting behaviour of the dust of the galaxies. But the resulting speed profiles, calculated now with the correct sign, are decreasing too much especially for the NGC 7541 galaxy.
But the most promising work would be the explanation of the disparity of the gravitational constant measurements. Indeed, the gravitational model of this document is predicting that the same measurement of *G*, done in two different places, will give completely different values, and that this difference can be actually calculated by the model.
Today this work consists first of getting the crucial information of the locations of two measurements of *G*, yielding different values. Thereafter, this work will be the evaluation of the presence of mountains in the neighbourhood of those locations. This will enable to calculate the theoretical ratio, and finally compare it to the measured one.

May we hope to solve someday the issue of the measurement of *G* ? Will we find a unique theoretical explanation for actual gravitation issues and physical mysteries ?

Topic	Distance range	Newton's law modification	Result	Next steps
Dark matter, velocities of the galaxies	>100 *kpc* *Groups of galaxies.*	1st	Excellent	Finding the exact ratio *G'/G* from experimental data.
Dark matter, galaxies speed profiles	1-100 *kpc* *A galaxy.*	2nd	Very encouraging.	Construction of the exact occulting equation, solving the sign issue and with better results than today.
Tully-Ficher relation	1-100 *kpc* *A galaxy*	2nd	Encouraging	Stars/dust ratio profile in a galaxy. Experimental data on the galaxies used in [11].
Sideral gravity	1 *kpc.*	1st	Very encouraging	More precise calculations. Finding and using the exact L_u / L_g ratio.
Pioneer anomaly	10-70 *AU*	1st and 2nd	Almost excellent	Modelisation of objects beyond Kuiper belt. Getting precise experimental data for x < 10 *AU*.
Saturn flyby	9.6-10.5 *AU*	1st	Interesting	Experimental data is needed.
Ephemerides	1-70 *AU* *Solar system.*	1st	Bad result for the four inner planets.	Obtaining exact accuracy of ephemerides for the four inner planets. Execute precise calculations. Estimation of the compensating effect.
Ephemerides: "missing asteroid mystery"	1-5 *AU* *Main belt of solar system.*	2nd	Encouraging.	Experimental data. Calculation of an estimation of the total mass of the "missing" asteroids.
Disparities of *G* measurements	10000 *km.*	2nd	Very encouraging	More precise experimental data. Specific *G* measurements.
PPN formalism	All ranges	-	Excellent	-

Table 17: Results of the gravitational model of the three elements theory

APPENDIX 1 CALCULATION OF GH' CONSTANT

Resolution of the third degree equation for Gh':

a y**3 + b y² + c y + d = 0

a = 3√(2/xs)/c
b = -1
c = 0
d = Gh

A = b/a
B = c/a
C = d/a

P = B - A²/3
Q = (A/27)(2A² - 9B) + C:

R = Q² + (4/27)P3

R is negative, see below.

u = (-Q/2 + (i/2)√(-R))**(1/3)
v = (-Q/2 - (i/2)√(-R))**(1/3)

j = -1/2 + i√3/2:
y = j²u + jv - A/3 This is the closest value from √Gh value, from the 3 real roots.

Gh' = y²

Numerical application:

a = 0,11830385 10**(-13)
b = -1
b = 0
d = 0,13271244 10**21
A = - 0,84528101 10**14
B = 0
C = 0,11217930 10**35
P = - 0,23816666 10**28
Q = - 0,44737269 10**41
R = - 0,10037193 10**76 R is negative
u = 0,281760 10**14 + 0,665113 10**10 i
v = 0,281760 10**14 - 0,665113 10**10 i
y = 0,115208 10**11

Ghp = 0,13273052 1021 m3/s**

APPENDIX 2 PLOTTING FIRST MODIFICATION CURVE

```
#################################################################
#                                                               #
#              FIRST CALCULATION OF PIONEER ANOMALY             #
#                                                               #
# This MAPLE program is calculating and plotting the curve of   #
# the PIONEER anomaly, using only the first modification of     #
# Newton's law.                                                 #
#                                                               #
#################################################################

#################################################################
######################### Initialisations #######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                       # Floating points precision.
unassign('x'): unassign('y'): unassign('Da'):

# ----------------- Cosmological values --------------------- #
day := 24*3600:                                       # in sec.
km := 10**3:                                              # m.
AU := 149597870 * km :              # Astronomical Unit in m.
c := 3 * 10**8:                           # light speed in m/s.
xs := 1429 * 10**9:                # Distance Saturn sun in m.

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################
x0 := 1 * xs:   # This is the distance from the sun at which Gh,
the
                # heliocentric gravitational constant is
calculated.
                # This value is seen on the curve of the Pioneer
                # measured anomaly (experimental curve).
                # It is very near to x0 = xs, the distance of
Saturn
                # from the sun, as seen from the experimental
curve.

# heliocentric gravitational constant inside the solar system.
Gh := (0.01720209895)**2 * AU**3 / day**2:

aa := 3*sqrt(2/x0)/c:
bb := -1:
cc := 0:
dd := Gh:
A := bb/aa:
B := cc/aa:
```

```
C := dd/aa:
P := B - A**2/3:
Q := (A/27)*( 2*A**2 - 9*B) + C:
R := evalf(Q**2 + (4/27)*P**3 ):                         # Negative.
U := evalf( root[3](-Q/2 + (I/2)*sqrt(-R)) ):
V := evalf( root[3](-Q/2 - (I/2)*sqrt(-R)) ):
jj := -(1/2) + I*sqrt(3)/2:
y3 := Re(evalf(jj**2*U + jj*V)):
x3 := evalf(y3 - A/3):
Ghp := evalf(x3**2):

#################################################################
# Calculation of Da, the Pioneer anomaly.                       #
#################################################################
A := Ghp - Gh:
B :=  3*Ghp*sqrt(2*Ghp)/c:
Da := (A - B/sqrt(x)) / x**2:

#################################################################
#        Plotting the theoretical Pioneer anomaly curve        #
#################################################################
x := AU * y:
plot( Da, y=x0/AU..50 );
```

APPENDIX 3 PLOTTING PIONEER ANOMALY CURVE

```
#################################################################
#                                                               #
#                CALCULATION OF PIONEER ANOMALY                 #
#                                                               #
# This MAPLE program is calculating and plotting the curve of   #
# the PIONEER anomaly.                                          #
#                                                               #
#################################################################

#################################################################
######################### Initialisations #######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                       # Floating points precision.
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

# ----------------- Cosmological values --------------------- #
s := 1:                                                    # s.
day := 24*3600:                                       # in sec.
gram := 10**(-3):                                         # kg.
cm := 10**(-2):                                            # m.
km := 10**3:                                               # m.
kpc := 3.08 * 10**19 :      # 1 kilo parsec expressed in meter.
pc := kpc/1000 :                 # 1 parsec expressed in meter.
mpc := 10**3 * kpc:                                        # m.
c := 3 * 10**8:                          # light speed in m/s.
LY := 365 * 24 * 3600 * c:             # 1 light year in meter.
AU := 149597870 * km :               # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).
Gp := 10 * G:        # Gravitational constant outside any galaxy.
rg := 473 * 10**18:                  # Ray of the galaxy in m.
rg_last := 0.9 * 10**(-2) * rg:                # Fitted value.
                 # Distance of last galaxy's contribution. in m.

# ----------------- Solar system values --------------------- #
# heliocentric gravitational constant inside the solar system:
Gh := (0.01720209895)**2 * AU**3 / day**2:
Mt := 5.97 * 10**24:                       # Earth mass in Kg.
M0 := 2 * 10**30 :                          # Sun mass in Kg.
R0 := M0 * G / c**2:          # Half Schwarschild ray for SUN.
xs := 1429 * 10**9:               # Distance Saturn sun in m.
rho0 := 0.003 * M0 / (LY**3): #Matter density near solar system.

# ------------------ Kuiper belt values --------------------- #
xkmin := 30 * AU:         # Kuiper min distance from the sun. m.
xkmax := 48 * AU:            # Kuiper max distance from the sun.
xkmean := (xkmin+xkmax)/2:   # Kuiper mean distance from the sun.
```

```
thetak := Pi*20/180:                   # Kuiper thickness angle.
xkthi := xkmean * thetak:                    # Kuiper thickness.
Mk := 0.3 * Mt:                            # Kuiper belt mass
value.
Vk := Pi*(xkmax**2 - xkmin**2) * xkthi: # Kuiper belt volume m3.
rhok := evalf(Mk/Vk):        # Kuiper belt matter density kg/m3.
#xKuiper :=  xkmin + xkmax:   # If only Kuiper belt xKuiper=81 AU
xKuiper :=  114*AU:       # Fitted value for taking into account
                          # the scattered disk and other
                          # transneptunian objects.

#################################################################
########### Calculation of Lk, the symmetric contribution #######
#################################################################

# ------------------ Calculation of fmax --------------------- #
Lk_on_Lg_max := (sqrt(xkmax**2 - xkmin**2)/rg_last) *
sqrt(rhok/rho0):

# ------------- ---- Calculation of Lk ----------------------- #
# Here is the final equation of the symmetric contribution coming
# from the Kuiper belt:
Lk_on_Lg := Lk_on_Lg_max * (2/Pi) * invfunc[tan]((2/Pi) *
                                                     xKuiper/x):
x := x0:      # Lk0_on_Lg : value of Lk where Gh is correct value
Lk0_on_Lg := Lk_on_Lg:
unassign('x'):

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################
x0 := 1 * xs: # This is the distance from the sun at which Gh the
              # heliocentric gravitational constant is calculated.
              # This value is seen on the curve of the Pioneer
              # measured anomaly (experimental curve).
              # It is very near to x0 = xs, the distance of Saturn
              # from the sun, as seen from the experimental curve.
              # The best value for x0 is x0 = 0.95 * xs,
              # (in place of x0 = xs). This last value yields the
              # value of 8.25 10**(-10) m/s².

# ----- Used method: iteration ------------------------------- #
Ghpmin := Gh/100:
Ghpmax := 100*Gh:
Ghp := Ghpmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
  unassign('Da'):

  Rp := Ghp/c**2:
```

```
  L1 := 1 + Lk_on_Lg + (1 + Lk0_on_Lg) * sqrt(8*Rp/x):
  L2 := 1 + Lk_on_Lg:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
                                            # Newton corrected law.
  ann := Gh / x**2 :         # Classical "Newton's acceleration".
  x := x0:

  if(evalf(arr) > evalf(ann)) then
    Ghpmax := Ghp:
    Ghp := (Ghpmin+Ghp)/2:
  else
    Ghpmin := Ghp:
    Ghp := (Ghpmax+Ghp)/2:
  end if:
end do:
Ghp_iter := Ghp:

# ------------------------------------------------------------- #
# ----- Printing the results ----------------------------------- #
# ------------------------------------------------------------- #
GhpApprox:
Ghp_3emedegree:
Ghp_iter:               # Finally this is the most precise value.


#################################################################
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################

unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

Rp := Ghp/c**2:
L1 := 1 + Lk_on_Lg + (1 + Lk0_on_Lg) * sqrt(8*Rp/x):
L2 := 1 + Lk_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:     # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :      # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                                   # Our "Newton's acceleration".
an := Gh/x**2 :              # Classical "Newton's acceleration".

Da := ar - an:                     # Theoretical PIONEER anomaly.

#################################################################
#        Plotting the theoretical Pioneer anomaly curve         #
#################################################################

# Plotting Da
x := AU * y:
```

```
plot( Da, y=x0/AU..50 );
```

APPENDIX 4 APPROXIMATING WITH SYMMETRIC CONTRIBUTIONS

As it has been showed above, the Kuiper's belt must be taken into account in the calculation of the relativistic operator. This contribution, as usual, will be considered as being the sum of its symmetric and asymmetric parts. Let's remind that the symmetric contribution is named L2, and the asymmetric is L1-L2, where L1 is the space-time deformation value coming from the left, and L2 the space-time deformation value coming from the right. In this notation it is supposed L1>L2 because the pin pointed mass gravitational object is supposed located on the left. We suppose that the probe is moving from the O point (the Earth) along the Ox axis, on the right.

It will be supposed that the probe is located on the ecliptic plane before the Kuiper belt.
We will try to prove that, in this case, the symmetric contributions coming from the Kuiper belt are equal to the contributions coming from the right.
In other words, that is to say that L1>L2. Therefore, the contributions coming from the left are only driving the (remaining) asymmetric part of the Kuiper belt contribution (L1-L2). But this asymmetric contribution is driving only the gravitational attractive force coming from this left part of the Kuiper belt.

For this proof, we just redo the calculation of this document, but taking into account the beginning of the Kuiper belt. The resulting equation is the following.

$$Lk\_on\_Lg\_max = \sqrt{[rk^2 - rk'^2]}/rg\_last \quad \sqrt{[\rho k/\rho 0]}$$

This equation above is equation (18). rk is the ending distance of the Kuiper belt, and rk' is the beginning distance of the Kuiper belt.
If we apply this equation with x different from 0 (x being the distance of the probe from the sun), we get:

$$L1 = \sqrt{[(x+rk)^2 - (x+rk')^2]}/rg\_last \quad \sqrt{[\rho k/\rho 0]}$$
$$L2 = \sqrt{[(x-rk)^2 - (x-rk')^2]}/rg\_last \quad \sqrt{[\rho k/\rho 0]}$$

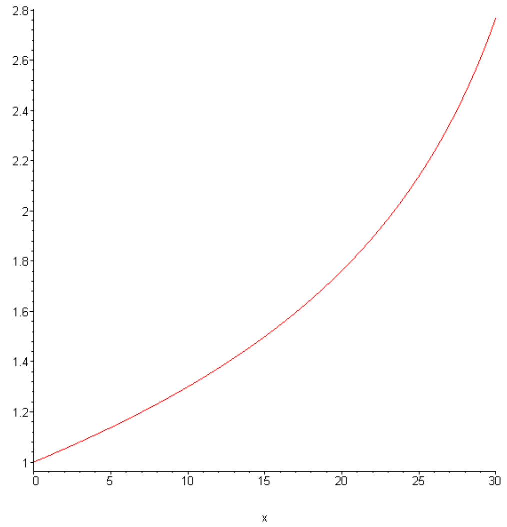
Where L1 is the contribution of the Kuiper belt coming from the left, and L2 coming from the right.
Now if we compare L1 with L2 we get:

$$L1/L2 = \sqrt{[(A + Bx) / (A - Bx)]}$$

with $A = rk^2 - rk'^2$, and $B = 2(rk - rk')$

Now if we plot this curve we get the following result.

```
xpk := 30;                          # Beginning of the Kuiper belt, in AU.
xk := 48;                           # Ending of the Kuiper belt, in AU.
A := xk**2 - xpk**2;    .
B := 2 * (xk - xpk);
L1_on_L2 := sqrt((A + B*x) / (A - B*x));    # Respective value of L1 from L2.
plot( L1_on_L2, x=0..xpk );         # Plotting until the beginning of the belt.
```



It is noticed that L1 is always greater than L2. Hence, we conclude that the symmetric contribution is always equal to L2. The asymmetric contribution L1-L2 is yielding only a gravitational attractive effect. It is noticed also that L1/L2 is weaker than 2.8.

It is supposed that the probe is located on the ecliptic plane before the Kuiper belt.
It has been proven that, in this case, the symmetric contribution coming from the Kuiper belt is equal to the contribution coming from the right.

APPENDIX 5 CALCULATION OF THE OORT CLOUD CONTRIBUTION

This appendix will try to show that the OORT cloud (symmetric) contribution is very weak in front the galactic symmetric contribution.

The following equation will be used as usual.

Lk_on_Lg_max	=	$\sqrt{[rk^2 - rk'^2]}$/rg_last $\sqrt{[\rho k/\rho 0]}$

Here are the MAPLE lines of code which must be added to the MAPLE program of appendix 3, and which are calculating this contribution ratio for the OORT cloud.

```
xomin := 3000 * AU:                          # OORT min distance from the sun. m.
xomax := 5000 * AU:                          # OORT max distance from the sun.
Mo := 5 * Mt:                                # OORT cloud mass value. kg.
Vo := (4/3) * Pi * (xomax**3 - xomin**3):    # OORT cloud volume. m3.
rhoo := evalf(Mo/Vo):                        # OORT cloud matter density. kg/m3.
Lo_on_Lg_max := (sqrt(xomax**2 - xomin**2)/rg_last) * sqrt(rhoo/rho0):
evalf(Lo_on_Lg_max);
```

And here is the result:

Loort / Lg = 2.5 10(-4)**

Let's remind the ratio which is used in the calculation of the relativistic operator :

Lkuiper / (Lg + Loort)

Now using the estimation above we can write:

Lkuiper / (Lg + Loort) $\cong$ Lkuiper / Lg

And the error of this approximation is roughly 0,02 %.
It is this approximation which has been used in this document, noticeably in the program of appendix 3.

