Consequences of Relational Time

A. K. T. Assis
Institute of Physics 'Gleb Wataghin', University of Campinas-UNICAMP
13083-859 Campinas, SP, BRAZIL
e-mail: assis@ifi.unicamp.br, homepage: http://www.ifi.unicamp.br/~assis/

There are two competing formulations of time in physics. Newton defended in the *Principia* the utilization of absolute time which, according to him "flows equably without relation to anything external." Leibniz, on the other hand, was against this concept and proposed relative time to replace it: " As for my opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions." Leibniz ideas were accepted and developed by Ernst Mach in his book *The Science of Mechanics*. Mach proposed to replace Newton's absolute time by the angle of rotation of the planets relative to the frame of fixed stars.

In this work we consider the implementation of relational time and its consequences for physics. We concentrate our analysis in a single phenomenon, namely, the flattening of the Earth due to its diurnal rotation. We consider the figure of the Earth in Newtonian mechanics. We point out some philosophical problems with this classical formulation. We then present the flattening of the Earth from the point of view of Relational Mechanics, which is a mathematical implementation of Mach's principle utilizing Weber's law for gravitation.

1. Introduction

Isaac Newton (1642-1727) presented two concepts of time in his book *Mathematical Principles of Natural Philosophy*, also known by its first Latin name, *Principia*, published originally in 1687, [1]:

> "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year."

Thus, in his axioms or laws of motion, use only absolute time.

Leibniz (1646-1716) never accepted the utilization in physics of Newton's absolute time. He maintained that time depends on things, being the order of successive phenomena. There is a famous correspondence between Leibniz and Clarke (1675-1729), a disciple of Newton, which took place between 1715 and 1716. Leibniz said the following in the fourth paragraph of his third letter to Clarke, [2]:

> "As for my opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions."

Ernst Mach (1838-1916) also rejected the employment of absolute time in physics. His points of view as regards time were presented clearly on pp. 273, 287 and 295 of his book *The Science of Mechanics*, published originally in 1883. He proposed to replace the time which appears in Newton's laws of motion by the angle of rotation of the planets with respect to the fixed stars. For instance, on p. 295 of his book he wrote the following, [3]:

> "We measure time by the angle of rotation of the earth, but could measure it just as well by the angle of rotation of any other planet."

We agree with Leibniz and Mach as regards the time concept. However, in this work we will utilize the expression "relational time" instead of "relative time." There are two main reasons for this choice: (a) To avoid confusion with the time concept which appears in Einstein's special and general theories of relativity. (b) To comply with Relational Mechanics, a formulation which implements Mach's principle quantitatively, [4] and [5].

In this work we consider the implementation of relational time in physics. Our goal is to consider the consequences arising with this implementation.

2. Relational Time

We consider the material bodies as the primary entities of physics. The basic and primitive concepts are: (a) Gravitational mass, (b) electrical charge, (c) distance between material bodies, (e) force or interaction between material bodies. We do not define these basic concepts, since we wish to avoid vicious circles. These primitive concepts are necessary to define more complex concepts.

It is observed that the positions of bodies among themselves change, they are not fixed. The changes of things lead to an abstract concept, that of relational time. Relational time is an abstraction created by man at which we arrive by means of the changes of things. It is a measure of duration by means of the mutual motions of bodies among themselves.

Mach has a very clear statement on page 273 of his book, [3]:

"It is utterly beyond our power to *measure* the changes of things by *time*. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one *definite* measure, all being interconnected. A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform. But the question whether a motion is *in itself* uniform, is senseless. With just as little justice, also, may we speak of "absolute time"-*of a time independent of* change. This absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value; and no one is justified in saying that he knows aught about it. It is an idle metaphysical conception."

We agree that we can only compare a motion with another motion. Here we want to analyze the consequences of this point of view for physics as a whole. Our comparison will be with Newtonian physics, in which the motion of bodies is considered to take place in absolute time, which has no relation to anything external.

3. Motion in Newtonian Mechanics

In this Section we consider motion according to Newtonian mechanics.