APPENDIX 6 EARTH FLYBY ANOMALIES

I

```
#################################################################
#                                                               #
#             CALCULATION OF EARTH FLY BY ANOMALY               #
#                                                               #
# This MAPLE program is calculating the Earth flyby anomaly.    #
#                                                               #
#################################################################

#################################################################
######################### Initialisations #######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                        # Floating points precision.
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

# ---------------- Cosmological values --------------------- #
km := 10**3:                                                 # m.
Mkm := 1000 * km:
c := 3 * 10**8:                              # light speed in m/s.
AU := 149597870 * km :                  # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).

# ---------------- Solar system values --------------------- #
Mt := 5.97 * 10**24 :                         # Earth mass in Kg.
Gt := G * Mt:
rt := 6378 * km:                                     # Earth ray.
xl := 384403 * km:                         # Moon Earth distance.
xsat := 25000 * km:  # Half great axis of artificial  satellite.

xGalI := rt + 956 * km:  # Min altitude of Galileo I near Earth.
v_inf_GalI := 8949:        # Galileo I
xGalII := rt + 303 * km:
v_inf_GalII := 8877:
xNEAR := rt + 532 * km:
v_inf_NEAR := 6851:
xCass := rt + 1172 * km:
v_inf_Cass := 16010:
xRosI := rt + 1954 * km:
v_inf_RosI := 3863:
xMess := rt + 2336 * km:
v_inf_Mess := 4056:

x0 := 16200 * km:
# Fitted value for Galilleo I
```

```
#################################################################
# Calculation of Gtp, the Earth gravitational constant          #
# valid outside the Earth location.                             #
#################################################################
# ----- Calculation of Gtp, with 3rd method: iteration -------- #

Gtpmin := Gt/100:
Gtpmax := 100*Gt:
Gtp := Gtpmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
  unassign('Da'):

  Rp := Gtp/c**2:
  L1 := 1 + sqrt(8*Rp/x):
  L2 := 1:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
                                          # Newton corrected law.
  ann := Gt / x**2 :         # Classical "Newton's acceleration".
  x := x0:

  if(evalf(arr) > evalf(ann)) then
    Gtpmax := Gtp:
    Gtp := (Gtpmin+Gtp)/2:
  else
    Gtpmin := Gtp:
    Gtp := (Gtpmax+Gtp)/2:
  end if:
end do:

#################################################################
#      Calculation of the corrected gravitational force         #
#################################################################

unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

Rp := Gtp/c**2:
L1 := 1 + sqrt(8*Rp/x):
L2 := 1:
cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                               # Our "Newton's acceleration".
an := Gt/x**2 :            # Classical "Newton's acceleration".

#################################################################
#                Added velocity at perigee                      #
#################################################################
```

```
print("---------- Galilleo I ----------");
DvReal := 0.0025;
xPerigee := xGalI:
v_inf := v_inf_GalI:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;

x := xPerigee:
Dv2 := (Gt**(3/2) / (c*vn*x)) * sqrt(2) * (3/sqrt(x0) - 2/sqrt(x) -
9* Gt**(3/2)/(vn**2*c*x*x0)) :
evalf((Dv-Dv2)/Dv);

#################################################################
print("---------- Galilleo II ----------");
DvReal := -0.002;
xPerigee := xGalII:
v_inf := v_inf_GalII:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;

x := xPerigee:
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9* Gt**(3/2)/(vn**2 * c * x * x0)) :
evalf((Dv-Dv2)/Dv);

#################################################################
print("---------- NEAR ----------");
DvReal := 0.0072;
xPerigee := xNEAR:
v_inf := v_inf_NEAR:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;

x := xPerigee:
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9* Gt**(3/2)/(vn**2 * c * x * x0)) :
```

```
evalf((Dv-Dv2)/Dv);

##################################################################
print("---------- Cassini ----------");
DvReal := -0.0017;
xPerigee := xCass:
v_inf := v_inf_Cass:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;

x := xPerigee:
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9* Gt**(3/2)/(vn**2 * c * x * x0)) :
evalf((Dv-Dv2)/Dv);

##################################################################
print("---------- Rosetta I ----------");
DvReal := 0.00067;
xPerigee := xRosI:
v_inf := v_inf_RosI:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;

x := xPerigee:
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9* Gt**(3/2)/(vn**2 * c * x * x0)) :
evalf((Dv-Dv2)/Dv);

##################################################################
print("---------- Messenger ----------");
DvReal := 0.000008;
xPerigee := xMess:
v_inf := v_inf_Mess:
unassign('x'):
x_inf := 1 * AU:
Epn := - evalf(Int( -an, x = x_inf.. xPerigee, digits=14 )):
Epr := - evalf(Int( -ar, x = x_inf.. xPerigee, digits=14 )):
vn := sqrt(v_inf**2 - 2*Epn):
vr := sqrt(v_inf**2 - 2*Epr):
Dv := vr - vn ;
```

```
x := xPerigee:
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9* Gt**(3/2)/(vn**2 * c * x * x0)) :
evalf((Dv-Dv2)/Dv);

unassign('x') :
Dv2 := (Gt**(3/2) / (c * vn * x)) * sqrt(2) * (3/sqrt(x0) -
2/sqrt(x) - 9*Gt**(3/2)/(vn**2 * c * x * x0)) :
plot(Dv2, x=rt..2*x0) ;
```

APPENDIX 7 SATURN FLYBY ANOMALY

```
#################################################################
#                                                               #
#         CALCULATION OF PIONEER SATURN FLYBY ANOMALY           #
#                                                               #
# This MAPLE program is calculating and plotting the curve of   #
# the anomaly of Saturn flyby during PIONEER 11 travel.         #
#                                                               #
#################################################################

#################################################################
####################### Initialisations #########################
#################################################################

# ----------------------------- MAPLE ------------------------ #
Digits := 50 :                       # Floating points precision.
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

# ------------------------- Units ---------------------------- #
s := 1:                                                    # s.
day := 24*3600:                                       # in sec.
gram := 10**(-3):                                         # kg.
cm := 10**(-2):                                            # m.
km := 10**3:                                               # m.
eps := 10**(-40):

# ----------------- Cosmological values --------------------- #
kpc := 3.08 * 10**19 :       # 1 kilo parsec expressed in meter.
pc := kpc/1000 :                  # 1 parsec expressed in meter.
mpc := 10**3 * kpc:                                        # m.
c := 3 * 10**8:                          # light speed in m/s.
AL := 365 * 24 * 3600 * c:              # 1 year light in meter.
AU := 149597870 * km :                 # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).
Gp := 10 * G:        # Gravitational constant outside any galaxy.
rg := 473 * 10**18:                    # Ray of the galaxy in m.
rg_last := 3.5 * 10**(-3) * 473 * 10**18:       # Fitted value.
                 # Distance of last galaxy's contribution. in m.

# ---------------- Solar system values ---------------------- #
# heliocentric gravitational constant inside the solar system.
Gh := (0.01720209895)**2 * AU**3 / day**2:
Mt := 5.97 * 10**24:                     # Earth mass in Kg.
xkmin := 30 * AU:        # Kuiper min distance from the sun. m.
xkmax := 48 * AU:           # Kuiper max distance from the sun.
xkmean := (xkmin+xkmax)/2:   # Kuiper mean distance from the sun.
thetak := Pi*20/180:                 # Kuiper thickness angle.
xkthi := xkmean * thetak:                  # Kuiper thickness.
Mk := (Mt/10):                  # Kuiper belt matter max value.
```

```
Vk := Pi*(xkmax**2 - xkmin**2) * xkthi: # Kuiper belt volume m3.
M0 := 2 * 10**30 :                               # Sun mass in Kg.
Ms := 2 * 10**27 :                            # Saturn mass in Kg.
Rs := Ms*G/c**2:                   # Schwartzchild ray for Saturn.
Gs := Ms*G:                   # Gravitational constant for Saturn.
xs := 1429 * 10**9:                 # Distance Saturn sun in m.
rs := 60268 * km:                                # Ray of Saturn.
ls := rs + 22000 * km:   # Dist Saturn center to Pioneer trajec.
R0 := M0 * G / c**2:           # Half Schwarschild ray for SUN.
rho0 := 0.003 * M0 / (AL**3): #Matter density near solar system.
rhok := evalf(Mk/Vk):        # Kuiper belt matter density kg/m3.

# ----------------- Luminous point's values ------------------ #
mp := me:           # Luminous point's mass = electron's mass !?
ep := mp * c**2:                # Luminous point energy at rest.
dk := (mp/rhok)**(1/3):  # Dist between 2 lum pts in Kuiper belt.
dg := (mp/rhog)**(1/3):  # Dist between 2 lum pts in the galaxy.
du := (mp/rhou)**(1/3):    # Dist between 2 lum pts in universe.

#################################################################
########### Calculation of Lk, the symmetric contribution #######
#################################################################

# ------------------ Calculation of fmax --------------------- #
Si_So := sqrt(Gp/G) - 1: # Intra gal/extra-galactic contributions
r := rho0 * Si_So**(-2): # Square of extragalactic contributions.
                  # proportional to the square of extra-galactic
                   # lights contributions. Linked to rho0 value.
                                            # Unit : Kg m(-3).
Lu_on_Lg := sqrt(r/rho0):   # Extra/intra galactic contributions.
Lk_on_Lg_max := (xkmax/rg_last) * sqrt(rhok/rho0):

# ------------------ Calculation of Lk ----------------------- #
xKuiper := 125 * AU:  # Fitted value. Greater than xkmax = 48 AU.
Lk_on_Lg := Lk_on_Lg_max * invfunc[tan]((2/Pi) * xKuiper/x)
                                                       / (Pi/2):
x := x0:      # Lk0_on_Lg : value of Lk where Gh is correct value
Lk0_on_Lg := Lk_on_Lg:
unassign('x'):

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################
x0 := 1 * xs: # This is the distance from the sun at which Gh the
              # heliocentric gravitational constant is calculated.
              # This value is seen on the curve of the Pioneer
              # measured anomaly (experimental curve).
              # It is very near to x0 = xs, the distance of Saturn
              # from the sun, as seen from the experimental curve.
              # The best value for x0 is x0 = 0.95 * xs,
```

```
              # (in place of x0 = xs). This last value yields the
              # value of 8.25 10**(-10) m/s².

Ghpmin := Gh/100:
Ghpmax := 100*Gh:
Ghp := Ghpmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('z'):
  unassign('ar'): unassign('an'): unassign('Da'):

  # -------------- Sun attraction force --------------------- #
  Rp := Ghp/c**2:
  Lu0 := 1 + Lu_on_Lg + Lk0_on_Lg:
  L1 := 1 + Lu_on_Lg + Lk_on_Lg +  Lu0 * sqrt(8*Rp/x):
  L2 := 1 + Lu_on_Lg + Lk_on_Lg:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):

  ann := Gh/x**2:                         # Newton's acceleration.

  # --------------- Comparison of acceleration forces  -------- #
  x := x0:            # Comparing the forces for the x0 distance.

  if(evalf(arr) > evalf(ann)) then
    Ghpmax := Ghp:
    Ghp := (Ghpmin+Ghp)/2:
  else
    Ghpmin := Ghp:
    Ghp := (Ghpmax+Ghp)/2:
  end if:
end do:


#################################################################
# Calculation of Ghs, the Saturn gravitational constant   #
# valid outside the solar system.                               #
#################################################################
x0s := 2*AU:            # Fitted value, in order to get a correct
                                        # Pioneer anomaly curve.
Gspmin := Gs/100:
Gspmax := 100*Gs:
Gsp := Gspmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('z'):
  unassign('ar'): unassign('an'): unassign('Da'):

  Rs := Gsp/c**2:
  L1s := 1 + sqrt(8*Rs/x):
  L2s := 1:
```

```
  cos2:= 4*L1s*L2s/(L1s+L2s)**2:   # Square of relativistic
operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):

  ann := Gs/x**2:                            # Newton's acceleration.

  # --------------- Comparison of acceleration forces  -------- #
  x := x0s:            # Comparing the forces for the x0 distance.

  if(evalf(arr) > evalf(ann)) then
    Gspmax := Gsp:
    Gsp := (Gspmin+Gsp)/2:
  else
    Gspmin := Gsp:
    Gsp := (Gspmax+Gsp)/2:
  end if:
end do:


#################################################################
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################

unassign('x'): unassign('y'): unassign('z'):
unassign('ar'): unassign('an'): unassign('Da'):

# --------------- Sun attraction force ---------------------- #
Rp := Ghp/c**2:
Lu0 := 1 + Lu_on_Lg + Lk0_on_Lg:
L1 := 1 + Lu_on_Lg + Lk_on_Lg + Lu0 * sqrt(8*Rp/x):
L2 := 1 + Lu_on_Lg + Lk_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):

# --------------- Saturn attraction force ------------------- #
Rsp := Gsp/c**2:
L1s := 1 + sqrt(8*Rsp/y):
L2s := 1:
cos2:= 4*L1s*L2s/(L1s+L2s)**2:      # Square of relat operator.
tg:= sqrt(1/cos2 - 1) :     # Slope of space inside space-time.
arrs :=  - c**2 * diff(tg,y) * tg * (1 - tg**2)**(-3/2):
y := sqrt((x-xs)**2 + ls**2):
coss := (x-xs)/y:

# --------------- Total attraction forces ------------------- #
arr := arr + coss*arrs:              # Corrected acceleration.
ann := Gh/x**2 + coss*Gs/y**2 :        # Newton's acceleration.

# --------------- Comparison of acceleration forces  -------- #
```

```
Da := arr - ann:                   # Theoretical PIONEER anomaly.

#################################################################
#        Plotting the theoretical Pioneer anomaly curve         #
#################################################################

# Plotting Da
x := AU * t:
plot( Da, t=(0.9998)*xs/AU..(1.0002)*xs/AU );
plot( Da, t=9.8..30 );
```

APPENDIX 8 PERIHELION ADVANCE

```
####################################################################
#                                                                  #
#              CALCULATION OF PERIHELION ADVANCE                   #
#                                                                  #
# This MAPLE program is calculating the perihelion advance for     #
# some planets in the solar system, in the context of the three    #
# elements theory.                                                 #
#                                                                  #
####################################################################

####################################################################
######################### Initialisations ##########################
####################################################################

# ---------------------------- MAPLE -------------------------- #
restart:
Digits := 50 :                      # Floating points precision.
DigInt := 12:
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

# ----------------- Cosmological values ---------------------- #
s := 1:                                                      # s.
day := 24*3600:                                         # in sec.
gram := 10**(-3):                                           # kg.
cm := 10**(-2):                                              # m.
km := 10**3:                                                 # m.
kpc := 3.08 * 10**19 :        # 1 kilo parsec expressed in meter.
pc := kpc/1000 :                   # 1 parsec expressed in meter.
mpc := 10**3 * kpc:                                          # m.
c := 3 * 10**8:                            # light speed in m/s.
AL := 365 * 24 * 3600 * c:              # 1 year light in meter.
AU := 149597870 * km :                 # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).
Gp := 10 * G:        # Gravitational constant outside any galaxy.
rg := 473 * 10**18:                    # Ray of the galaxy in m.
rg_last := 0.6 * 10**(-2) * 473 * 10**18:         # Fitted value.
                 # Distance of last galaxy's contribution. in m.

# ----------------- Solar system values ---------------------- #
Mt := 5.97 * 10**24:                        # Earth mass in Kg.
M0 := 2 * 10**30 :                           # Sun mass in Kg.
R0 := M0 * G / c**2:           # Half Schwarschild ray for SUN.
xs := 1429 * 10**9:                # Distance Saturn sun in m.
rho0 := 0.003 * M0 / (AL**3): #Matter density near solar system.
# Heliocentric gravitational constant inside the solar system:
Gh := (0.01720209895)**2 * AU**3 / day**2:

# ------------------ Kuiper belt values ---------------------- #
```

```
xkmin := 30 * AU:         # Kuiper min distance from the sun. m.
xkmax := 48 * AU:            # Kuiper max distance from the sun.
xkmean := (xkmin+xkmax)/2:   # Kuiper mean distance from the sun.
thetak := Pi*20/180:                   # Kuiper thickness angle.
xkthi := xkmean * thetak:                    # Kuiper thickness.
xKuiper := 125 * AU:  # Fitted value. Greater than xkmax = 48 AU.
Mk := 0.3 * Mt:                                # Kuiper belt mass
value.
Vk := Pi*(xkmax**2 - xkmin**2) * xkthi: # Kuiper belt volume m3.
rhok := evalf(Mk/Vk):        # Kuiper belt matter density kg/m3.

#################################################################
########### Calculation of Lk, the symmetric contribution #######
#################################################################

# ------------------ Calculation of fmax ---------------------- #
Si_So := sqrt(Gp/G) - 1: # Intra gal/extra-galactic contributions
r := rho0 * Si_So**(-2): # Square of extragalactic contributions.
                  # proportional to the square of extra-galactic
                   # lights contributions. Linked to rho0 value.
                                              # Unit : Kg m(-3).
Lu_on_Lg := sqrt(r/rho0):   # Extra/intra galactic contributions.
Lk_on_Lg_max := (xkmax/rg_last) * sqrt(rhok/rho0):

# ------------------ Calculation of Lk ------------------------ #
Lk_on_Lg := Lk_on_Lg_max * invfunc[tan]((2/Pi) * xKuiper/x)
                                                     / (Pi/2):
x := x0:      # Lk0_on_Lg : value of Lk where Gh is correct value
Lk0_on_Lg := Lk_on_Lg:
unassign('x'):

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################
x0 := 1 * xs: # This is the distance from the sun at which Gh the
              # heliocentric gravitational constant is calculated.
              # This value is seen on the curve of the Pioneer
              # measured anomaly (experimental curve).
              # It is very near to x0 = xs, the distance of Saturn
              # from the sun, as seen from the experimental curve.
              # The best value for x0 is x0 = 0.95 * xs,
              # (in place of x0 = xs). This last value yields the
              # value of 8.25 10**(-10) m/s².

# ----- 3rd method: iteration -------------------------------- #
Ghpmin := Gh/100:
Ghpmax := 100*Gh:
Ghp := Ghpmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
```

```
  unassign('Da'):

  Rp := Ghp/c**2:
  L1 := 1 + Lu_on_Lg + Lk_on_Lg
                    + (1 + Lu_on_Lg + Lk0_on_Lg) * sqrt(8*Rp/x):
  L2 := 1 + Lu_on_Lg + Lk_on_Lg:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
                                          # Newton corrected law.
  ann := Gh / x**2 :         # Classical "Newton's acceleration".
  x := x0:

  if(evalf(arr) > evalf(ann)) then
    Ghpmax := Ghp:
    Ghp := (Ghpmin+Ghp)/2:
  else
    Ghpmin := Ghp:
    Ghp := (Ghpmax+Ghp)/2:
  end if:
end do:


#################################################################
#        Calculation of three elements perihelion advance      #
#################################################################
PerihelionCalcul := proc( planet, w, v )

  local u, M, a0, a1, a2, a3, a4, NumRelat, k, M0, d0, d1, d2, d3,
        d4, d5, d6, x, Num3elt, AdvRelat, AdvRelatArcC, DeltaRad,
        DeltaArcC:

  u := (v+w)/2 + (1/2)*(v-w)*cos(psi):

  ############# Explanation of series calculations ##############
  #restart:
  #du_dphi_2 := (v+w)*(1-2*M*v)*(1-2*M*w)/(2*M) - (1-2*M*u)*(v+w-
2*M*(v**2+v*w+w**2)+2*M*u**2)/(2*M):
  #u := x**2:
  #poly2 := (u-w)*(v-u):
  #PolyRes := du_dphi_2 / poly2 :
  #Res := PolyRes**(-1/2):
  #series(Res, x, 10);

  ######################### Relativity ##########################
  M := Gh/c**2:
  a0 := 1/((1-2*M*v-2*M*w)^(1/2)):
  a1 := 1/(1-2*M*v-2*M*w)^(3/2)*M:
  a2 := 3/2/(1-2*M*v-2*M*w)^(5/2)*M^2:
  a3 := 5/2/(1-2*M*v-2*M*w)^(7/2)*M^3:
  a4 := 35/8/(1-2*M*v-2*M*w)^(9/2)*M^4:
```

```
  NumRelat := a0 + a1*u + a2*u**2 + a3*u**3 + a4*u**4:

  ######################### 3 elts ##########################
  k := 3 * sqrt(2*Ghp)/c:
  M0 := (Ghp/c**2):

  d0 :=  1/((1-2*M0*v-2*M0*w)^(1/2)):

  d1 := -2/3/(1-2*M0*v-2*M0*w)^(3/2)*(v+w)*M0*k:

  d2 := 1/(1-2*M0*v-2*M0*w)^(1/2)*(M0/(1-2*M0*v-
2*M0*w)+2/3*(v+w)^2*M0^2*k^2/(1-2*M0*v-2*M0*w)^2):

  d3 := 1/(1-2*M0*v-2*M0*w)^(1/2)*(-2/3*M0*k/(1-2*M0*v-2*M0*w)-
2*(v+w)*M0^2*k/(1-2*M0*v-2*M0*w)^2-20/27*(v+w)^3*M0^3*k^3/(1-
2*M0*v-2*M0*w)^3):

  d4 := 1/(1-2*M0*v-2*M0*w)^(1/2)*(10/3*(v+w)^2*M0^3*k^2/(1-2*M0*v-
2*M0*w)^3+4/3*(v+w)*M0^2*k^2/(1-2*M0*v-2*M0*w)^2+3/2*M0^2/(1-
2*M0*v-2*M0*w)^2+70/81*(v+w)^4*M0^4*k^4/(1-2*M0*v-2*M0*w)^4):

  d5 := 1/(1-2*M0*v-2*M0*w)^(1/2)*(-140/27*(v+w)^3*M0^4*k^3/(1-
2*M0*v-2*M0*w)^4-20/27*(v+w)^2*M0^3*k^3/(1-2*M0*v-2*M0*w)^3-
10/3*(v+w)*M0^3*k/(1-2*M0*v-2*M0*w)^3-10/9*(4/3*(v+w)*M0^2*k^2/(1-
2*M0*v-2*M0*w)^2+3/2*M0^2/(1-2*M0*v-2*M0*w)^2)*(v+w)*M0*k/(1-
2*M0*v-2*M0*w)-2*M0^2/(1-2*M0*v-2*M0*w)^2*k-
28/27*(v+w)^5*M0^5*k^5/(1-2*M0*v-2*M0*w)^5):

  d6 := 1/(1-2*M0*v-2*M0*w)^(1/2)*(70/81*(v+w)^3*M0^4*k^4/(1-
2*M0*v-2*M0*w)^4+35/6*(v+w)^2*M0^4*k^2/(1-2*M0*v-2*M0*w)^4-7/6*(-
20/27*(v+w)^2*M0^3*k^3/(1-2*M0*v-2*M0*w)^3-10/3*(v+w)*M0^3*k/(1-
2*M0*v-2*M0*w)^3-10/9*(4/3*(v+w)*M0^2*k^2/(1-2*M0*v-
2*M0*w)^2+3/2*M0^2/(1-2*M0*v-2*M0*w)^2)*(v+w)*M0*k/(1-2*M0*v-
2*M0*w))*(v+w)*M0*k/(1-2*M0*v-2*M0*w)+40/9*(v+w)*M0^3*k^2/(1-
2*M0*v-2*M0*w)^3+5/3*(4/3*(v+w)*M0^2*k^2/(1-2*M0*v-
2*M0*w)^2+3/2*M0^2/(1-2*M0*v-2*M0*w)^2)*M0/(1-2*M0*v-
2*M0*w)+2/3*M0^2*k^2/(1-2*M0*v-2*M0*w)^2+70/9*(v+w)^4*M0^5*k^4/(1-
2*M0*v-2*M0*w)^5+308/243*(v+w)^6*M0^6*k^6/(1-2*M0*v-2*M0*w)^6):

  x := u**(1/2):
  Num3elt := d0 + d1*x + d2*x**2 + d3*x**3 + d4*x**4 + d5*x**5 +
d6*x**6:

  ######################### Perihelion advance ###################
  AdvRelat := evalf(Int(NumRelat, psi = 0..2*Pi, digits=30 ) -
2*Pi);
  AdvRelatArcC := evalf(AdvRelat * (0.103/(5*10**(-7))) * 415);
  DeltaRad := evalf(Int((Num3elt - NumRelat), psi = 0..2*Pi,
digits=15 )):
  DeltaArcC := evalf(DeltaRad * (0.103/(5*10**(-7))) * 415):
```

```
  print(planet);
  print("Advance Relat/Newton");
  print(AdvRelatArcC);
  print("Advance 3elts/Relat");
  print(DeltaArcC);

end proc:


########################### Mercure #############################
a := 5.8 * 10**(12) * cm:
e := 0.206:
p := a * (1 - e**2):
v := (1+e)/p:
w := (1-e)/p:
PerihelionCalcul("Mercure", w, v);

########################### Saturn #############################
rp := 1.51450 * 10**9 * km:
rm := 1.35255 * 10**9 * km:
w := 1/rp:
v := 1/rm:
print("\n");
PerihelionCalcul("Saturn", w, v);
```

APPENDIX 9 SIDERAL GRAVITY CALCULATION

```
#------------------------------------------------------------
# MAPLE program
# This program is estimating the sideral gravity
# in the context of the three elements theory.
#------------------------------------------------------------
# INIT
#------------------------------------------------------------
# DISTANCES
c := 3 * 10**8 :
km := 1000 :
pc := 3.08568025 * 10**16 :
kpc := 1000*pc:
AU := 149598000 * km :
YL := c * 365 * 24 * 3600:
G := 1 :

# SUN
M0 := 1.9891 * 10**30 :
r0 := 1392000 * km :

# GALAXY
rg_last := 4.257 * 10**18 :  # Maximum active ray in the
                             # galaxy. Empirical value.
                             # Fitted with Pioneer anomaly.
rhog := 5.7 * 10**(-22):     # Matter density near solarsys.
xs := 8*kpc:                 # Sun galactic center distance.

#----------------------------------------------------------
# Calculating the solid angle used by the balance.
#----------------------------------------------------------

# 1) Stars locally.          # There are NbStarsLocal stars
NbStarsLocal := 65:          # inside a sphere which ray
RayStarsLocal := 5 * pc:     # equal to RayStarsLocal.
                             # Wikipedia Internet source:
#http://fr.wikipedia.org/wiki/Liste_d%27%C3%A9toiles_proches

# 2) Calculation solid angle for the attracting lines of
# the balance.
DistMass := (23.4 + 2*10)/2:
RayMass := (23.4 / 2.5)/2:
AngleBalance := invfunc[tan] ( RayMass / DistMass ):
SolidAngleBalance := 2 * %pi * (1 - cos(AngleBalance)):
                # Solid angle calculated below in comments.

#----------------------------------------------------------
# Calculating the relative amplitude for contributions.
#----------------------------------------------------------
```

```
ContribStarCalculation := proc( r )
  local Lu, Lg, alpha, rg_last_real, SolidAngleMax,
        SolidAngleByStar, NbStars, RelatAmpli, RelatAmpliG:

  # 3) Calculation of rg_last_real.
  Lu := 1:                      # Extragalactic contribution.
  Lg := r/xs:                   # Intragalactic contribution.
  if(r=0)          # Meaning Extragalactic contribution = 0.
    then alpha := 0:
    else alpha := Lu/Lg:
  end if:  # Here alpha always = extragalcont/intragalcont.
  rg_last_real := rg_last/(1+alpha): # Ray proport to contr
                      # see parag “The real rg_last value”.
  # 4) Calculation solid angle for one star.
  NbStars := NbStarsLocal * (rg_last_real / RayStarsLocal)**3:
  SolidAngleMax := 4 * %pi :      # 4 pi = 2 pi (1-cos(pi)).
  SolidAngleByStar := SolidAngleMax / NbStars:

  # 5) Calculation relative amplitude of contributions
  # signal when Earth is rotating
  # around the fixed system of stars.
  RelatAmpli := (1/(1+alpha)) * SolidAngleByStar /
                                           SolidAngleBalance:
  # 6) Calculation relative amplitude of G.
  RelatAmpliG := 2 * RelatAmpli;          # To be compared
                                          # with 0.054 %
  #-----------------------------------------------------------
  # Theoretical appendix.
  #-----------------------------------------------------------
  # Calculation of function SolidAngle (SolidA) as a function
  # of the angle (theta)
  #SolidA := sum of 0 to theta de (2 pi sin(r) dr)
  #SolidA := sum of 0 to theta de (2 pi L² t dt)
  #SolidA := 2 pi sum of 0 to theta of (sin(t) dt)     # L:=1
  #SolidA := 2 pi  [- cos(t)] 0 to theta
  #SolidA := 2 pi  (1 - cos(theta))

end proc:


print("0 kpc");
r := 0:   # Around 1 kpc ? Distance at which Lu=Lg....
          # Difficult to estimate!
print("Ratio extragal/gal");
print("0");
print("Relative G error:");
ContribStarCalculation( r );
print("\n");

print("0.5 kpc");
r := 0.5*kpc:
```

```
print("Ratio extragal/gal");
print(xs/r);
print("Relative G error:");
ContribStarCalculation( r );
print("\n");

print("1 kpc");
r := 1*kpc:
print("Ratio extragal/gal");
print(xs/r);
print("Relative G error:");
ContribStarCalculation( r );
print("\n");

print("2 kpc");
r := 2*kpc:
print("Ratio extragal/gal");
print(xs/r);
print("Relative G error:");
ContribStarCalculation( r );
print("\n");

print("4 kpc");
r := 4*kpc:
print("Ratio extragal/gal");
print(xs/r);
print("Relative G error:");
ContribStarCalculation( r );
print("\n");

#-----------------------------------------------------------
# Printing speed profiles corresponding to r=1kpc and r=4kpc.
#-----------------------------------------------------------
r:= 1 * kpc:
unassign('x'): Digits:= 30: m:=1: M:= 7 * 10**36: G:= 6.6742867 *
10**(-11): c:= 3 * 10**8:
R:= M*G/c**2: kpc:= 3.08  * 10**19:     # Initialisations.
e:= sqrt(8*R/x) / (1 + r/x) :  # "relativistic coefficient".
cos2:= (1 + e) / ((1 + e/2)**2) :   # Square of relat operator.
tg:= sqrt( 1/cos2  -   1 )  :             # Slope of space inside
st.
F:= - m * c**2 * diff(tg,x) * tg * (1  -   tg**2)**(-3/2):
FN:= m * c**2 * R/(x**2) :       # Classical "Newton's law".
v1kpc:= sqrt( F * x / m ) :       # Tangential speed v.
vn:= sqrt( FN * x / m ) :         # Classical Newton's law.

r:= 4 * kpc:
unassign('x'): Digits:= 30: m:=1: M:= 7 * 10**36: G:= 6.6742867 *
10**(-11): c:= 3 * 10**8:
R:= M*G/c**2: kpc:= 3.08  * 10**19:
e:= sqrt(8*R/x) / (1 + r/x) :
```

```
cos2:= (1 + e) / ((1 + e/2)**2) :
tg:= sqrt( 1/cos2  -   1 )  :
F:= - m * c**2 * diff(tg,x) * tg * (1  -   tg**2)**(-3/2) :
FN:= m * c**2 * R/(x**2) :
v4kpc:= sqrt( F * x / m ) :
x := y*kpc:
plot([v1kpc, v4kpc, vn], y=1..15, color=[red,green,blue],
style=[line,line], numpoints=1000, title="Speed profiles",
legend=["1 kpc", "4 kpc", "Newton'law"], labels=["Distance from the
galactic center in kpc", "Speed in km/sec"]);
```

APPENDIX 10 KUIPER CONTRIBUTION

Here are the calculations done for construction of the equation of the symmetric contribution coming from the Kuiper belt.

$Lk/Lg(x) = \sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[(rk - x)^2 - (rk' - x)^2]}$

$d\, Lk/Lg(x) / dx = \sqrt{[\rho k/\rho g]} /rg \quad (½)\, [(rk - x)^2 - (rk' - x)^2]^{**(-½)}$
$[\, -2(rk - x) + 2(rk' - x)\,]$

$d\, Lk/Lg(x) / dx = \sqrt{[\rho k/\rho g]} /rg \quad [(rk - x)^2 - (rk' - x)^2]^{**(-½)} \;\; (rk' - rk)$

$d\, Lk/Lg(x) / dx = ((rk' - rk)/rg) \sqrt{[\rho k/\rho g]} \;/\; \sqrt{[(rk - x)^2 - (rk' - x)^2]}$

$d\, Lk/Lg/dx \text{ in } x=0 = ((rk' - rk)/rg) \sqrt{[\rho k/\rho g]} \;/\; \sqrt{[rk^2 - rk'^2]}$

$d\, Lk/Lg/dx \text{ in } x=0 = ((rk' - rk)/rg) \sqrt{[\rho k/\rho g]} \;/\; \sqrt{[(rk - rk')\, (rk + rk')]}$

$d\, Lk/Lg/dx \text{ in } x=0 = - \sqrt{[\rho k/\rho g]} \;\; \sqrt{[(rk - rk')/(rk + rk')]} \;/\; rg$

$f = A - B\, x$

with:

$A = \sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[rk^2 - rk'^2]}$

$B = (\sqrt{[\rho k/\rho g]} \;\; \sqrt{[(rk - rk')/(rk + rk')]} \;/\; rg\,)$

f = 0 for:

$\sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[rk^2 - rk'^2]} \; - \; (\sqrt{[\rho k/\rho g]} \;\; \sqrt{[(rk - rk')/(rk + rk')]} \;/\; rg\,) \;\; x = 0$

$x = \sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[rk^2 - rk'^2]} \;\; / \;\; (\sqrt{[\rho k/\rho g]} \;\; \sqrt{[(rk - rk')/(rk + rk')]} \;/\; rg\,)$

$x = \sqrt{[rk^2 - rk'^2]} \;\; / \;\; \sqrt{[(rk - rk')/(rk + rk')]}$

$x = \sqrt{[\, (rk - rk')(rk + rk') \;/\; ((rk - rk')/(rk + rk'))\,]}$

$x = \sqrt{[\, (rk + rk') \;/\; (1/(rk + rk'))\,]}$

$x = \sqrt{[\, (rk + rk')^2]}$

$x = rk + rk'$

$h = (2/Pi)\, hmax\, \arctan(\, (2/Pi)\, xKuiper/x\,)$

$hmax = \sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[rk^2 - rk'^2]}$

For x=0 ?

$h = (2/Pi)hmax\, \arctan((2/Pi)xKuiper/x)$

$(Pi/2)\, h/hmax = \arctan((2/Pi)xKuiper/x)$

$\tan(\, (Pi/2)\, h/hmax\,) = (2/Pi)xKuiper/x$

$\tan(\, (Pi/2)\, h/hmax\,) = \text{infinite}$

$(Pi/2)\, h/hmax\,) = Pi/2$

$h = hmax$

dh/dx? => For x=0 ?