Let us analyze the flattening of the Earth. The Earth has an average mass density, ρ_E, given by $\rho_E = 5.5\times10^3 kg/m^3$. Due to its diurnal rotation around the North-South direction, the Earth takes essentially the form of an ellipsoid of revolution. With a period of one day, the angular velocity of the Earth relative to an inertial frame of reference is given by $\omega_d = 7.3\times10^{-5} rad/s$. In Proposition XIX of Book III of the *Principia* Newton calculated the figure of the Earth. He concluded that its diameter at the equator was to its diameter from pole to pole as 230 to 229. That is, the diameter from East to West should be 0.4% larger than the diameter from North to South. Let us call d_E the Earth's diameter at the equator, d_P its diameter from pole to pole, and *f* this fractional flattening. According to Newtonian mechanics and utilizing the International System of Units (in which the universal gravitational constant *G* has the value $G = 6.7\times10^{-11} m^3/kg^1 s^2$), the fractional flattening is given by:

$$f \equiv \frac{d_E - d_P}{d_P} \approx \frac{15\omega_d^2}{16\pi G\rho_E} \approx 0.004 \qquad (1)$$

This prediction was confirmed later on by geodetic measurements.

The fractional flattening *f* is inversely proportional to the Earth's average mass density. This flattening is also proportional to the square of the dynamical angular velocity of the Earth relative to absolute space, measured by absolute time. That is, it is proportional to ω_d^2.

4. Questionable Aspects of Absolute Motion

There are several aspects of Newtonian mechanics which are questionable. We analyze each one of them here, concentrating our analysis in the figure of the Earth, as this is a concrete example.

(a) The flattening *f* is inversely proportional to ρ_E. If we could increase or decrease this mass density, the flattening would decrease or increase, respectively. But increase or decrease ρ_E in comparison to what? If there is no other mass density to compare to, this statement makes no sense. From a Machian perspective, the flattening should be proportional to the ratio ρ_0/ρ_E, where ρ_0 is the mass density of some other body for comparison. This "other body" should not be arbitrary. That is, it should affect causally the flattening of the Earth.

(b) According to Newton, this fractional change depends upon the angular rotation of the Earth relative to absolute space (or relative to an inertial frame of reference, as stated in modern textbooks). In principle the distant universe composed of stars and galaxies could disappear without affecting *f*. This consequence is not intuitive. After all, if the Earth were alone in the universe, it would not make sense to speak of its rotation. According to a Machian perspective, the flattening of the Earth should disappear if the distant stars and galaxies also disappeared. That is, somehow *f* should be directly proportional to the mean gravitational matter density of the universe. This should be the meaning of ρ_0 in the previous item.

This aspect has been clearly seen by Clarke in his fifth reply to Leibniz, [2]:

> "It is affirmed [by Leibniz], that motion necessarily implies a relative change of situation in one body, with regard to other bodies: and yet no way is shown to avoid this absurd consequence, that then the mobility of one body depends on the existence of other bodies; and that any single body existing alone, would be incapable of motion; or that the parts of a circulating body, (suppose the sun,) would lose the *vis centrifuga* arising from their circular motion, if all the extrinsic matter around them were annihilated."

(c) If the Earth could rotate faster or slower, its flattening would increase or decrease, respectively. But rotate faster or slower relative to what? How can we know that the Earth is rotating faster or slower, if there is no other motion to compare to?

(d) Newton believed that it was possible to distinguish absolute motions from relative ones. In the Scholium in the beginning of Book I of the *Principia* he made a very interesting discussion related to two globes connected by a cord. We quote his words here, but replacing the globes by the Earth, and replacing the tension of the cord by the flattening of the Earth. Due to these replacements of words, the next quotation goes in italic, instead of utilizing quotation marks:

> *It is indeed a matter of great difficulty to discover, and to effectually to distinguish, the true motions of particular bodies from the apparent. […] Yet the thing is not altogether desperate […] As the Earth revolves about its center of gravity, we can, from its flattening, discover its endeavor to recede from the axis of its motion, and from thence we can compute the quantity of its circular motion. And thus we can find both the quantity and determination of this circular motion, even in an immense vacuum, where there is nothing external or sensible with which the Earth could be compared. But now, if in that space some remote bodies were placed that kept always a given position to one another, as the fixed stars do in our regions, we could not indeed determine from the relative translation of the Earth among those bodies, whether the motion did belong to the Earth or to the bodies. But if we observed the figure of the Earth, and found that its flattening was that very flattening which the motion of the Earth required, we might conclude the motion to be in the Earth, and the bodies to be at rest.*

That is, according to Newton there two situations which are kinematically equivalent: (I) The fixed stars at rest and the Earth spinning once a day; and (II) the Earth at rest and the set of fixed stars spinning around it once a day. Considering only this relative rotation between the Earth and the fixed stars, it is not possible to know which body is really in motion.

However, Newton believed these two situations could be distinguished dynamically. In situation (I) the Earth would be flattened at the poles, while in situation (II) the figure of the Earth would be spherical. In Newtonian mechanics the Earth's flattening is a function of its absolute motion relative to absolute space, as measured by absolute time.