$dh/dx = (2/Pi)hmax \;\; 1/(1 + ((2/Pi)xKuiper/x)^2)\, (2/Pi)\, xKuiper\, (-1/x^2)$

$dh/dx = (2/Pi)hmax(2/Pi)xKuiper(-1/x^2)/\, (1 + ((2/Pi)xKuiper/x)^2)$

$dh/dx = -\, (2/Pi)^2 hmax\, xKuiper \;/\; (x^2 + ((2/Pi)xKuiper)^2)$

$dh/dx \text{ en } 0 = -\, (2/Pi)^2 hmax\, xKuiper \;/((2/Pi)xKuiper)^2$

$dh/dx \text{ en } 0 = -\, hmax\, xKuiper \;/xKuiper^2$

$dh/dx \text{ en } 0 = -\, hmax \;/\; xKuiper$

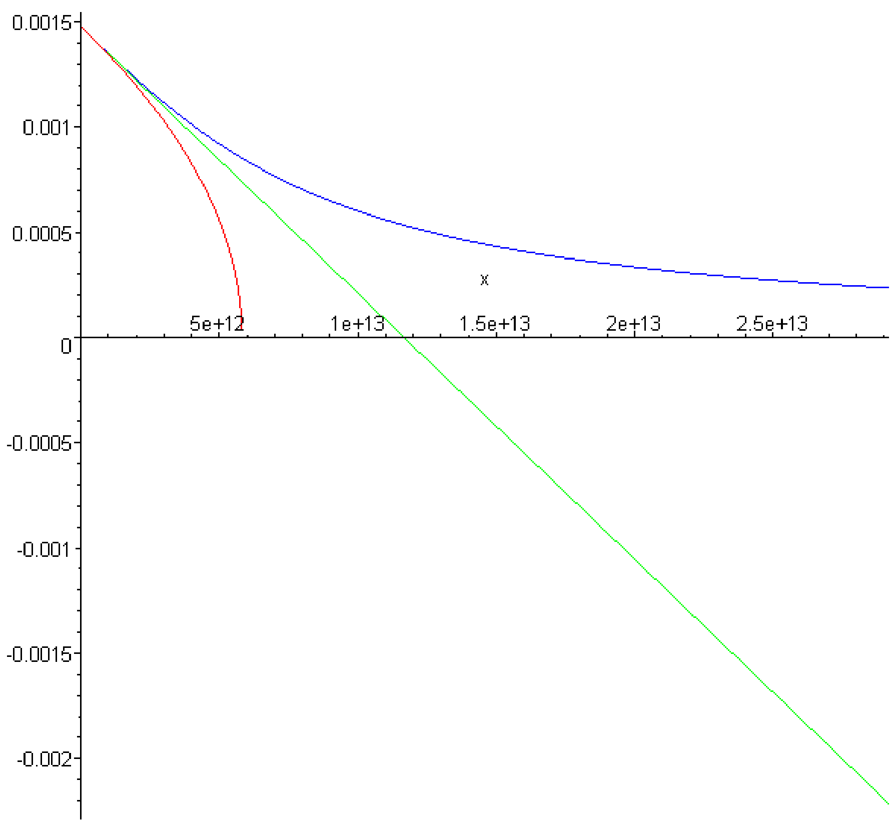
$dh/dx \text{ en } 0 = -\, \sqrt{[\rho k/\rho g]}/rg \quad \sqrt{[rk^2 - rk'^2]} \;\; / \;\; (rk + rk')$

dh/dx en 0 = - √[ρk/ρg]/rg √[(rk - rk')/(rk + rk')]
dh/dx en 0 = - B

Below are the lines of codes which must be added at the end of the program of appendix 3 in order to see the curve of the Kuiper belt contribution, as well as other curves such as the tangent of this curve for x=0.

```
unassign('x'):
rkp := xkmin:
rk := xkmax:
rhog := rho0:
f := (sqrt(rhok/rhog) / rg_last)   * sqrt((rk-x)**2 - (rkp-x)**2):
A := (sqrt(rhok/rhog) / rg_last)   *  sqrt(rk**2 - rkp**2):
B := (sqrt(rhok/rhog) / rg_last)   *  sqrt((rk-rkp)/(rk+rkp)):
d := A - B*x:
xKuiper :=  xkmin + xkmax:
Lk_on_Lg := Lk_on_Lg_max * (2/Pi) * invfunc[tan]((2/Pi) * xKuiper/x):
plot([f,d,Lk_on_Lg],x=0..5*(rk+rkp)/2, color=[red,green,blue]);
```

Below are the resulting curves. In red is the equation (15). In green is the tangent of this curve for x=0. In blue is the final Kuiper belt contribution, which share the same tangent in x=0 with the red and the green curve. The contribution for the Kuiper belt which is used in the program of appendix 3 is the same except for the value xKuiper, which is not equal to xkmin+xkmax = 78 AU, like here, but which is equal to 114 AU, which is a fitted value for taking into account the scattered disk and other transneptunian objects.



APPENDIX 11 GALAXIES SPEED PROFILES: SOLVING THE SIGN ISSUE

```
#################################################################
#                                                               #
#        CALCULATION OF THE VELOCITY CURVES OF NGC3310          #
#         NGC1068 NGC157 NGC7541 AND NGC7331 GALAXIES           #
#         TAKING INTO ACCOUNT AN OCCULTATION MECHANISM          #
#                                                               #
# This MAPLE program is calculating and plotting the curve of   #
# rotation velocity of the galaxy stars, as a function of       #
# their distance x from the galactic center.                    #
#                                                               #
# This calculation is done for the NGC3310 NGC1068              #
# NGC157 NGC7541 and NGC7331 galaxies.                          #
#                                                               #
# It is done using uniquely the <<solution for the              #
# "dark matter mystery" based on Euclidean relativity>>         #
# article. Refer to this article in vixra april 2009            #
# F.Lassiaille : viXra.org, Relativity and Cosmology,           #
# viXra:0912.0048.                                              #
#                                                               #
# The matter density values are memorized in the "rho" arrays.  #
# They has been printed and measured on the printed papers.     #
# Those values are convert in Kg/AL3, then smoothed with a      #
# polynomial interpolation.                                     #
#                                                               #
# After this, the calculation is done.                          #
# Equations for calculating the resulting curve is explained    #
# and determined in the article mentioned above.                #
#                                                               #
# Determination of the constant <<r>> used in these equations   #
# is made using the informations coming from another "dark      #
# matter" mystery, the velocities of galaxies inside their      #
# groups (see article). It uses also the value of 10 for        #
# G'/G, where G' is the extra-galactic gravitational            #
# constant, outside the studied galaxy. G is our                #
# gravitational constant, inside our milky-way galaxy.          #
#                                                               #
# The curve for the calculated gravitational force shows        #
# great variations along the x axis. There are, roughly,        #
# as much positive (attractive) values as negative values.      #
# This is why the following usual formula, v² = F x / m, is not #
# correct in this case, because this formula supposes a         #
# positive (attractive) value of F.                             #
#                                                               #
# For trying to solve simply this problem, i smoothed the       #
# force curve with a FSmoothFactor, and raised it up enough     #
# (+F_pull) in order to get only positive values for F.         #
# The FSmoothFactor value has been fitted in order to get the   #
# best resulting compared speed values for the 2 maximum of the #
```

```
# NGC3310 galaxy speed curve.                                     #
#                                                                 #
# After that the speed is calculated with v = sqrt(F x/m).        #
# Since the gravitational force was raised up with a value        #
# F_pull, i substract a value sqrt(F_pull x) after calculating    #
# the speed.                                                      #
# This is the calculation done with ProgMode set to 10 and 11.    #
#                                                                 #
# The resulting curve has the same shape and the same values as   #
# the measured one.                                               #
#                                                                 #
# A more correct calculation of the speed, using only             #
# the initial curve of the force, is done with ProgMode = 24.     #
# In this case the calculation is a simulation of the             #
# trajectories of the stars. Each simulated star is starting      #
# on the same "Ox" axis and at the same speed.                    #
# For each simulated star, the program stops the star trajectory#
# simulation as soon as this one reach the "Or" axis,             #
# and calculates the rotational speed of this star at this        #
# instant. The "Or" axis is a straight line starting from the     #
# origin (0,0), and makes an "a" angle with "Ox", the initial     #
# axis (starting from the origin also) from where the simulated #
# stars has started.                                              #
# This "a" angle, the initial speed, and some more parameters     #
# are set depending of the galaxy the program is working on.      #
#                                                                 #
# At the end i execute exactly the same program but with the      #
# NGC1068 galaxy. Again i obtain nearly the same values and the #
# same shape. Also i executed exactly the same program with       #
# the NGC 157 NGC 7541 and NGC 7331 galaxies. But here i dont     #
# get any estimation of galactic center masses. However, the      #
# global shape is retrieved also in this case.                    #
#                                                                 #
#################################################################
#################################################################


#################################################################
######################### Initialisations #######################
#################################################################

# --------------------------- MAPLE ------------------------- #
with(CurveFitting):
Digits := 50 :                      # Floating points precision.
MAXVAL := 10**100000000:
unassign('DensPoly'): unassign('n'):
unassign('l'):        unassign('ll'):       unassign('rho'):
unassign('x'):        unassign('xprime'):   unassign('e'):
unassign('tg'):       unassign('cos2'):     unassign('F'):
unassign('v'):
```

```
# ----------------- Cosmological values --------------------- #
kpc := 3.08 * 10**19 :       # 1 kilo parsec expressed in meter.
c := 3 * 10**8 / kpc:                    # light speed in kpc/s.
AL := 365 * 24 * 3600 * c:               # 1 year light in kpc.
pc := kpc/1000 :                 # 1 parsec expressed in meter.
M0 := 2 * 10**30 :                             # Sun mass in Kg.
G := 6.6742867 *10**(-11)/kpc**3:  # Gravit const m3 kg(-1) s(-2)
Gp := 10 * G:        # Gravitational constant outside any galaxy.
rho0 := 0.003 * M0 / (AL**3):  # Matter density near solar system
                                               # in Kg / kpc 3.

# ----------- Corrected Newton's law calculation ------------- #
Si_So := sqrt(Gp/G) - 1: # Intra gal/extra-galactic contributions
r := rho0 * Si_So**(-2):   # Square of extragalactic contribution
                   # proportional to the square of extra-galactic
                   # lights contributions. Linked to rho0 value.
                   # Unit : Kg m(-3).
k_factor := sqrt(r) + sqrt(rho0):  # In order to have the normal
               # classical Newton's law in the case of our solar
               # system. This value supposes that the evolution
               # of mass density around the sun is weak
               # and negligeable in front o rho0.
# - Fitted parameters for the contributions: ------------------
# Those parameters have been fitted in order to get the best
# possible curve for the speed profiles of the galaxies.
K0 := 0.2:     # For intra-galactic contribution galactic center.
K1 := 1:                      # For intra-galactic contribution.
K2 := 65:                     # For extra-galactic contribution.
R1 := 0.4 * 10**34:                # For intra-galactic contrib.

# ---------- Discretising the curve of the force ------------ #
MUL := 10:        # This multiplies the NPoly number to get the
MUL := 50:        # This multiplies the NPoly number to get the
           # number of points for discretising the force curve.



#################################################################
#                         PROGRAM MODE                          #
#################################################################

# ----- WHAT DO YOU WANT TO CALCULATE WITH THIS PROGRAM ? ----- #
ProgMode := 10:   # 0:   Speed profile brute force (neg val = 0).
                  # 10:  Relative speed profile: force F smoothed
                  #      raised up, speed calculated from this F,
                  #      and sqrt(Fup * x) substracted.
                  # 11:  Same as 10 but with a much greater value
                  #      for the mass of the galactic center,
                  #      for weakening the effect of the
                  #      variations of matter density.
```

```
                  # 20:  Speed profile: dynamic calculation
                  #      from the star trajectories below.
                  # 24:  Same as 20 but along a specific axis.
                  # 30:  Star trajectories in 2D.
                  # 40:  Plotting the matter density.
                  #      (after interpolation).
                  # 50:  Plotting e: "relativistic ratio".
                  # 60:  Plotting cos2: square of the
                  #      relativistic operator.
                  # 70:  Plotting tg: tan = slope of space line.
                  # 80:  Plotting the gravitationnal force.
                  # 90:  Plotting integral mass profile.
                  # 100: Plotting curve of space inside
                  #      space-time.

# ------------------------- Parameters ------------------------ #
# ------------------- if ProgMode := 10 or 11 ----------------- #
FSmoothFactor := 1.6 * 10**(-4):    # Reduce amplitude of force.
# -------------------- if ProgMode := 11 --------------------- #
MFactor := 0.3 * 10**20:                      # Increasing factor.



#################################################################
#################### Calculation procedure ######################
#################################################################
# M:          Mass of the galactic center, in Kg.
# rho:        Matter density values, in Kg/m3.
# IMin:       First indice of valid values in the rho array.
# IMax:       Last indice of valid values in the rho array.
# xmin:       Distance in m corresponding to imin.
# xmax:       Distance in m corresponding to imax.
# xminpl:     Plotting min distance from the galactic center in m.
# xmaxpl:     Plotting max distance from the galactic center in m.
# NPoly:      Nbpts polynomial smoothing of matter density curve.
# HSpeed:     A constant used for raising or lowing the speed.
#             The shape of the curve in not affected by this value.
#             Only the whole curve is raised up or lowered.
# ProgMode:   Very important : what has to be done.
#             See above all the possible values.
# NbStep:     Only if ProgMode = 20<->30 (simulation of star
#             trajectories). Nb of step of simulation.
# NbTraject: Only if ProgMode = 20<->30. Nb of simulated stars.
# v0:         Only if ProgMode = 20<->30. Initial speed. Kpc/s.
# dy:         Only if ProgMode = 20<->30. Sampling space lgth. Kpc
# AxisAngle: Only if ProgMode = 24. Angle of the axis for the
#             calculation of the speed profile. This axis in the
#             line starting from the galactic center. The angle
#             is between this axis and the axis from which stars
#             are initially starting with v0 speed. In rad.
# Title:      for writing the title of the figure when plotting.
```

```
GalaxySpeedCalcul := proc( M, rho, IMin, IMax, xmin,
                           xmax, xminpl, xmaxpl, NPoly,
                           HSpeed, ProgMode, NbStep, NbTraject,
                           v0, dy, AxisAngle, Title )

# This procedure ensures that the calculation is exactly the   #
# same for each galaxy.                                        #

local xminm, xmaxm, xminplm, xmaxplm, Delta, DeltaKpc, M2, R,
      Step, lp, DensPoly, Lend, f, F, v, IMinForce, IMaxForce,
      StepForce, n, x, Xar, F_arr, F_mean, F_min, F_max, F_pull,
      FSmooth, imin2, imax2, v_arr, Fup, Fupa, Fups, imin3,
      imax3, Fold, vmin, vmax, vup, imin4, imax4, ll, k, x0, dt,
      xx, yy, i, vx, vy, rr, vv, lv, l, y, lxy, IntDensPoly:

# ---------------- Scaling factor ------------------------- #
xminm := xmin*kpc:
xmaxm := xmax*kpc:
xminplm := xminpl*kpc:
xmaxplm := xmaxpl*kpc:
Delta := (xmaxm-xminm)/(IMax-IMin):  # Distance in m between 2
                         # consecutive measurements on my paper.
DeltaKpc := Delta/kpc:

# ---------------- Cosmological value --------------------- #
if(ProgMode = 11 ) then
  M2 := M * MFactor:       # Weakening effect of matter density.
else:
  M2 := M:                                   # No change for M.
end if:
R := M2 * G / c**2:    # Schwarschild ray for considered galaxy.


################## Polynomial interpolation ##################
# That's mandatory because calculation of gravitational force
# is very sensitive to low variations. This sensitivity can be
# demonstrated with calculations, and is seen with this program.

Step := floor((IMax-IMin)/(NPoly-1)):
                        # Nb values skipped between 2 poly pts.
lp := [[xmin + DeltaKpc*nn*Step, rho[IMin + nn*Step]]
                                          $nn=0..(NPoly-1)]:
unassign('x'):
DensPoly := PolynomialInterpolation(lp, x) :

########### Corrected Newton's law calculation #################

# ---------------- Symmetric contribution -------------------
Lend := xmaxpl:               # Location of the end of the galaxy.
f :=   K0/x**(3/4) # Near galactic center, behaviour is not the
```

```
                  # same… Fitted added contrib. for the moment.
     + sqrt( K1*(DensPoly/R1)**(-1/3) # Intra-galactic contrib.
     + exp(-K2*(1-x/Lend)) ):         # Extra-galactic contrib.

# ----------------- Gravitational force ---------------------
F := (k_factor**2/r) * (M*G/x**2) * (f + 2*x*diff(f,x)) / f**3:

########### Calculation and plotting depending of the mode ####
unassign('x'):
if( ProgMode = 0 )              # If NO smoothing of the force
then              # plot speed curve with positive values of F.

  ################## Plotting the v curve #####################
  unassign('x'):                                # x is in kpc.
  v := sqrt( abs(max(F,0) * x )) * kpc :   # neg value F => v=0
  plot( v, x=xminpl..xmaxpl );         # v in m/s and x in kpc.

elif( ProgMode = 10 or ProgMode = 11 ) then      # If smoothing
        # gravit force for getting repulsing effect on speed
                                              # of the stars.

  # ---------- Discretising the curve of the force ---------- #
  # in order to be able to smooth it.
  IMinForce := 1:       # Each value between xmaxpl and xminpl
  IMaxForce := NPoly * MUL: # discretised with Npoly*MUL points.
  StepForce := (xmaxpl-xminpl) / (IMaxForce - IMinForce):

  for n from IMinForce to IMaxForce
  do
    x := xminpl + (n - IMinForce) * StepForce:
    Xar[n] := x:               # For performance memorize this.
    F_arr[n] := F(x):                         # Discretisation.
  end do:

  # --------------- Smoothing the force ------------------- #

  # --- Calculating the min, max and mean of F curve -------- #
  F_mean := 0:
  F_min := MAXVAL:
  F_max := -MAXVAL:
  for n from IMinForce to IMaxForce
  do
    F_mean := F_arr[n]:
    F_min := min(F_arr[n], F_min):
    F_max := max(F_max, F_arr[n]):
  end do:
  F_mean := F_mean/(IMaxForce-IMinForce+1):
  unassign('x'):

  # --- Calculating value for raising up the whole curve ---- #
  F_pull := max( F_max-F_mean, F_mean-F_min ) * FSmoothFactor:
```

```
# --- Actual smotthing of the curve ----------------------- #
unassign('x'):
for n from IMinForce to IMaxForce
do
  FSmooth[n] := F_pull + (F_arr[n]-F_mean) * FSmoothFactor:
end do:

# --- Cutting the edges if negatives values  ------------- #
imin2 := IMinForce:
for n from IMinForce to IMaxForce
do
  if(FSmooth[n] >= 0)
  then
    imin2 := n:
    break:
  end if;
end do:

imax2 := IMaxForce:
for n from IMaxForce to IMinForce by -1
do
  if(FSmooth[n] >= 0)
  then
    imax2 := n:
    break:
  end if;
end do:

# ---------- Calculation of the speed -------------------- #
# using simple smoothing of the gravitational force and    #
# usual equation v² = F x / m.                             #

unassign('x'):
for n from imin2 to imax2
do
  v_arr[n] := sqrt( FSmooth[n] * Xar[n] ):
         # This speed has globally the sqrt(F_pull x) shape.

  # ----- Substracting "raising up force" term ----------- #
  Fup := F_pull - F_mean*FSmoothFactor:
  Fupa := abs(Fup):
  Fups := sign(Fup):
  v_arr[n] := v_arr[n] - Fups * sqrt(Fupa * Xar[n]):
end do:

# --- Calculating boundaries for calculating the min ----- #
imin3 := imin2:
for n from imin2+1 to imax2
do
  if(v_arr[n] < v_arr[n-1])
```

```
    then
      imin3 := n-1:
      break:
    end if;
  end do:

  imax3 := imax2:
  Fold := -MAXVAL:
  for n from imax2-1 to imin2 by -1
  do
    if(v_arr[n] < v_arr[n+1])
    then
      imax3 := n+1:
      break:
    end if;
  end do:

  if(imin3 = imax3)                              # No minimum.
  then
    imin3 := imin2:                # Stay with old boundaries.
    imax3 := imin3:
  else      # There is a minimum. Boundaries imin 4 and imax 4
  end if:        # = maximum on the left and max on the rigth.

  # --- Calculating the min, max of v curve ---------------- #
  vmin := MAXVAL:
  vmax := -MAXVAL:
  for n from imin3 to imax3
  do
    vmin := min( v_arr[n], vmin ):
    vmax := max( v_arr[n], vmax ):
  end do:

  # --- Calculating value for raising up the whole curve --- #
  if(vmin < 0)
  then
    # ---- Using the fitting parameter for raising up whole curve
    vup := - vmin + (vmax-vmin) * HSpeed:
  else                              # Here no use for raising up.
    vup := 0:
  end if:

  # --- Raising up curve and cutting edges if neg values  -- #
  imin4 := imin2:
  for n from imin2 to imax2
  do
    if(v_arr[n]+vup >= 0)
    then
      imin4 := n:
      break:
    end if;
```

```
  end do:

  imax4 := imax2:
  for n from imax2 to imin2 by -1
  do
    if(v_arr[n]+vup >= 0)
    then
      imax4 := n:
      break:
    end if;
  end do:


  ################## Plotting the curve ######################

  # ----------------------- Speed curve -------------------- #
  ll := [[Xar[nn], (v_arr[nn] + vup)/(v_arr[imax4] + vup)]
                                            $nn=imin4..imax4]:
  plot(ll, Title=xminpl..xmaxpl,
              labels=[Title, Relative_speed]);

elif( ProgMode = 20 ) then   # If simulating dynamic in order
                             # to calculate speed of the stars.

  Step := (xmaxpl - xminpl)/(NbTraject-1):
  for k from 1 to NbTraject
  do
    x0 := xminpl + (k-1)*Step:
    x0 := 0.001 + (k-1)*Step:
    dt := dy/v0:
    xx[1] := x0:
    yy[1] := 0:
    xx[2] := xx[1]:
    yy[2] := yy[1] + dy:
    for i from 3 to NbStep
    do
      x := evalf( sqrt(xx[i-1]**2 + yy[i-1]**2) ):
      xx[i] := evalf( - xx[i-2]
                     + xx[i-1] * ( 2 - (dt**2*F(x))/(2*x) )):
      yy[i] := evalf( - yy[i-2]
                     + yy[i-1] * ( 2 - (dt**2*F(x))/(2*x) )):
    end do:

    vx := (xx[NbStep] - xx[NbStep-1])/dt:
    vy := (yy[NbStep] - yy[NbStep-1])/dt:
    rr[k] := evalf( sqrt(xx[NbStep]**2 + yy[NbStep]**2) ):
    if(yy[NbStep] = 0) then
      vv[k] := 0:
    else
      vv[k] := ((xx[NbStep]*vx + yy[NbStep]*vy) / rr[k])
                                   * (xx[NbStep]/yy[NbStep])
```

```
                                        - rr[k]*vx/yy[NbStep]:
    end if:
  end do:

  lv := [[rr[kk], vv[kk]*kpc] $kk=1..NbTraject]:
  plot(lv, Title=xminpl..xmaxpl, style=line );

elif( ProgMode = 24 ) then   # Same as 20 but with calculation
                             # of speed along an axis.

  Step := (xmaxpl - xminpl)/(NbTraject-1):

  l := 1:
  for k from 1 to NbTraject
  do
    x0 := xminpl + (k-1)*Step:
    x0 := 0.001 + (k-1)*Step:
    dt := dy/v0:
    xx[1] := x0:
    yy[1] := 0:
    xx[2] := xx[1]:
    yy[2] := yy[1] + dy:

    for i from 3 to NbStep
    do
      x := evalf( sqrt(xx[i-1]**2 + yy[i-1]**2) ):
      xx[i] := evalf( - xx[i-2]
                   + xx[i-1] * ( 2 - (dt**2*F(x))/(2*x) )):
      yy[i] := evalf( - yy[i-2]
                   + yy[i-1] * ( 2 - (dt**2*F(x))/(2*x) )):

      if( yy[i]/xx[i] > sin(AxisAngle) ) then
        i := i+1:
        break:
      end if:

    end do:

    if( i = NbStep+1) then
      next:                       # Don't reach the axis : out.
    else:                         # Reached the axis : memorize it.
      vx := (xx[i-1] - xx[i-2])/dt:
      vy := (yy[i-1] - yy[i-2])/dt:
      x := xx[i-1]:
      y := yy[i-1]:
      rr[l] := evalf( sqrt(x**2 + y**2) ):
      if(y = 0) then
        vv[l] := 0:
      else
        vv[l] := ((x*vx+y*vy)/rr[l])*x/y - rr[l]*vx/y:
      end if:
```

```
      l := l + 1:
    end if:
  end do:

  lv := [[rr[ll], vv[ll]*kpc] $ll=1..l-1]:
  plot(lv, Title=xminpl..xmaxpl, style=line );
  plot(lv, Title=xminpl..xmaxpl, style=point );

elif( ProgMode = 30 ) then  # Plotting star trajectories in 2D

  Step := (xmaxpl - xminpl)/(NbTraject-1):
  for k from 1 to NbTraject
  do
    x0 := xminpl + (k-1)*Step:
    dt := dy/v0:
    xx[k,1] := x0:
    yy[k,1] := 0:
    xx[k,2] := xx[k,1]:
    yy[k,2] := yy[k,1] + dy:

    for i from 3 to NbStep
    do
      x := evalf( sqrt(xx[k,i-1]**2 + yy[k,i-1]**2) ):
      xx[k,i] := evalf( - xx[k,i-2]
                     + xx[k,i-1] * ( 2 - (dt**2*F(x))/(2*x) )):
      yy[k,i] := evalf( - yy[k,i-2]
                     + yy[k,i-1] * ( 2 - (dt**2*F(x))/(2*x) )):
    end do:
  end do:

  lxy := [ [[xx[kk,nn], yy[kk,nn]] $nn=1.. NbStep] $kk=1..NbTraject
]:
  plot(lxy, style=line);

elif( ProgMode = 40 ) then                # For matter density.
  #plot( DensPoly, x=xminpl..xmaxpl );
  plot( DensPoly, x=1.5..xmaxpl );
elif( ProgMode = 50 ) then                          # For e.
  plot( e, x=xminpl..xmaxpl );
elif( ProgMode = 60 ) then                          # For cos2.
  plot( cos2, x=xminpl..xmaxpl );
elif( ProgMode = 70 ) then                          # For tan.
  plot( tg, x=xminpl..xmaxpl );
elif( ProgMode = 80 ) then                          # For F.
  plot( F, x=xminpl..xmaxpl );
elif( ProgMode = 90 ) then       # For integral mass profile.
  IntDensPoly := int(DensPoly, x);
  plot(IntDensPoly, x=xminpl..xmaxpl );       # Does not work.
elif( ProgMode = 100 ) then          # Space inside space-time.
   # TO BE WRITTEN #
else
```

```
end if:


 #plot( f, x=xminpl..xmaxpl );         # v in m/s and x in kpc.

  #plot( sqrt(DensPoly), x=1.5..xmaxpl );
#plot( f, x=1.5..xmaxpl );
#plot( f, x=1.5..xmaxpl );




end proc:               # end of "GalaxyProgModeul" procedure.



###############################################################
################### NGC3310 and NGC1068 #######################
###############################################################

# ----------------- Matter density values ------------------- #
# Those values are read on a printed paper of the density     #
# matter of the galaxy. Hence their unit is the mm.           #

# ------------------------- NGC3310 ------------------------- #

# Below is the value extrapolated from my paper               #
# supposing that the density matter has a constant slope      #
# between 0 and 1 kpc.                                        #
rho_NGC3310[1] := 700:    rho_NGC3310[2] := 500:
rho_NGC3310[3] := 400:    rho_NGC3310[4] := 300:
rho_NGC3310[5] := 200:

# Below are the values from my paper, plain line curve.       #
rho_NGC3310[6] := 147:
rho_NGC3310[7] := 94:     rho_NGC3310[8] := 63:
rho_NGC3310[9] := 36:     rho_NGC3310[10] := 18:
rho_NGC3310[11] := 8:     rho_NGC3310[12] := 3.3:
rho_NGC3310[13] := 2.3:   rho_NGC3310[14] := 0.8:
rho_NGC3310[15] := 0.18:  rho_NGC3310[16] := 0.08:
rho_NGC3310[17] := 0.07:  rho_NGC3310[18] := 0.08:
rho_NGC3310[19] := 0.2:   rho_NGC3310[20] := 1.2:
rho_NGC3310[21] := 1.4:   rho_NGC3310[22] := 2.5:
rho_NGC3310[23] := 3:     rho_NGC3310[24] := 4:
rho_NGC3310[25] := 4.5:   rho_NGC3310[26] := 5:
rho_NGC3310[27] := 5.7:   rho_NGC3310[28] := 6.5:
rho_NGC3310[29] := 6.55:  rho_NGC3310[30] := 6.6:
rho_NGC3310[31] := 6.9:   rho_NGC3310[32] := 5.2:
rho_NGC3310[33] := 5.1:   rho_NGC3310[34] := 5:
rho_NGC3310[35] := 4.5:   rho_NGC3310[36] := 4:
rho_NGC3310[37] := 3.85:  rho_NGC3310[38] := 3.7:
```

```
rho_NGC3310[39] := 2.9:   rho_NGC3310[40] := 2.1:
rho_NGC3310[41] := 2.05:  rho_NGC3310[42] := 2:

# Below are the values from my paper, plain dashed line.     #
rho_NGC3310[43] := 1.9:   rho_NGC3310[44] := 1.8:
rho_NGC3310[45] := 1.78:  rho_NGC3310[46] := 1.77:
rho_NGC3310[47] := 1.77:  rho_NGC3310[48] := 1.76:
rho_NGC3310[49] := 1.76:  rho_NGC3310[50] := 1.75:
rho_NGC3310[51] := 1.75:  rho_NGC3310[52] := 1.74:

IMIN_NGC3310 := 5:          # First valid indice for rho array.
IMAX_NGC3310 := 52:          # Last valid indice for rho array.
LMIN_NGC3310 := 0.69:       # Distance corresponding to IMin.
LMAX_NGC3310 := 7.25:         # Distance corresponding to Imax.
LMINPL_NGC3310 := 0.01:     # Min value for x, for plot. In Kpc.
LMAXPL_NGC3310 := 6.57:    # Max value for x, for plot. In Kpc.
NPOLY_NGC3310 := 10:  # Nb points for polynomial interpolation.
HSPEED_NGC3310 := 1.02:                        # Fitting value.
M_NGC3310 := 0.07 * 10**11 * M0 :  # NGC3310 center mass in Kg.
NBSTEP_NGC3310 := 75 :       # Nb of steps for star simulation.
NBTRAJ_NGC3310 := 3000 :     # Nb of stars for star simulation.
V0_NGC3310 := 150*10**3/kpc:       # Init speed for star simul.
DY_NGC3310 := 1/50 :            # Sampling space period in kpc.
AXANG_NGC3310 := 0.33 :     # Angle of axis for profile in rad.

# ------------------------- NGC1068 ------------------------- #

# Below is the value extrapolated from my paper                #
# supposing that the density matter has a constant slope      #
# between 0 and 1 kpc.                                         #
rho_NGC1068[1] := 1000:    rho_NGC1068[2] := 800:
rho_NGC1068[3] := 600:    rho_NGC1068[4] := 400:
rho_NGC1068[5] := 350:    rho_NGC1068[6] := 300:
rho_NGC1068[7] := 220:

# Below are the values from my paper, plain line curve.       #
rho_NGC1068[8] := 160:
rho_NGC1068[9] := 124:    rho_NGC1068[10] := 99:
rho_NGC1068[11] := 77:    rho_NGC1068[12] := 62.5:
rho_NGC1068[13] := 48.5:  rho_NGC1068[14] := 40:
rho_NGC1068[15] := 32:    rho_NGC1068[16] := 26.5:
rho_NGC1068[17] := 23:    rho_NGC1068[18] := 20:
rho_NGC1068[19] := 18:    rho_NGC1068[20] := 17:
rho_NGC1068[21] := 16:    rho_NGC1068[22] := 15.3:
rho_NGC1068[23] := 14.6:  rho_NGC1068[24] := 14.5:
rho_NGC1068[25] := 14.7:  rho_NGC1068[26] := 15:
rho_NGC1068[27] := 15.2:  rho_NGC1068[28] := 15.5:
rho_NGC1068[29] := 15.8:  rho_NGC1068[30] := 16:
rho_NGC1068[31] := 16.5:  rho_NGC1068[32] := 16.8:
rho_NGC1068[33] := 17.05: rho_NGC1068[34] := 17.3:
rho_NGC1068[35] := 17.5:  rho_NGC1068[36] := 17.8:
```

```
rho_NGC1068[37] := 18:    rho_NGC1068[38] := 18.1:
rho_NGC1068[39] := 18.1:  rho_NGC1068[40] := 18.2:

# For information only : below are the value extrapolated     #
# from my paper using a simple straight line with the last    #
# right measured slope.                                       #
rho_NGC1068[41] := 18.21:  rho_NGC1068[42] := 18.22:
rho_NGC1068[43] := 18.23:  rho_NGC1068[44] := 18.24:
rho_NGC1068[45] := 18.25:  rho_NGC1068[46] := 18.26:
rho_NGC1068[47] := 18.27:  rho_NGC1068[48] := 18.28:
rho_NGC1068[49] := 18.29:  rho_NGC1068[50] := 18.3:

# ---- Below is the value extrapolated from my paper -------- #
# supposing that the density matter is lowering after 20 kpc. #
# This indice 46 is the only used value by polynomial         #
# interpolation in this extrapolated range.                   #

rho_NGC1068[46] := 10:        # Needed value when ProgMode = 10
                                             # (smoothed force).
rho_NGC1068[46] := 18:        # Most natural extrapolated value
             # when ProMode = 10 (simulated stra trajectories).

IMIN_NGC1068 := 1:           # First valid indice for rho array.
IMAX_NGC1068 := 50:          # Last valid indice for rho array.
LMIN_NGC1068 := 0.1388:        # Distance corresponding to IMIN.
LMAX_NGC1068 := 6.97:          # Distance corresponding to IMAX.
LMINPL_NGC1068 := 0.01:          # Best value for ModProg = 24.
LMINPL_NGC1068 := 0.1:           # Best value for ModProg = 10.
LMAXPL_NGC1068 := 6:         # Max value for x, for plotting.
NPOLY_NGC1068 := 10:   # Nb points for polynomial interpolation.
HSPEED_NGC1068 := 23.5:                         # Fitting value.
M_NGC1068 := 10**10 * M0 :          # NGC1068 center mass in Kg.
#M_NGC1068 := 7.5 * 10**10 * M0 :  # For having measured values.
NBSTEP_NGC1068 := 480 :       # Nb of steps for star simulation.
NBTRAJ_NGC1068 := 3000 :      # Nb of stars for star simulation.
V0_NGC1068 := 80*10**3/kpc:         # Init speed for star simul.
DY_NGC1068 := 1/400:             # Sampling space period in kpc.
AXANG_NGC1068 := 0.16 :      # Angle of axis for profile in rad.

# ----------- Matter density unit conversion --------------- #
alpha := 1.35 * 10**(-24): # Ratio matter density/paper_length
                           # unit : g cm(-3) mm(-1)
alpha2 := 10**3 * kpc**3:  # Ratio for converting matter density
                           # from g cm(-3) to kg kpc(-3).

MatDensUnitConv := proc( rho, IMin, IMax )
  local n:
  for n from IMin to IMax
  do
    rho[n] := rho[n] * alpha * alpha2:
  end do:        # converting from mm on paper to Kg/m3 unit.
```