This is a questionable interpretation of this flattening. From a Machian perspective, there is only the rotation of the Earth relative to the frame of distant stars and galaxies. The flattening of the Earth should be directly proportional to this relative rotation. Whenever the rotation of the Earth relative to the frame of distant galaxies is the same, the same flattening should arise, no matter which bodies were in motion. Consider for the moment the existence of an arbitrary frame of reference R. When the frame of distant galaxies is at rest in this frame and the Earth rotates once a day in this frame of reference, its 0.004 flattening appears, as in situation (I) above. From a Machian perspective, the same flattening should also arise if the Earth remained stationary in R, while the frame of distant galaxies rotated once a day around the North-South axis of the Earth, as in situation (II) above.

This has been clearly seen by Mach. When discussing Newton's bucket experiment he said the following:

[3, p. 279]: "Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces." The analogous statement applied to the Earth's rotation and taking into account the distant galaxies, not known by Mach, might be as follows: *Try to fix the Earth and rotate the heaven of distant galaxies and then prove the absence of the Earth's flattening.*

Although he could not create a working mechanics implementing this idea, Mach believed this was possible. On page 284 of his book he said, [3]: "The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise."

5. Motion in Relational Mechanics

We now consider the same motion according to Relational Mechanics, which is based upon Weber's law for gravitation, [6]. The equation of motion of any test particle is due to its interaction with the distant galaxies. We need to integrate Weber's law over the whole universe. The size of the known universe is given by Hubble's radius $R_0 = c/H_0 \approx 10^{26}\,m$, where $c = 3\times10^8\,m$ is the value of light velocity in vacuum and $H_0 \approx 3\times10^{-18}\,s^{-1}$ is Hubble's constant. If the universe is infinite, R_0 may represent a characteristic length of gravitational interactions. For instance, it might represent the effective length of gravitational interactions due to an exponential decay in the gravitational force.

The flattening of the Earth is given by, [4] and [5]:

$$f \equiv \frac{d_E - d_P}{d_P} \approx \frac{5\alpha}{8}\frac{\omega_{EU}^2}{H_0^2}\frac{\rho_0}{\rho_E} \approx 0.004 \qquad (1)$$

In this Equation α is a dimensionless number. Its value is 6 if we work with a finite universe and integrate Weber's law for gravitation until Hubble's radius. If we work with Weber's law and an exponential decay in gravitation, we can integrate up to infinity. In this last situation we get $\alpha = 12$.

The flattening is proportional to $\rho_0 \approx 3\times10^{-27}\,kg/m^3$, the average gravitational mass density of the distant universe. The values of R_0, ρ_0 and H_0 are not yet known with great precision. But the order of magnitude of these quantities is compatible with the observed flattening of 0.004. We can also utilize this observed value of *f*, together with the known values of ρ_E and ω_{EU}^2, to derive the value of $5\alpha\rho_0/8H_0^2$.

The flattening is also inversely proportional to the average gravitational mass density of the Earth, $\rho_E = 5.5\times10^3\,kg/m^3$. The important aspect to emphasize here is that only the ratio ρ_0/ρ_E is relevant for Relational Mechanics. We can decrease the flattening by increasing ρ_E (supposing a planet made of liquid mercury, for instance), or by hypothetically decreasing ρ_0. In principle, if we could annihilate the extrinsic matter around the Earth, making $\rho_0 \to 0$, the Earth's flattening would also disappear, $f \to 0$. That is, Relational Mechanics implements mathematically the consequence which Clarke considered absurd in his correspondence with Leibniz.

The flattening is proportional to ω_{EU}^2, that is, to the square of ω_{EU}. This symbol represents the angular velocity of the Earth relative to the distant universe. This means that only the relative rotation between the Earth and the frame of distant galaxies is relevant in Relational Mechanics. This consequence is completely Machian. There will be the same flattening of the Earth no matter if the Earth rotates relative to an arbitrary frame of reference while the distant universe remains stationary in this frame, as in situation (I) above, or if the distant universe rotates in the opposite direction relative to this frame of reference while the Earth remains stationary in this frame, as in situation (II) above. That is, provided the quantitative relative rotation between the Earth and the distant universe is the same in both cases, the same flattening of the Earth arises. If there is one relative turn per day, so that $\omega_{EU} = 7.3\times10^{-5}\,rad/s$, then $f = 0.004$ in situations (I) and (II). The flattening of the Earth cannot be considered anymore as a proof of the real or absolute rotation of the Earth, as Newton thought.