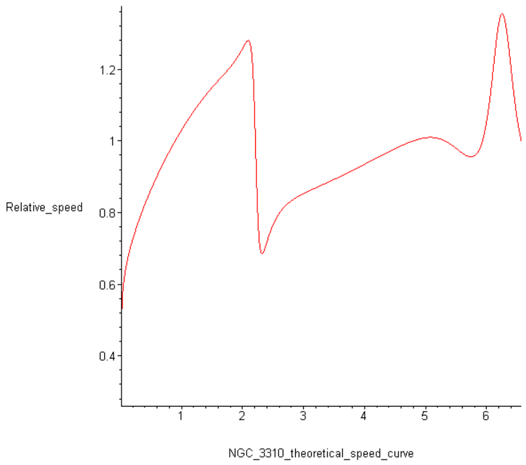
```
end proc:

MatDensUnitConv (rho_NGC3310, IMIN_NGC3310, IMAX_NGC3310):
MatDensUnitConv (rho_NGC1068, IMIN_NGC1068, IMAX_NGC1068):

# ------------------ Speed curve calculations ---------------- #
GalaxySpeedCalcul( M_NGC3310, rho_NGC3310, IMIN_NGC3310,
                   IMAX_NGC3310, LMIN_NGC3310, LMAX_NGC3310,
                   LMINPL_NGC3310, LMAXPL_NGC3310,
                   NPOLY_NGC3310, HSPEED_NGC3310, ProgMode,
                   NBSTEP_NGC3310, NBTRAJ_NGC3310,
                   V0_NGC3310, DY_NGC3310, AXANG_NGC3310,
                   NGC_3310_theoretical_speed_curve );
GalaxySpeedCalcul( M_NGC1068, rho_NGC1068, IMIN_NGC1068,
                   IMAX_NGC1068, LMIN_NGC1068, LMAX_NGC1068,
                   LMINPL_NGC1068, LMAXPL_NGC1068, NPOLY_NGC1068,
                   HSPEED_NGC1068, ProgMode,
                   NBSTEP_NGC1068, NBTRAJ_NGC1068,
                   V0_NGC1068, DY_NGC1068, AXANG_NGC1068,
                   NGC_1068_theoretical_speed_curve );



################################################################
######################### NGC157 ###############################
################################################################

# ----------------- Matter density values ------------------- #
# Those values are read on a printed paper of the density     #
# matter of the galaxy. Hence their unit is the mm.           #

rho_NGC157[1] := 69.5:    rho_NGC157[2] := 66:
rho_NGC157[3] := 58.5:    rho_NGC157[4] := 50:
rho_NGC157[5] := 41.5:    rho_NGC157[6] := 33.5:
rho_NGC157[7] := 25:      rho_NGC157[8] := 18.5:
rho_NGC157[9] := 12.5:    rho_NGC157[10] := 8.8:
rho_NGC157[11] := 5:      rho_NGC157[12] := 3:
rho_NGC157[13] := 2.1:    rho_NGC157[14] := 1.4:
rho_NGC157[15] := 1.1:    rho_NGC157[16] := 1.05:
rho_NGC157[17] := 1:      rho_NGC157[18] := 0.6:
rho_NGC157[19] := 0.3:    rho_NGC157[20] := 0.3:
rho_NGC157[21] := 0.6:    rho_NGC157[22] := 3:

IMIN_NGC157 := 1:            # First valid indice for rho array.
IMAX_NGC157 := 22:            # Last valid indice for rho array.
LMIN_NGC157 := 0:              # Distance corresponding to IMin.
LMAX_NGC157 := 26.35:          # Distance corresponding to IMax.
LMINPL_NGC157 := 0.8:           # Min value for x, for plotting.
LMAXPL_NGC157 := 21.5:          # Max value for x, for plotting.
NPOLY_NGC157 := 6:     # Nb points for polynomial interpolation.
HSPEED_NGC157 := 4.8:                           # Fitting value.
```

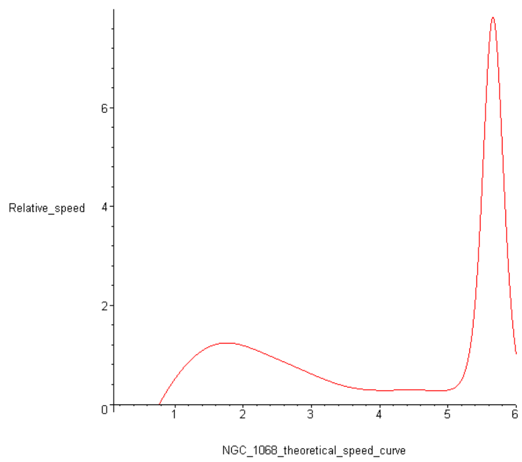
```
M_NGC157 := 5 * 10**11 * M0:        # Fitted NGC157 center mass.
                      # No correct available measured value. Kg.
NBSTEP_NGC157 := 200 :       # Nb of steps for star simulation.
NBTRAJ_NGC157 := 500 :       # Nb of stars for star simulation.
V0_NGC157 := 180*10**3/kpc:     # Initial speed for star simul.
DY_NGC157 := 1/20 :             # Sampling space period in kpc.
AXANG_NGC157 := 0.4 :       # Angle of axis for profile in rad.

# ----------- Matter density unit conversion --------------- #
paperhigh := 69.5:  # Paper length for rel dens matter = 1 (mm).
alpha := 0.57 * M0 * 10**9 / paperhigh:
                           # Ratio matter density/paper_length
                           # unit : Kg kpc(-3) mm(-1)
for n from IMIN_NGC157 to IMAX_NGC157
do
  rho_NGC157[n] := rho_NGC157[n] * alpha:
end do:             # converting from mm on paper to Kg/m3 unit.
         # Remark : this matter density is quite the same range
         # of values as the one of the NGC3310.

# ----------------- Speed curve calculations --------------- #
GalaxySpeedCalcul( M_NGC157, rho_NGC157, IMIN_NGC157,
                   IMAX_NGC157, LMIN_NGC157, LMAX_NGC157,
                   LMINPL_NGC157, LMAXPL_NGC157, NPOLY_NGC157,
                   HSPEED_NGC157, ProgMode,
                   NBSTEP_NGC157, NBTRAJ_NGC157,
                   V0_NGC157, DY_NGC157, AXANG_NGC157,
                   NGC_157_theoretical_speed_curve );



################################################################
######################## NGC7541 ###############################
################################################################

# ---------------- Matter density values ------------------ #
# Those values are read on a printed paper of the density     #
# matter of the galaxy. Hence their unit is the mm.           #

# Below values of matter density on paper for 0 < x < 2 Kpc.  #
rho_NGC7541[1] := 126:    rho_NGC7541[2] := 124.8:
rho_NGC7541[3] := 120.5:  rho_NGC7541[4] := 114:
rho_NGC7541[5] := 104.5:  rho_NGC7541[6] := 86:
rho_NGC7541[7] := 78.5:   rho_NGC7541[8] := 75:
rho_NGC7541[9] := 73.8:   rho_NGC7541[10] := 72.5:
rho_NGC7541[11] := 71.5:  rho_NGC7541[12] := 70.5:
rho_NGC7541[13] := 70:

IMIN_NGC7541 := 1:          # First valid indice for rho array.
IMAX_NGC7541 := 91:          # Last valid indice for rho array.
```

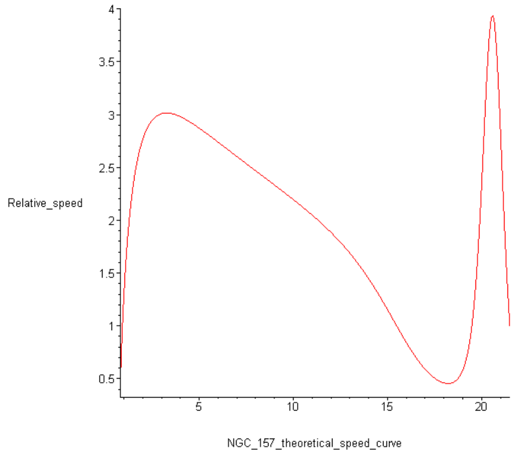
```
# Below are the values of matter density on the paper for x   #
# between 2 and 25 kpc. I see a straight line on the paper    #
# for x in this range of values.                              #
for n from 14 to IMAX_NGC7541
do
  rho_NGC7541[n] := (34-70)*(n-13)/(91-13) + 70:
end do:

LMIN_NGC7541 := 0:            # Distance corresponding to IMin.
LMAX_NGC7541 := 25:           # Distance corresponding to IMax.
LMINPL_NGC7541 := 0.3:           # Best value for ModProg = 24.
LMINPL_NGC7541 := 0.45:          # Best value for ModProg = 10.
LMINPL_NGC7541 := 0.2:           # Best value for ModProg = 11.
LMAXPL_NGC7541 := 23.2:        # Max value for x, for plotting.
NPOLY_NGC7541 := 31:   # Nb points for polynomial interpolation.
HSPEED_NGC7541 := 3.8:                        # Fitting value.
M_NGC7541 := 4.6 * 10**10 * M0:        # NGC7541 center mass !?
            # No available measured value. Kg. =total_mass/10 ?

# Warning: with the values below, this prog must run around
# 20 hours on a dual core 2 G Hz PC (but no parallelization on
# the dual core!).
NBSTEP_NGC7541 := 4200 :      # Nb of steps for star simulation.
NBTRAJ_NGC7541 := 2000 :      # Nb of stars for star simulation.
V0_NGC7541 := 287*10**3/kpc:     # Initial speed for star simul.
DY_NGC7541 := 1/78 :             # Sampling space period in kpc.
AXANG_NGC7541 := 0.94 :      # Angle of axis for profile in rad.

# ----------- Matter density unit conversion --------------- #
alpha := 2 * 10**36:    # Ratio matter density/paper_length
         # FITTED VALUE BECAUSE NO MATTER DENSITY AVALAIBLE HERE
                     # for this galaxy. Unit : Kg m(-3) mm(-1).
for n from IMIN_NGC7541 to IMAX_NGC7541
do
  rho_NGC7541[n] := rho_NGC7541[n] * alpha:
end do:             # converting from mm on paper to Kg/m3 unit.

# ----------------- Speed curve calculations --------------- #
GalaxySpeedCalcul( M_NGC7541, rho_NGC7541, IMIN_NGC7541,
                   IMAX_NGC7541, LMIN_NGC7541, LMAX_NGC7541,
                   LMINPL_NGC7541, LMAXPL_NGC7541, NPOLY_NGC7541,
                   HSPEED_NGC7541, ProgMode,
                   NBSTEP_NGC7541, NBTRAJ_NGC7541,
                   V0_NGC7541, DY_NGC7541, AXANG_NGC7541,
                   NGC_7541_theoretical_speed_curve );


################################################################
######################### NGC7331 ##############################
################################################################
```

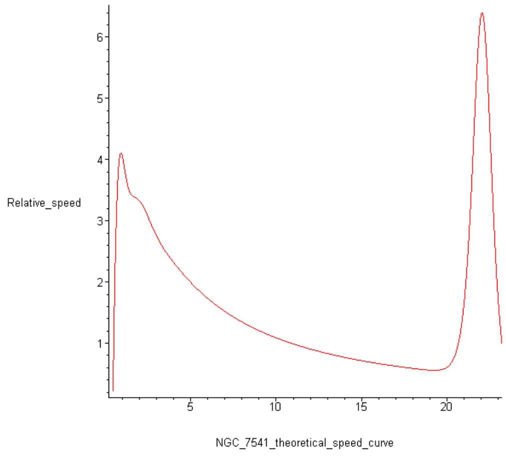
```
# ---------------- Matter density values ------------------ #
# Those values are read on a printed paper of the density     #
# matter of the galaxy. Hence their unit is the mm.           #

# Below values of matter density on paper for 0 < x < 14 Kpc. #
rho_NGC7331[1] := 145:    rho_NGC7331[2] := 128:
rho_NGC7331[3] := 115:    rho_NGC7331[4] := 97:
rho_NGC7331[5] := 85:     rho_NGC7331[6] := 64:
rho_NGC7331[7] := 62.7:   rho_NGC7331[8] := 54:
rho_NGC7331[9] := 48:     rho_NGC7331[10] := 41.5:
rho_NGC7331[11] := 35:    rho_NGC7331[12] := 30:
rho_NGC7331[13] := 25:    rho_NGC7331[14] := 21.5:
rho_NGC7331[15] := 17.5:  rho_NGC7331[16] := 15:
rho_NGC7331[17] := 11.5:  rho_NGC7331[18] := 9.5:
rho_NGC7331[19] := 8:     rho_NGC7331[20] := 6.5:
rho_NGC7331[21] := 4.5:   rho_NGC7331[22] := 4:
rho_NGC7331[23] := 2.5:   rho_NGC7331[24] := 1.8:
rho_NGC7331[25] := 1.5:   rho_NGC7331[26] := 1.2:
rho_NGC7331[27] := 1:     rho_NGC7331[28] := 0.6:
rho_NGC7331[29] := 0.3:   rho_NGC7331[30] := 0.24:
rho_NGC7331[31] := 0.2:   rho_NGC7331[32] := 0.16:
rho_NGC7331[33] := 0.12:  rho_NGC7331[34] := 0.113:
rho_NGC7331[35] := 0.11:  rho_NGC7331[36] := 0.103:
rho_NGC7331[37] := 0.1:

IMIN_NGC7331 := 1:          # First valid indice for rho array.
IMAX_NGC7331 := 81:          # Last valid indice for rho array.

# Below are the values of matter density on the paper for x   #
# between 15 and 35 kpc. I see a straight line on the paper   #
# for x in this range of values.                              #
for n from 38 to IMAX_NGC7331
do
  rho_NGC7331[n] := (0.01-0.1)*(n-37)/(IMAX_NGC7331-37) + 0.1:
end do:

LMIN_NGC7331 := 0:             # Distance corresponding to IMin.
LMAX_NGC7331 := 33:            # Distance corresponding to IMax.
LMINPL_NGC7331 := 0.01:            # Best value for ModProg = 24.
LMINPL_NGC7331 := 0.1:            # Best value for ModProg = 10.
LMINPL_NGC7331 := 0.2:            # Min value for x, for plotting.
LMAXPL_NGC7331 := 17.5: # No more because pb with polyn interpol
NPOLY_NGC7331 := 14:   # Nb points for polynomial interpolation.
HSPEED_NGC7331 := 7:                            # Fitting value.
M_NGC7331 := 0.43 * 10**11 * M0 :   # NGC7331 center mass in Kg?
                              # No available measured value. Kg.
# With values below: calculation for around 20 hours.
NBSTEP_NGC7331 := 3600 :      # Nb of steps for star simulation.
NBTRAJ_NGC7331 := 3000 :      # Nb of stars for star simulation.
V0_NGC7331 := 280*10**3/kpc:     # Initial speed for star simul.
```

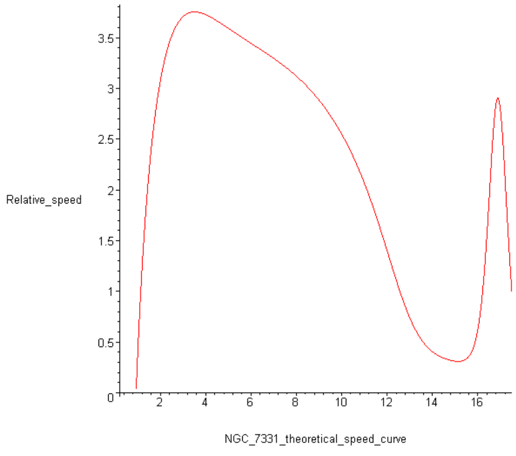
```
DY_NGC7331 := 1/200 :           # Sampling space period in kpc.
AXANG_NGC7331 := 0.72 :       # Angle of axis for profile in rad.

# ------------ Matter density unit conversion ---------------- #
alpha := 3.6*M0*10**9/145:    # Ratio matter density/paper_lgth
                    # for this galaxy. Unit : Kg kpc(-3) mm(-1).
for n from IMIN_NGC7331 to IMAX_NGC7331
do
  rho_NGC7331[n] := rho_NGC7331[n] * alpha:
end do:             # converting from mm on paper to Kg/m3 unit.

# ------------------ Speed curve calculations ---------------- #
GalaxySpeedCalcul( M_NGC7331, rho_NGC7331, IMIN_NGC7331,
                   IMAX_NGC7331, LMIN_NGC7331, LMAX_NGC7331,
                   LMINPL_NGC7331, LMAXPL_NGC7331, NPOLY_NGC7331,
                   HSPEED_NGC7331, ProgMode,
                   NBSTEP_NGC7331, NBTRAJ_NGC7331,
                   V0_NGC7331, DY_NGC7331, AXANG_NGC7331,
                   NGC_7331_theoretical_speed_curve );
# F.Lassiaille may 2010.
```






Relative_speed
NGC_157_theoretical_speed_curve


Relative_speed
NGC_7541_theoretical_speed_curve


Relative_speed
NGC_7331_theoretical_speed_curve

APPENDIX 12 PIONEER ANOMALY BEFORE SATURN

```
#################################################################
#                                                               #
#                CALCULATION OF PIONEER ANOMALY                 #
#                       BEFORE SATURN                           #
#                    FOR THE EIGTH PLANETS                      #
#                                                               #
# This MAPLE program is calculating the PIONEER anomaly,        #
# before the location of Saturn, taking into account:           #
#                      Kuiper belt                              #
#                      AND                                      #
#                      Main belt of asteroids,                  #
# supposing that Newton's law is perfect in the location of     #
# the Earth.                                                    #
#                                                               #
#################################################################
# The equation used is Om = sqrt(G(M+m)/x**3), which is the     #
# classical equation for the rotational speed of a couple of    #
# planets. We get also t = 2 Pi/Om, where t is the orbital      #
# revolution period. Hence we get dt/t = ½ da/a, where a is the #
# gravitational acceleration, because of the two equations      #
# above, and because the real acceleration <<a>> is             #
# proportional to the real gravitational constant <<G>>:        #
# dt/t = dOm/Om = ½ dG/G = ½ da/a.                              #
# That’s why is written below : evalf((1/2)*Da/an);             #
#################################################################

#################################################################
######################### Initialisations #######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                     # Floating points precision.
unassign('x'): unassign('y'): unassign('z'): unassign('ar'):
unassign('an'): unassign('Da'):

# ----------------- Cosmological values ---------------------- #
# --- Constants ---
s := 1:                                                     # s.
day := 24*3600:                                       # in sec.
km := 10**3:                                                # m.
kpc := 3.08 * 10**19 :       # 1 kilo parsec expressed in meter.
pc := kpc/1000 :                 # 1 parsec expressed in meter.
mpc := 10**3 * kpc:                                         # m.
c := 3 * 10**8:                           # light speed in m/s.
LY := 365 * 24 * 3600 * c:               # 1 light year in meter.
AU := 149597870 * km :                  # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).
rg := 473 * 10**18:                     # Ray of the galaxy in m.
rg_last := 0.9 * 10**(-2) * rg:                    # Fitted value.
```

```
                 # Distance of last galaxy's contribution. in m.

# ---------------- Solar system values --------------------- #
# heliocentric gravitational constant inside the solar system:
Gh := (0.01720209895)**2 * AU**3 / day**2:
Mt := 5.97 * 10**24:                           # Earth mass in Kg.
M0 := 2 * 10**30 :                            # Sun mass in Kg.
R0 := M0 * G / c**2:          # Half Schwarschild ray for SUN.
# Semi-major axis of the planets:
xme := 57909176 * km:                   # Distance Mercure sun in m.
xv := 108208930 * km:                     # Distance Venus sun in m.
xe := 149597887.5 * km:                 # Distance Earth sun in m.
xm := 227936637 * km:                 # Distance Mars sun in m.
xj := 778412027 * km:                 # Distance Jupiter sun in m.
xs := 1433449370 * km:                 # Distance Saturn sun in m.
xu := 2876679082 * km:                 # Distance Uranus sun in m.
xn := 4498253000 * km:                 # Distance Neptune sun in m.
rho0 := 0.003 * M0 / (LY**3):  # Matter density near solar system.
tmme := 87.969257 * day:      # Mercury orbital period. JPL value.
tmv := 224.70079922 * day:     # Venus orbital period. JPL value.
tmm := 686.98 * day:      # Mars orbital period. JPL value.
tmt := 365.25636 * day:         # Earth orbital period. JPL value.
tmj := 4332.820 * day:          # Jupiter orbital period. JPL
value.
tms := 10755.698 * day:         # Saturn orbital period. JPL value.
tmu := 30687.153 * day:         # Uranus orbital period. JPL value.
tmn := 60190.029 * day:         # Neptune orbital period. JPL
value.
# ----------------- Kuiper belt values --------------------- #
xkmin := 30 * AU:         # Kuiper min distance from the sun. m.
xkmax := 48 * AU:            # Kuiper max distance from the sun.
xkmean := (xkmin+xkmax)/2:   # Kuiper mean distance from the sun.
thetak := Pi*20/180:                  # Kuiper thickness angle.
xkthi := xkmean * thetak:                   # Kuiper thickness.
Mk := 0.3 * Mt:                             # Kuiper belt mass
value.
Vk := Pi*(xkmax**2 - xkmin**2) * xkthi: # Kuiper belt volume m3.
rhok := evalf(Mk/Vk):        # Kuiper belt matter density kg/m3.
#xKuiper :=  xkmin + xkmax:   # If only Kuiper belt xKuiper=81 AU
xKuiper :=  114*AU:       # Fitted value for taking into account
                          # the scattered disk and other
# ----------------- Main belt values --------------------- #
xmmin := 2.2 * AU:         # Main min distance from the sun. m.
xmmax := 3.3 * AU:            # Main max distance from the sun.
xmmean := (xmmin+xmmax)/2:   # Main mean distance from the sun.
thetam := Pi*36/180:             # Main thickness angle in ra.
xmthi := xmmean * thetam:                   # Main thickness.
Mm := 3.3 * 10**21:                                     # kg.
Vm := Pi*(xmmax**2 - xmmin**2) * xmthi: # Main belt volume m3.
rhom := evalf(Mm/Vm):        # Main belt matter density kg/m3.
xMain :=  xmmin + xmmax:                       # Middle of it.
```

```
x0 := xe:     # This is the distance from the sun at which Gh the
              # gravitational heliocentric constant, is
              # exactly fitting Newton's law. This xSmin value
              # correspond to the best value for minimizing all
              # orbital periods for all planets.

#################################################################
########### Calculation of Lk, Kuiper symmetric contribution ####
#################################################################
# ----------------- Calculation of fmax --------------------- #
Lk_on_Lg_max := (sqrt(xkmax**2 - xkmin**2)/rg_last) *
sqrt(rhok/rho0):
# ------------- ---- Calculation of Lk ------------------------ #
# Here is the final equation of the symmetric contribution coming
# from the Kuiper belt:
Lk_on_Lg := Lk_on_Lg_max * (2/Pi) * invfunc[tan]((2/Pi) *
                                                     xKuiper/x):
#################################################################
########### Calculation of Lm, main belt symm contribution ######
#################################################################
Lm_on_Lg := (sqrt(abs((xmmax-x)**2 - (xmmin-x)**2))/rg_last) *
                                                sqrt(rhom/rho0):
                                       # Direct value is better !
#################################################################
########### Calculation of Ls, the total solar system ###########
################## symmetric contribution #######################
#################################################################
Ls_on_Lg := Lk_on_Lg + Lm_on_Lg:   # Addition because coming from
                           # numerous luminous points contributions.
x := x0:      # Lm0_on_Lg : value of Lm where Gh is correct value
Ls0_on_Lg := Ls_on_Lg:
unassign('x'):

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################

# ----- Used method: iteration ------------------------------ #
Ghpmin := Gh/100:
Ghpmax := 100*Gh:
Ghp := Ghpmin:
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
  unassign('Da'):

  Rp := Ghp/c**2:
  L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
  L2 := 1 + Ls_on_Lg:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
```

```
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
                                          # Newton corrected law.
  ann := Gh / x**2 :         # Classical "Newton's acceleration".
  x := x0:

  if(evalf(arr) > evalf(ann)) then
    Ghpmax := Ghp:
    Ghp := (Ghpmin+Ghp)/2:
  else
    Ghpmin := Ghp:
    Ghp := (Ghpmax+Ghp)/2:
  end if:
end do:
Ghp_iter := Ghp:

#################################################################
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):
Rp := Ghp/c**2:
L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
L2 := 1 + Ls_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:     # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :      # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                                   # Our "Newton's acceleration".
an := Gh/x**2 :              # Classical "Newton's acceleration".
Da := ar - an:                     # Theoretical PIONEER anomaly.
x := AU * y:

#################################################################
#                             Mercury
#################################################################
print("Mercury relative error");
y := xme/AU:              # Distance from Mercury to the sun in AU.
an := Gh/xme**2:          # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                             Venus
#################################################################
print("Venus relative error");
y := xv/AU:              # Distance from Venus to the sun in AU.
an := Gh/xv**2:          # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                             Earth
#################################################################
print("Earth relative error");
```

```
y := xe/AU:              # Distance from Earth to the sun in AU.
an := Gh/xe**2:          # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.
t := 365*24:             # Nb of hours in a year.
Dt := t * (1/2) * Da/an:  # Nb of hours for anomaly for each year.
print("Earth error in hours (per year)");
evalf(Dt);

#################################################################
#                              Mars
#################################################################
print("Mars relative error");
y := xm/AU:              # Distance from Mars to the sun in AU.
an := Gh/xm**2:          # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
########### Calculation of Ls, the total solar system ###########
################## symmetric contribution #######################
#################################################################
Ls_on_Lg := Lk_on_Lg:      # No more main belt contribution after
                           # the location of Mars.
x := x0:      # Lm0_on_Lg : value of Lm where Gh is correct value
Ls0_on_Lg := Ls_on_Lg:
unassign('x'):

#################################################################
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):
Rp := Ghp/c**2:
L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
L2 := 1 + Ls_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:     # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :      # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                                   # Our "Newton's acceleration".
an := Gh/x**2 :              # Classical "Newton's acceleration".
Da := ar - an:                     # Theoretical PIONEER anomaly.
x := AU * y:

#################################################################
#                              Jupiter
#################################################################
print("Jupiter relative error");
y := xj/AU:              # Distance from Jupiter to the sun in AU.
an := Gh/xj**2:          # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
```

```
#                               Saturn
#################################################################
print("Saturn relative error");
y := xs/AU:             # Distance from Saturn to the sun in AU.
an := Gh/xs**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                               Uranus
#################################################################
print("Uranus relative error");
y := xu/AU:             # Distance from Uranus to the sun in AU.
an := Gh/xu**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                               Neptune
#################################################################
print("Neptune relative error");
y := xn/AU:             # Distance from Neptune to the sun in AU.
an := Gh/xn**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.
```

APPENDIX 13 PIONEER ANOMALY BEFORE SATURN: OUTER PLANETS

```
#################################################################
#                                                               #
#                CALCULATION OF PIONEER ANOMALY                 #
#                       BEFORE SATURN                           #
#             FOR THE FOUR OUTER PLANETS ONLY                   #
#                                                               #
# This MAPLE program is calculating the PIONEER anomaly,        #
# before the location of Saturn, taking into account:           #
#                      Kuiper belt                              #
#                      AND                                      #
#                      Main belt of asteroids,                  #
# supposing that Newton's law is perfect for a fitted location  #
# in order to minimize the maximum yielded relative error.      #
#                                                               #
#################################################################
# The equation used is Om = sqrt(G(M+m)/x**3), which is the     #
# classical equation for the rotational speed of a couple of    #
# planets. We get also t = 2 Pi/Om, where t is the orbital      #
# revolution period. Hence we get dt/t = ½ da/a, where a is the #
# gravitational acceleration, because of the two equations      #
# above, and because the real acceleration <<a>> is             #
# proportional to the real gravitational constant <<G>>:        #
# dt/t = dOm/Om = ½ dG/G = ½ da/a.                              #
# That’s why is written below : evalf((1/2)*Da/an);             #
#################################################################

#################################################################
########################## Initialisations ######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                       # Floating points precision.
unassign('x'): unassign('y'): unassign('z'): unassign('ar'):
unassign('an'): unassign('Da'):

# ----------------- Cosmological values ---------------------- #
# --- Constants ---
s := 1:                                                     # s.
day := 24*3600:                                        # in sec.
km := 10**3:                                                # m.
kpc := 3.08 * 10**19 :       # 1 kilo parsec expressed in meter.
pc := kpc/1000 :                # 1 parsec expressed in meter.
mpc := 10**3 * kpc:                                         # m.
c := 3 * 10**8:                            # light speed in m/s.
LY := 365 * 24 * 3600 * c:              # 1 light year in meter.
AU := 149597870 * km :                 # Astronomical Unit in m.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).
rg := 473 * 10**18:                    # Ray of the galaxy in m.
```

```
rg_last := 0.9 * 10**(-2) * rg:                  # Fitted value.
                 # Distance of last galaxy's contribution. in m.

# ----------------- Solar system values --------------------- #
# heliocentric gravitational constant inside the solar system:
Gh := (0.01720209895)**2 * AU**3 / day**2:
Mt := 5.97 * 10**24:                            # Earth mass in Kg.
M0 := 2 * 10**30 :                             # Sun mass in Kg.
R0 := M0 * G / c**2:           # Half Schwarschild ray for SUN.
# Semi-major axis of the planets:
xme := 57909176 * km:                     # Distance Mercure sun in m.
xv := 108208930 * km:                       # Distance Venus sun in m.
xe := 149597887.5 * km:                   # Distance Earth sun in m.
xm := 227936637 * km:                   # Distance Mars sun in m.
xj := 778412027 * km:                   # Distance Jupiter sun in m.
xs := 1433449370 * km:                   # Distance Saturn sun in m.
xu := 2876679082 * km:                   # Distance Uranus sun in m.
xn := 4498253000 * km:                   # Distance Neptune sun in m.
rho0 := 0.003 * M0 / (LY**3):  # Matter density near solar system.
tmme := 87.969257 * day:      # Mercury orbital period. JPL value.
tmv := 224.70079922 * day:     # Venus orbital period. JPL value.
tmm := 686.98 * day:      # Mars orbital period. JPL value.
tmt := 365.25636 * day:         # Earth orbital period. JPL value.
tmj := 4332.820 * day:          # Jupiter orbital period. JPL
value.
tms := 10755.698 * day:         # Saturn orbital period. JPL value.
tmu := 30687.153 * day:         # Uranus orbital period. JPL value.
tmn := 60190.029 * day:         # Neptune orbital period. JPL
value.
# ------------------ Kuiper belt values --------------------- #
xkmin := 30 * AU:         # Kuiper min distance from the sun. m.
xkmax := 48 * AU:            # Kuiper max distance from the sun.
xkmean := (xkmin+xkmax)/2:   # Kuiper mean distance from the sun.
thetak := Pi*20/180:                  # Kuiper thickness angle.
xkthi := xkmean * thetak:                   # Kuiper thickness.
Mk := 0.3 * Mt:                             # Kuiper belt mass
value.
Vk := Pi*(xkmax**2 - xkmin**2) * xkthi: # Kuiper belt volume m3.
rhok := evalf(Mk/Vk):        # Kuiper belt matter density kg/m3.
#xKuiper :=  xkmin + xkmax:   # If only Kuiper belt xKuiper=81 AU
xKuiper :=  114*AU:       # Fitted value for taking into account
                          # the scattered disk and other
# ------------------ Main belt values --------------------- #
xmmin := 2.2 * AU:         # Main min distance from the sun. m.
xmmax := 3.3 * AU:            # Main max distance from the sun.
xmmean := (xmmin+xmmax)/2:   # Main mean distance from the sun.
thetam := Pi*36/180:             # Main thickness angle in ra.
xmthi := xmmean * thetam:                   # Main thickness.
Mm := 3.3 * 10**21:                                     # kg.
Vm := Pi*(xmmax**2 - xmmin**2) * xmthi: # Main belt volume m3.
rhom := evalf(Mm/Vm):        # Main belt matter density kg/m3.
```

```
xMain :=  xmmin + xmmax:                          # Middle of it.

#################################################################
#################################################################
############## Calculation of the best value for x0 #############
#################################################################
#################################################################

# ------------------------- Ghp calculation ---------------------- #
#a y**3   +  b y²  +   c y   +   d    =   0
a := 3*sqrt(2/z)/c:
b := -1:
cc := 0:
d := Gh:
A := b/a:
B := cc/a:
C := d/a:
P := B - A**2/3:
Q := (A/27)*(2*A**2 - 9*B) + C:
R := Q**2 + (4/27)*P**3:
#R is negative.
u := (-Q/2 + (I/2)*sqrt(-R))**(1/3):
v := (-Q/2 - (I/2)*sqrt(-R))**(1/3):
j := -1/2 + I*sqrt(3)/2:
y := j**2*u + j*v - A/3:
             # Closest value from  Gh value, from the 3 real roots.
Ghp := evalf(y**2):

# ---------------- Roughly Pioneer Anomaly calculation -----------
#
AA := Ghp - Gh:
BB :=  (3*Ghp/c) * sqrt(2*Ghp):
Da2 :=  (AA - BB/sqrt(x)) / x**2:    # Approximated acceler
anomaly.
an := Gh/x**2:                                 # Newton's
acceleration.
RelDa := Da2/an:       # Relative approximated acceleration
anomaly.

# ---------------- Pioneer Anomaly maximum as a function of x0=z -
#
xl := (25/16) * BB**2/AA**2:
amax :=  (AA - BB/sqrt(xl)) / xl**2:
#plot( abs(amax), z=xme..xt );        # Allways decreasing function
.

# ----------------- Least square calculation -------------------- #
pow := 10:  # Finally power 4 is better than square method: the
best
           # should be power infinite (maximum norme)
           # but power greater than or equal to 5 take too much
```

```
           # time to compute.
S := 0:
x := xj:  S := evalf(S + abs(RelDa)**pow):
x := xs:  S := evalf(S + abs(RelDa)**pow):
x := xu:  S := evalf(S + abs(RelDa)**pow):
x := xn:  S := evalf(S + abs(RelDa)**pow):
unassign('x'):

# -------------- Minimum of Least square sum ---------------- #
print("Least square function:");
plot( 10**45 * abs(S), z=10.389*AU..10.39*AU );
          # After watching this,
xSmin := 10.39 * AU:       # Min value of S is for this x value.
x0 := xSmin: # Let's calculate now with it, exact anomaly values.

#################################################################
########### Calculation of Lk, Kuiper symmetric contribution ####
#################################################################
# ----------------- Calculation of fmax --------------------- #
Lk_on_Lg_max := (sqrt(xkmax**2 - xkmin**2)/rg_last) *
sqrt(rhok/rho0):
# ------------- ---- Calculation of Lk ----------------------- #
# Here is the final equation of the symmetric contribution coming
# from the Kuiper belt:
Lk_on_Lg := Lk_on_Lg_max * (2/Pi) * invfunc[tan]((2/Pi) *
                                                   xKuiper/x):
#################################################################
########### Calculation of Lm, main belt symm contribution ######
#################################################################
Lm_on_Lg := (sqrt(abs((xmmax-x)**2 - (xmmin-x)**2))/rg_last) *
                                              sqrt(rhom/rho0):
                                     # Direct value is better !
#################################################################
########### Calculation of Ls, the total solar system ###########
################## symmetric contribution #######################
#################################################################
Ls_on_Lg := Lk_on_Lg:   # Addition because coming from
                        # numerous luminous points contributions.
x := x0:      # Lm0_on_Lg : value of Lm where Gh is correct value
Ls0_on_Lg := Ls_on_Lg:
unassign('x'):

#################################################################
# Calculation of Ghp, the heliocentric gravitational constant   #
# valid outside the solar system.                               #
#################################################################

# ----- Used method: iteration ------------------------------- #
Ghpmin := Gh/100:
Ghpmax := 100*Gh:
Ghp := Ghpmin:
```

```
for i from 1 to 100 do
  unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
  unassign('Da'):

  Rp := Ghp/c**2:
  L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
  L2 := 1 + Ls_on_Lg:
  cos2:= 4*L1*L2/(L1+L2)**2:   # Square of relativistic operator.
  tg:= sqrt( 1/cos2 - 1) :    # Slope of space inside space-time.
  arr :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
                                          # Newton corrected law.
  ann := Gh / x**2 :          # Classical "Newton's acceleration".
  x := x0:

  if(evalf(arr) > evalf(ann)) then
    Ghpmax := Ghp:
    Ghp := (Ghpmin+Ghp)/2:
  else
    Ghpmin := Ghp:
    Ghp := (Ghpmax+Ghp)/2:
  end if:
end do:
Ghp_iter := Ghp:

#################################################################
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):
Rp := Ghp/c**2:
L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
L2 := 1 + Ls_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:     # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :      # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                                   # Our "Newton's acceleration".
an := Gh/x**2 :               # Classical "Newton's acceleration".
Da := ar - an:                     # Theoretical PIONEER anomaly.
x := AU * y:

#################################################################
########### Calculation of Ls, the total solar system ###########
################## symmetric contribution #######################
#################################################################
Ls_on_Lg := Lk_on_Lg:     # No more main belt contribution after
                          # the location of Mars.
x := x0:      # Lm0_on_Lg : value of Lm where Gh is correct value
Ls0_on_Lg := Ls_on_Lg:
unassign('x'):

#################################################################
```

```
#     Calculation the theoretical Pioneer anomaly function      #
#################################################################
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):
Rp := Ghp/c**2:
L1 := 1 + Ls_on_Lg + (1 + Ls0_on_Lg) * sqrt(8*Rp/x):
L2 := 1 + Ls_on_Lg:
cos2:= 4*L1*L2/(L1+L2)**2:     # Square of relativistic operator.
tg:= sqrt( 1/cos2 - 1) :      # Slope of space inside space-time.
ar :=  - c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2) :
                                   # Our "Newton's acceleration".
an := Gh/x**2 :              # Classical "Newton's acceleration".
Da := ar - an:                     # Theoretical PIONEER anomaly.
x := AU * y:

#################################################################
#                              Jupiter
#################################################################
print("Jupiter relative error");
y := xj/AU:             # Distance from Jupiter to the sun in AU.
an := Gh/xj**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                              Saturn
#################################################################
print("Saturn relative error");
y := xs/AU:             # Distance from Saturn to the sun in AU.
an := Gh/xs**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                              Uranus
#################################################################
print("Uranus relative error");
y := xu/AU:             # Distance from Uranus to the sun in AU.
an := Gh/xu**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.

#################################################################
#                              Neptune
#################################################################
print("Neptune relative error");
y := xn/AU:             # Distance from Neptune to the sun in AU.
an := Gh/xn**2:         # Newton acceleration.
evalf((1/2)*Da/an);  # Relat anomaly for orbital revolution time.
```