Relational Mechanics implements mathematically Mach's idea according to which the Ptolemaic and Copernican modes of view are equivalent not only kinematically, but also dynamically. That is, the same flattening of the Earth arises not only in the Copernican world view in which the distant universe is at rest and the Earth rotates once a day, but also in the Ptolemaic world view in which the Earth is stationary and the distant universe rotates around it once a day.

6. An Open Question in Relational Mechanics

Here we want to discuss something which has not yet been completely clarified by Relational Mechanics.

If the Earth might turn in relation to the distant universe 3 times faster than usual, its flattening would be 9 times larger, namely, $f \approx 0.012$. The reason is that f is proportional to ω_{EU}^2. But when we say that the Earth is rotating 3 times faster than normal, we need to compare it with something else. The comparison should not be with our wrist clock. In order to understand this conclusion, we can consider an astronaut in a spaceship recording a video. The video should include the rotation of the Earth and other motions in the universe. Let us suppose that the normal rate of this video is 30 frames per second (fps).

By watching this video in fast motion, with 90 fps, we would find all velocities increased to three times their normal values. However, the flattening would remain the same in the fast motion video, namely, $f \approx 0.004$. The reason is that not only the Earth would appear to us spinning 3 times faster than usual, but the same would happen to all other velocities recorded in this video (sound velocity, a projectile motion, the velocity of a satellite, the angular velocity of galaxies etc.)

The flattening would also remain the same in a slow motion video. Even if the astronaut takes a picture of the Earth, so that the Earth appears stationary, its flattening will remain.

The same conclusion is reached by any inhabitant of the Earth. The ground below our foot does not move relative to us during a whole day, so that the Earth appears as stationary in relation to us. Despite this fact, it remains flattened at the poles.

The conclusion is that the amount of flattening is not a function of our clock or measuring time device.

From a Machian point of view, the amount of flattening should be proportional to a ratio of two motions. (a) The square of the angular velocity of the Earth in relation to the frame of distant galaxies. (b) The square of another velocity related to other motions in the universe. The open question is that up to now we don't know what are these other motions in the universe which might be connected with the flattening of the Earth.

The flattening of the Earth in Relational Mechanics is proportional to the square of the angular velocity of the Earth relative to the universal frame of reference. The flattening is also inversely proportional to the square of Hubble's constant. This suggests that Hubble's constant should be connected to other motions in the universe. Somehow Hubble's constant must be like an average frequency of oscillation and/or rotation of the matter in the universe; or the average angular velocity of the galaxies in the universe; or the average angular velocity of microscopic particles inside the Earth or spread around the universe; or it might be related with light velocity; or …

We postulate that if everything did move faster or slower, increasing or decreasing its pace by the same rate, no effects would arise in the behavior of bodies. For instance, the flattening of the Earth should remain the same if it could rotate three times faster than usual, provided all other motions did also increase three times their pace.

On the other hand, it is known that the centrifugal effects have larger magnitudes when the spinning body rotates faster relative to other motions in the universe. A Machian perspective suggests the opposite effect. That is, if the Earth could keep its pace of rotation, while all other motions in the universe did move slower, the Earth's flattening should also increase. In an hypothetical situation in which we could stop all other motions in the universe (the rotation of galaxies, the rotation of electrons and so on), while the Earth were still spinning in relation to the set of distant galaxies, its flattening should tend to infinity. That is, the Earth would explode in this hypothetical situation.

But what other motions are specifically connected with the flattening of the Earth? The spin of the electrons? The average rotation of the galaxies relative to the frame of distant galaxies? The velocity of photons? The vibrations of atoms?

This is an open question which requires further investigation.

References

[1] I. Newton. **Mathematical Principles of Natural Philosophy**. University of California Press, Berkeley, 1934. Cajori edition.

[2] H. G. Alexander (ed.). **The Leibniz-Clarke Correspondence**. Manchester University Press, Manchester, 1984.

[3] E. Mach. **The Science of Mechanics – A Critical and Historical Account of Its Development**. Open Court, La Salle, 1960.

[4] A. K. T. Assis, **Relational Mechanics**. Apeiron, Montreal, 1999. ISBN: 0968368921. http://www.ifi.unicamp.br/~assis/.

[5] A. K. T. Assis, "The principle of physical proportions", *Annales de la Fondation Louis de Broglie* **29**: 149-171 (2004).

[6] A. K. T. Assis, **Weber's Electrodynamics** (Kluwer Academic Publishers, Dordrecht, 1994). ISBN: 0792331370.