APPENDIX 14 JPL EPHEMERIDES ERROR

```
#################################################################
#                                                               #
#              CALCULATION OF JPL MAXIMUM CUMULATED             #
#       RELATIVE ERRORS IN RIGHT ASCENSION, WHEN USING THE      #
#            SIMPLIFIED ALGORITHM OF JPL EPHEMERIDES.           #
#                                                               #
# Those calculations are based on data coming from:             #
#                                                               #
#         http://ssd.jpl.nasa.gov/?planet_pos                   #
#                                                               #
#         http://ssd.jpl.nasa.gov/txt/aprx_pos_planets.pdf      #
#                                                               #
# The calculated values are always expressed in terms of        #
# relative errors for the periodic revolution times for         #
# the eigth planets:                                            #
#              RelatError = Lag/RevolutionTime                  #
# where RevolutionTime is the time for one orbit revolution,    #
# and Lag is the time for the yielded lag error during          #
# one orbit revolution.                                         #
#                                                               #
# For each planet, the value is a cumulative relative error     #
# between :                                                     #
#                                                               #
#   - Newton equation calculated with semi major axis of the    #
#     planets's orbits: omega = sqrt(G(M+m)/a3),                #
#                                                               #
#     and                                                       #
#                                                               #
#   - JPL full integrated algorithm ephemerides calculation.    #
#                                                               #
# Those values are, for each planet, the sum of the absolute    #
# values of:                                                    #
#                                                               #
#   1) relative error between:                                  #
#                                                               #
#        * Newton value,                                        #
#                                                               #
#        and                                                    #
#                                                               #
#        * JPL value obtained with the JPL rotation speed value #
#          available in the table 1 of the JPL simplified       #
#          algorythm. Those rotation speed values are available #
#          in table 1, column L, of:                            #
#          http://ssd.jpl.nasa.gov/txt/aprx_pos_planets.pdf     #
#                                                               #
#   2) relative error of this JPL simplified algorithm and full #
#     integrated JPL algorythm. Those errors are available in   #
#     http://ssd.jpl.nasa.gov/?planet_pos                       #
#                                                               #
```

```
#################################################################

#################################################################
######################### Initialisations #######################
#################################################################

# ---------------------------- MAPLE ------------------------- #
Digits := 50 :                        # Floating points precision.
unassign('x'): unassign('y'): unassign('ar'): unassign('an'):
unassign('Da'):

# ---------------- Cosmological values --------------------- #
day := 86400:                        # Number of sec in a day.
cty := 36525*day:                    # Number of sec in a century.
km := 1000:
AU := 149597870 * km :                   # Astronomical Unit in m.
Gh := (0.01720209895)**2 * AU**3 / day**2:       # CODATA value.
G := 6.6742867 * 10**(-11):   # Gravit constant m3 kg(-1) s(-2).

# ---------------- Solar system values --------------------- #

# --- Distances of the planets from the sun
xme := 57909176 * km:                 # Distance Mercure sun in m.
xv := 108208930 * km:                 # Distance Venus sun in m.
xe := 149597887.5 * km:               # Distance Earth sun in m.
xe := 149597887.5 * km:               # Distance Earth sun in m.
xm := 227936637 * km:                 # Distance Mars sun in m.
xj := 778412027 * km:                 # Distance Jupiter sun in m.
xs := 1433449370 * km:                # Distance Saturn sun in m.
xu := 2876679082 * km:                # Distance Uranus sun in m.
xn := 4498253000 * km:                # Distance Neptune sun in m.

# --- Planets's masses
Mme := 3.3022 * 10**23:                 # Mercury mass in kg.
Mv := 4.8685 * 10**24:                  # Venus mass in kg.
Me := 5.9736 * 10**24:                  # Earth mass in kg.
Mm := 641.85 * 10**21:                  # Mars mass in kg.
Mj := 1.8986 * 10**27:                  # Jupiter mass in kg.
Ms := 5.6846 * 10**26:                  # Saturn mass in kg.
Mu := 86.810 * 10**24:                  # Uranus mass in kg.
Mn := 102.43 * 10**24:                  # Neptune mass in kg.

# --- Planets's rotation speed as seen from JPL tables.
# Data source: http://ssd.jpl.nasa.gov/txt/aprx_pos_planets.pdf
Lme := 149472.67411175: # Mercury rotation speed. Degree/century.
Lv := 58517.81538729:   # Venus rotation speed. Degree/century.
Le := 35999.37244981:   # Earth rotation speed. Degree/century.
Lm := 19140.30268499:   # Mars rotation speed. Degree/century.
Lj := 3034.74612775:    # Jupiter rotation speed. Degree/century.
Ls := 1222.49362201:    # Saturn rotation speed. Degree/century.
Lu := 428.48202785:     # Uranus rotation speed. Degree/century.
```

```
Ln := 218.45945325:     # Neptune rotation speed. Degree/century.

# --- Relative errors for Rigth Ascension,
# with the simplified JPL calculation (from JPL HTML page).
# Data source: http://ssd.jpl.nasa.gov/?planet_pos
dme2 := 15:              # JPL approx err Mercury RA, arcsec.
dv2 := 20:               # JPL approx err Venus RA, arcsec.
de2 := 20:               # JPL approx err Earth RA, arcsec.
dm2 := 40:               # JPL approx err Mars RA, arcsec.
dj2 := 400:              # JPL approx err Jupiter RA, arcsec.
ds2 := 600:              # JPL approx err Saturn RA, arcsec.
du2 := 50:               # JPL approx err Uranus RA, arcsec.
dn2 := 10:               # JPL approx err Neptune RA, arcsec.

#################################################################
################### Calculation with JPL DATA ###################
#################################################################

print("Cumulated relative errors for orbital revolution time coming
from JPL simplified algorythm"):

# ------- Calculation procedure
PlanetMaxErrCalcul := proc( Planet, Dist, Mass, RotSpeed, JPLErr )
  local RotSpeed0, RotSpeed1, dRotSpeed1, dRotSpeed2, dRotSpeed,
        NbTurn, NbTurnYear, NbYear:
  description " # Cumulative relative error";
  print(Planet):                       # For the eigth planets.
  RotSpeed0 := sqrt((Gh+G*Mass)/Dist**3):     # Newton equation.
  RotSpeed1 := RotSpeed * (Pi/180) / cty:    # Types conversion.
  dRotSpeed1 := (RotSpeed1-RotSpeed0)/RotSpeed0:  # RelErr Newton.
  NbTurnYear := RotSpeed1*365*24*60*60/(2*Pi):
  NbYear := 2050-1800:
  NbTurn := NbYear * NbTurnYear:
  dRotSpeed2 := JPLErr/(380*60*60*NbTurn): #RelErr of this calcul.
  dRotSpeed := abs(dRotSpeed1) + abs(dRotSpeed2):
  evalf(dRotSpeed);   # Cumulative relative error between Newton
end proc:             # equation (with semi major axes) and
                      # JPL simplified calculation.

# ------- Planets Calculation
PlanetMaxErrCalcul( "Mercury", xme, Mme, Lme, dme2 );
PlanetMaxErrCalcul( "Venus",   xv,  Mv,  Lv,  dv2 );
PlanetMaxErrCalcul( "Earth",   xe,  Me,  Le,  de2 );
PlanetMaxErrCalcul( "Mars",    xm,  Mm,  Lm,  dm2 );
PlanetMaxErrCalcul( "Jupiter", xj,  Mj,  Lj,  dj2 );
PlanetMaxErrCalcul( "Saturn",  xs,  Ms,  Ls,  ds2 );
PlanetMaxErrCalcul( "Uranus",  xu,  Mu,  Lu,  du2 );
PlanetMaxErrCalcul( "Neptune", xn,  Mn,  Ln,  dn2 );
```

APPENDIX 15 MATTER DENSITY AND SPEED PROFILES

```
#------------------------------------------------------------
# MAPLE program
# This program is plotting the galaxy speed profiles
# for different global matter density profiles.
# It is used noticeably for the Tully-Ficher study.
#------------------------------------------------------------
# INIT
#------------------------------------------------------------
Digits:= 30: m:=1: M:= 7 * 10**36: G:= 6.6742867 *  10**(-11):
c:= 3 * 10**8: R:= M*G/c**2: kpc:= 3.08  * 10**19: r:= 1 * kpc:

#------------------------------------------------------------
# CALCULATION AND PLOTTING PROCEDURE
#------------------------------------------------------------
SpeedProfile := proc( betha )
  local rho, L, e, cos2, tg, F, v:
  unassign('x'):
  rho := 1/x**betha:
  L := r*sqrt(rho):
  e:= sqrt(8*R/x) / (1 + L):    # Famous "relat coefficient".
  cos2:= (1 + e) / ((1 + e/2)**2):  # Square of relat operat.
  tg:= sqrt(1/cos2 - 1):         # Slope of space inside st.
  F:= - m * c**2 * diff(tg,x) * tg * (1 - tg**2)**(-3/2):
  v:= sqrt(F*x/m):  # Centrifugal force and tangential speed.
  plot(v, x=0.01*kpc..15*kpc);  # Plotting from 1 to 15 kpc.
end proc:

#------------------------------------------------------------
# CALCULATIONS AND PLOTTING THE CURVES
#------------------------------------------------------------
SpeedProfile(2);                             # Betha = 2
SpeedProfile(2/3);                           # Betha = 2/3
```

APPENDIX 16 SPECIFIC MEASUREMENTS OF G

17.1. FIRST EXPERIMENTATION: SAME MEASUREMENT AT DIFFERENT LOCATIONS

This experimentation is conceived for validating or unvalidating the gravitational model of the three elements theory.
Its aim is to confirm an important prediction of this model.

A) The idea

The idea is to explain the mystery of the disparity of the measurements of G, the gravitational constant.
Indeed, this mystery is explained by an application of the equation of *G* obtained by this model.
This equation is predicting that *G* depends of the surrounding distribution of matter during the measurement, along the "attracting lines". An attracting line is a straight line in space which connects 2 space points. Each of them must be part of, respectively, each of the 2 attracted masses.
The equation of *G* is showing that the *G* value is depending of those attracting lines and the presence or absence of matter along those lines.
For example, in the non-realistic but limit case of pin pointed masses, it exists only one attracting line. If those objects are located exactly at the same altitude on earth, then this line is an horizontal line. Moreover, if those objects are located in the bottom of a valley, then this line is encountering an important quantity of matter. This matter is the matter of the surrounding mountains. On the contrary, if those objects are located on the top of a hill, then it is possible that this attracting line will not encounter any matter at all on earth. Faraway, of course, this line will probably encounter asteroids, planets, or celestial objects such as stars or gas.

B) The value to be measured

The model is predicting a difference in the *G* value between the two extreme cases above.
The value of this relative difference is around 0.001. This value is in accordance with available experimental data.
Indeed, the greatest difference between the measured *G* values is around 0.0065.

C) The experimentation

Hence a specific measurement is needed. This experimentation is the following.

1) Realisation of an apparatus, such as the attracting lines between the attracting objects are horizontal, as much as possible. An apparatus such as the one used by Cavendish in 1798 should be enough. It seems to be mandatory to use a laser beam in order to measure the angular deviation. This laser beam will reflect on a little mirror attached to the pendulus beam. One measurement consist of two distinct angular measurements, one with, and another without the presence of the attracting objects nearby.

2) Execution of two measurements, on distinct locations. For example, one measurement can be done in the bottom of a valley, and the other one at the top of a hill, or at the top of a mountain pass. In the case of a pass, it will be unsured that the attracting lines are encountering as less as possible the nearby mountains (attracting lines perpendicular to the top line of the pass).

3) Only the relative difference of the two measured values matters.
This implies that the apparatus is not required to be very precise for an absolute measurement of *G*.

D) Accuracy

A first experimentation might consists of only detecting this difference between the two measurements. If a difference is ever noticed, systematically, then it will be possible on a second step, to get an experimental estimation of the apparatus precision.
Concretely, a precision on *G* of around 0.002 should be enough. Hence, this should allows to detect a relative difference of 0.004 between the two measurements, which is less than the extreme value of 0.0065 above.

The apparatus is not required to be very precise for an absolute measurement of *G*.
Indeed, only a difference between the two values of *G* must be detected.

17.2. SECOND EXPERIMENTATION: TESTING VIOLATION OF ADDITIVITY AT EARTH SCALE

A) The idea

The idea here is not really to validate nor invalidate the model. The aim is to search a violation of gravitational forces additivity at earth scale.
The model retrieves the vector additivity (linearity) when combining parallel gravitational forces, if and only if the contributions are combining themselves using a "square root of the sum of the squares" rule.
But in the present document the model actually uses the additive operator for combining Kuiper contribution, with galactic-and-extragalactic contributions. It uses also the additive operator for combining stars contribution, with galactic-and-extragalactic contributions.
Moreover, for sideral gravity calculations, the additive operator has also been used. But here a "square root of the sum of the squares" would not allows to retrieve sideral gravity experimental data. Hence, the order of magnitude for the relative amplitude of *G* should be squared, and equal to the square of 0.054 %. This tends to shows that there is a violation of gravitational forces additivity at galaxy scales. This experimentation aims to detect now such a violation at earth scale.

B) Done before?

Of course the first action is checking wether or not this search has been done before.

C) The experimentation

A possible experimentation is the following.

1) Realisation of an apparatus, such as the attracting lines between the attracting objects are quite (globally) horizontal.

2) Execution of the measurement of *G* is this configuration: G_2

3) Pulling up the torsion fiber, raising up the balance (the attracted objects) over the attracting objects. Execution of the measurement of *G* in this configuration: G_3

4) Pushing down the torsion fiber, droping down the balance (the attracted objects) below the attracting objects. Execution of the measurement of *G* in this configuration: G_4
5) Possible results predicted by the model :

- $G_2 = G_3 = G_4$ no violation,
- $G_4 < G_2 < G_3$ if violation of additivity.

This means that any other order in those G values is predicted, by the model, has being impossible.

D) Accuracy

The apparatus is not required to be very precise for an absolute measurement of *G*.
Indeed, only an increasing law of *G* must be detected.

17.3. THIRD EXPERIMENTATION: TESTING VIOLATION OF ADDITIVITY AT "RELATIVISTIC DISTANCES"

A) The idea

The model predicts the following rule. There is a violation of gravitational forces additivity, at "relativistic distances". "Relativistic distances" means that the attracting objects are closed to each other at relativistic distances: distance x between the two objects closed to $R_1 = M_1 G / c^2$ or $R_2 = M_2 G / c^2$ distances.

B) Impossible?

This kind of experimentation seams to be impossible to realize at first glance.

C) The experimentation

To be worked!

17.4. CONCLUSION

It seams that these experimentations are really simple as compared to the one executed today for the measurement of *G* (using for example interferometry or sophisticated pendulus).

The first one is the only one yielding validation or invalidation of the model. Let's do it!

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FIGURES

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